OCR
Oxford Cambridge and RSA

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods
Question Paper

Tuesday 22 May 2018 – Afternoon
Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

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Answer **all** the questions.

1 The coordinates of point A are (1.05, 2.71) and the coordinates of point B are (1.07, 3.08). In each case the x and y values have been rounded to two decimal places. The gradient of the line AB is m.

Find the range of values of m, commenting on your answer. **[4]**

2 The table in Fig. 2 shows some values of $\log_3 x$ which are correct to 6 decimal places.

x	2	2.25	2.5	2.75	3
$\log_3 x$	0.630930	0.738140	0.834044	0.920799	1

Fig. 2

(i) Use Simpson's rule to calculate an approximation to $\int_2^3 \log_3 x \, dx$, giving your answer correct to 6 decimal places. **[3]**

(ii) Explain why it is unlikely that the answer to part **(i)** is in fact accurate to 6 decimal places. **[1]**

3 The spreadsheet output in Fig. 3 shows Table 1 and Table 2.

	A	B	C	D	E	F	G
1	x	2	2.1	2.01	2.001	2.0001	2.00001
2	f(x)	0.64	0.625876972	0.638573473	0.639857204	0.639985719	0.639998572
3				Table 1			
4							
5	h	0.1	0.01	0.001	0.0001	0.00001	
6	dy/dx	–0.1412303	–0.142652654	–1.14279594	–0.142810279	–0.14281171	
7	difference	–0.0014224	–0.000143287	–1.4339E-05	–1.43402E-06		
8	ratio	0.1007378	0.100073651	0.10000709			
9				Table 2			

Fig. 3

Table 1 shows values of a function, $y = \mathrm{f}(x)$, for different values of x. Table 2 shows approximations to $\frac{dy}{dx}$ at $x = 2$, along with the differences between successive approximations and the ratio of these differences.

The formula in cell C6 is

= (D2 – $B2)/C5 .

Equivalent formulae are in cells D6, E6 and F6.

(i) Explain why the symbol \$ is used. **[1]**

(ii) State what method is being used to approximate $\frac{dy}{dx}$ at $x = 2$. **[1]**

(iii) Use extrapolation to find the value of $\frac{dy}{dx}$ at $x = 2$ as accurately as you can, justifying your answer. **[4]**

(iv) Calculate an approximation to the value of f(2.05). **[2]**

4 The equation $e^x - x^2 - 2x = 0$ has a root α, where $0 < \alpha < 1$ and a root β, where $2 < \beta < 3$.

(i) Show how to obtain the iterative formula

$$x_{r+1} = \frac{e^{x_r} - x_r^2}{2}.$$

[2]

Fig. 4 shows part of the curve $y = \frac{e^x - x^2}{2}$ and part of the straight line $y = x$.

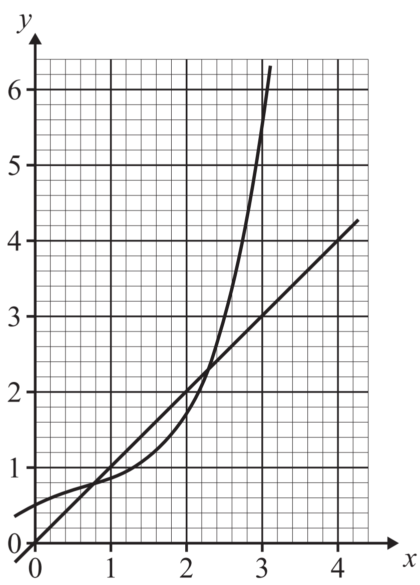

Fig. 4

(ii) Explain why the iteration in part **(i)** will

- successfully find α if a suitable starting value is chosen,
- fail to find β however close the starting value is to the root. **[2]**

(iii) Use the iteration in part **(i)** to find α correct to 6 significant figures. **[2]**

The relaxed iteration

$$x_{r+1} = (1-\lambda)x_r + \lambda\left(\frac{e^{x_r} - x_r^2}{2}\right)$$

is used to find β.

(iv) Use $x_0 = 2$ with $\lambda = -0.8$ to find the value of β correct to 6 decimal places. **[3]**

(v) Determine what happens when the relaxed iteration is used with $\lambda = 0.8$ and a starting value of 2. **[2]**

5 The value of shares in Sunfield plc was £2.21 per unit on the first day it came under new management. One week later the value of one unit of shares was £4.00 and after three weeks the value of one unit was £7.34. The information is summarised in Fig. 5.1, where x is the value in pounds per unit, and t is the time in weeks since coming under new management.

t	0	1	3
x	2.21	4.00	7.34

Fig. 5.1

The Public Relations Director uses this information to propose a quadratic model connecting the value of a unit of shares, in pounds, and the time, in weeks, since coming under new management.

(i) Use Lagrange's interpolation formula to find a quadratic model for these data, giving your answer in the form $x = at^2 + bt + c$, where, a, b and c are constants to be found. **[4]**

(ii) A shareholder notes that the value of a unit was £5.73 after two weeks. Determine whether the model is consistent with this information. **[2]**

(iii) According to the model, how will the value of a unit change in the long run? **[1]**

After seven weeks the management analysed the data and presented the findings in a difference table. The results are shown in Fig. 5.2.

t	x	Δ	Δ^2	Δ^3
0	2.21			
		1.79		
1	4.00		–0.06	
		1.73		–0.06
2	5.73		–0.12	
		1.61		–0.07
3	7.34		–0.19	
		1.42		–0.08
4	8.76		–0.27	
		1.15		–0.07
5	9.91		–0.34	
		0.81		–0.06
6	10.72		–0.40	
		0.41		
7	11.13			

Fig. 5.2

A shareholder proposes a cubic model for the data.

(iv) Explain whether the information in the difference table supports this proposal. **[1]**

(v) Use Newton's forward difference interpolation formula to show that the shareholder's model is

$$x = -0.01t^3 + 1.8t + 2.21.$$

[4]

(vi) Identify a limitation of the shareholder's model. **[1]**

6 **(i)** Show that the equation $0.1x^3 - 2x + 3 = 0$ has a root α, where $3 < \alpha < 4$. **[1]**

The method of false position is used to find α.
The spreadsheet output in Fig. 6.1 shows some of the results.

	A	B	C	D	E	F	G
1	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$		
2	0	3	–0.3	4	1.4	3.176471	–0.14789
3	1	3.176471	–0.14789	4	1.4	3.255155	–0.06114
4	2	3.255155	–0.06114	4	1.4	3.286321	–0.02345
5	3	3.286321	–0.02345	4	1.4	3.298076	–0.00873
6	4	3.298076	–0.00873	4	1.4	3.302428	–0.00322
7	5	3.302428	–0.00322	4	1.4	3.304028	–0.00118
8	6	3.304028	–0.00118	4	1.4	3.304614	–0.00043
9	7	3.304614	–0.00043	4	1.4	3.304829	–0.00016
10	8	3.304829	–0.00016	4	1.4	3.304908	–5.8E-05
11	9	3.304908	–5.8E-05	4	1.4	3.304937	–2.1E-05

Fig. 6.1

The spreadsheet formula in cell F2 is

= (B2*E2–D2*C2)/(E2–C2) .

(ii) What is being calculated in this cell? **[1]**

(iii) Write down a suitable spreadsheet formula for cell G2. **[1]**

(iv) Explain why the values in column G are necessary. **[1]**

The spreadsheet formula in cell B3 is

= IF(G2<0,F2,B2) .

(v) Explain the purpose of this formula. **[1]**

(vi) Write down the value of α to an accuracy that appears justified. **[1]**

Further analysis is carried out. This is shown in the spreadsheet output in Fig. 6.2.

	J	K	L	M
1	x_r	$x_{r+1} - x_r$		
2	3			
3	3.176471	0.176471		
4	3.255155	0.078684	0.445876	2.526632
5	3.286321	0.031166	0.396089	5.033923
6	3.298076	0.011756	0.377196	12.10282
7	3.302428	0.004352	0.370184	31.48985
8	3.304028	0.0016	0.367603	84.47262
9	3.304614	0.000587	0.366657	229.2012
10	3.304829	0.000215	0.36631	624.5195
11	3.304908	7.87E-05	0.366183	1704.301

Fig. 6.2

The spreadsheet formula in cell L4 is

= K4/K3

and the spreadsheet formula in cell M4 is

= K4/(K3^2) .

Equivalent formulae are in cells L5 to L11 and M5 to M11.

(vii) Explain what the values in columns L and M tell you about the order of convergence of the sequence of approximations to α found using the method of false position in this case. **[2]**

7 Fig. 7.1 shows part of the curve $y = 2^x - x^2 + 3$.

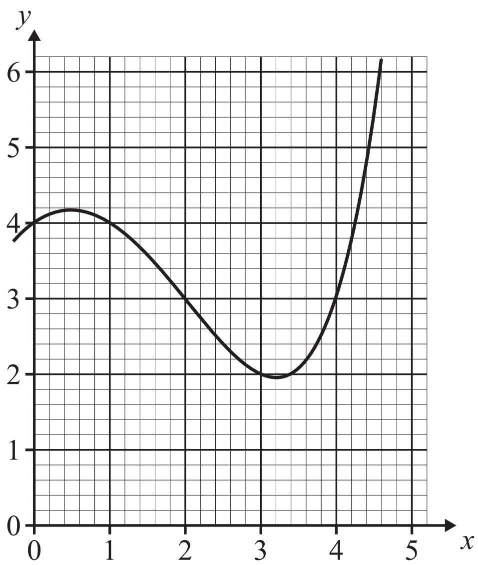

Fig. 7.1

(i) State, with a reason, whether using the trapezium rule to approximate $\int_3^4 (2^x - x^2 + 3)\,dx$ will give an under-estimate or an over-estimate. **[1]**

The spreadsheet output in Fig. 7.2 was generated in order to evaluate $\int_3^4 (2^x - x^2 + 3)\,dx$.

The values in columns N and O are estimates of the integral using the midpoint rule and the trapezium rule respectively.

	M	N	O
1	*n*	*Mn*	*Tn*
2	1	2.063708499	2.5
3	2	2.171499782	2.281854249
4	4	2.199007398	2.226677016
5	8	2.205919726	2.212842207
6	16	2.207650029	2.209380966
7	32	2.208082743	2.208515498

Fig. 7.2

(ii) Using only values from column N and/or column O, give a suitable spreadsheet formula for cell O3. **[2]**

(iii) Use the entries in cells N7 and O7 to write down the value of $\int_3^4 (2^x - x^2 + 3)\,dx$ as accurately as you can, explaining your reasoning. **[2]**

Further analysis shows that the ratio of differences of the midpoint rule approximations and the trapezium rule approximations converge rapidly to the value expected from theory.

(iv) Explain whether it is reasonable to assume that the ratio of differences of a sequence of approximations generated using Simpson's rule would also converge to the value predicted by theory. **[1]**

(v) (*A*) Use the information in Fig. 7.2 to obtain two Simpson's Rule estimates of $\int_3^4 (2^x - x^2 + 3)\,dx$, S_{32} and S_{64}. **[3]**

(*B*) Use extrapolation to find the value of $\int_3^4 (2^x - x^2 + 3)\,dx$ as accurately as you can, justifying the precision quoted. **[3]**

END OF QUESTION PAPER