Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Further Mathematics (9FM0)
Paper 3B: Further Statistics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

AL FM Stats 1 1906 Mark Scheme Final

Qu	Scheme	Marks	AO
1(a)	[Let X = no. of prizes Andreia wins] $X \sim B(40, 0.02)$	M1	3.3
	[Require $P(X \geqslant 3) = 1 - P(X \leqslant 2)$] = 0.04567… awrt **0.0457**	A1	1.1b
		(2)	
(b)	[Let Y = no. of the bar when Barney wins] $Y \sim \text{NegBin}(3, 0.02)$	M1	3.3
	[$P(Y = 40)$ =] $\binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
	= 0.0028071… awrt **0.00281**	A1	1.1b
		(3)	
(c)	$E(Y) = \frac{3}{0.02}$ = **150**	B1	1.1b
		(1)	
		(6 marks)	

	Notes
(a)	M1 for selecting a suitable model i.e. $B(40, p)$ where p is any probability Written or used, may be implied by a correct ans or 0.037429… from $P(X = 3)$ A1 for awrt 0.0457 (correct answer only 2/2)
(b)	1st M1 for selecting a suitable model (NB(3, 0.02)) May be implied by a correct expression 2nd M1 for use of model to form a correct expression
SC	$p \neq 0.02$ Allow prob of the form $\binom{39}{2} p^3(1-p)^{37}$ where $0 < p < 1$ scores M0M1
	A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]
(c)	B1 for 150

Qu	Scheme	Marks	AO
2(a)	{Let C = no of calls in a 20 min period} $C \sim \text{Po}(\ldots)$	M1	3.3
	80 calls per 4-hour period gives $\frac{20}{3}$ per 20 mins i.e. $C \sim \text{Po}(\frac{20}{3})$	M1	3.4
	$[\text{P}(C > 4)] = 1 - \text{P}(C \leqslant 4)$		
	$= 0.79437\ldots$ awrt **<u>0.794</u>**	A1	1.1b
		(3)	
(b)	{X = no. of 5 min periods with no calls } $X \sim \text{B}(4, \text{e}^{-\frac{5}{3}})$	M1	3.3
	$\text{P}(X = 3) = 0.02186125\ldots$ awrt **<u>0.0219</u>**	A1	1.1b
		(2)	
(c)	P(exactly one call) $\text{e}^{-\frac{5}{3}} \times \frac{5}{3}$ or $\text{e}^{-5} \times 5$	M1	2.1
	P(exactly one call in each break) = $\left(\text{e}^{-\frac{5}{3}} \times \frac{5}{3}\right) \times \left(\text{e}^{-5} \times 5\right)$	M1	1.1b
	$= 0.0106052\ldots$ awrt **<u>0.0106</u>**	A1	1.1b
		(3)	
		(8 marks)	

	Notes
(a)	1st M1 for selecting a Poisson model – written or used. May be implied by 2nd M1 or a correct Answer.
	2nd M1 for the correct Poisson $\text{Po}(\frac{20}{3})$ or Po(6.67) or better seen <u>and</u> writing or using $1 - \text{P}(C \leqslant 4)$
	A1 for awrt 0.794 (correct ans with no incorrect working scores 3/3)
(b)	M1 for selecting a correct model B(4, 0.189) or better (calc: 0.188875…)
	A1 for using the model to get awrt 0.0219 (correct ans with no incorrect working scores 2/2)
(c)	1st M1 for <u>a</u> correct prob of 1 call (expressions in e or values) (allow 0.31479… or awrt 0.315 <u>or</u> 0.033689… or awrt 0.0337)
	2nd M1 for a correct probability statement or expression. E.g. $\text{P}(S = 1 \mid S \sim \text{Po}(\frac{5}{3})) \times \text{P}(T = 1 \mid T \sim \text{Po}(5))$
SC	e.g. $F \sim \text{Po}(\lambda)$ used in (b) to find $\text{P}(F = 0)$ Then if we see $Y \sim \text{Po}(3\lambda)$ and statement $\text{P}(F = 1) \times \text{P}(Y = 1)$ award M0M1
	A1 for awrt 0.0106 (correct ans with no incorrect working scores 3/3)

Qu	Scheme	Marks	AO
3.	{ Let X = the number when the spinner is spun} $\mu = \underline{\mathbf{3}}$	B1	1.1b
	$\left[\mathrm{E}(X^2)=\right]0.3+4\times0.1+9\times0.2+16\times0.1\ +25\times0.3$ [= 11.6 or $\frac{58}{5}$]	M1	1.1b
	$\sigma^2\left[=11.6-3^2=\right]$ $\underline{\mathbf{2.6}}$	A1	1.1b
	$\bar{X} \approx\sim \mathrm{N}\left(\text{"3"},\sqrt{\frac{\text{"2.6"}}{80}}^{\,2}\right)$	M1	2.1
		A1ft	1.1b
	$\mathrm{P}(\bar{X}>3.25)=[\mathrm{P}(Z>1.3867...)=]0.0827589...(\text{calc})$ awrt $\underline{\mathbf{0.0828}}$	A1	3.4
		(6 marks)	

Notes

B1 for stating or using mean = 3

1st M1 for using the given model to attempt $\mathrm{E}(X^2)$ with at least 3 correct products seen

1st A1 for Var(X) = 2.6 or $\sigma=\sqrt{2.6}=1.6124...$ (awrt 1.61)

ALT **Use of pgf** (B1 when mean = 3 seen) (M1 when correct $\mathrm{G}''(t)$ seen with attempt at $\mathrm{G}''(1)$)

$$\mathrm{G}(t)=0.3t+0.1t^2+0.2t^3+0.1t^4+0.3t^5$$

$$\mathrm{G}'(t)=0.3+0.2t\ +0.6t^2+0.4t^3+1.5t^4$$

$$\mathrm{G}''(t)=0.2+1.2t+1.2t^2+6t^3 \text{ leading to } \mathrm{G}''(1)=8.6$$

2nd M1 for use of CLT – must use $\bar{X}$ and normal <u>or</u> sight of $\mathrm{N}\left(\text{"3"},\sqrt{\frac{\text{"2.6"}}{80}}^{\,2}\right)$ with any letter

2nd A1ft for a correct mean and variance, ft their 3 and their 2.6

This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to P(Z > 1.39) Must see correct use of Z

NB $\frac{2.6}{80}=0.0325$ and $\sqrt{\frac{2.6}{80}}=0.18027...$ so allow e.g. N(3, awrt $(0.180)^2$)

3rd A1 for using the normal model to find probability awrt 0.0828

ALT **Use of** $\sum X$ (If see clear attempt at P(ΣX > 260) condone P(ΣX > 260.5) then:

2nd M1 for ΣX ~ N(…) <u>or</u> any letter ~N(“240”, $\sqrt{\text{"2.6"}\times80}^{\,2}$)

2nd A1ft for mean = “3”$\times$80 = 240 <u>and</u> variance = “2.6”$\times$80 = 208

May see P(ΣX > 260.5) = 0.077597… but it will only score 2nd M1 2nd A1ft and **3rd A0**

Qu	Scheme	Marks	AO
4(a)	[T = no. of oak trees in a square] T ~ Binomial	M1	3.3
	$T \sim \mathrm{B}(6, p)$	A1	1.1b
		(2)	
(b)	Expected frequency for 6 is less than 5 so pool: new E_i = 13.08	M1	2.1

$\frac{(O_i-E_i)^2}{E_i}$	0.051	2.51	0.0654	3.84	1.85
$\frac{O_i^2}{E_i}$	4.521	29.617	21.805	7.599	24.771

Qu	Scheme	Marks	AO
	$\sum \frac{(O_i-E_i)^2}{E_i} = 8.313$	M1,A1	1.1b x2
	p needed estimating ($\hat{p} = 0.55$) so $\nu = 5 - 2 = 3$; cv 7.815	B1,B1ft	1.1b x2
	Significant result, so Liam's model is not suitable	M1,A1	1.1b2.2b
		(7)	
(c)	[R = no. of oak trees in a square for Simone's model] $R \sim \mathrm{Po}(3.3)$	M1	3.3
	Correct expression for s or t using Poisson	M1	3.4
	s = **17.67** and t = **9.62**	A1,A1	1.1b x2
		(4)	
(d)	H_0: Poisson is a good fit (for no. of oak trees per square) H_1: Poisson is not a good fit (for no. of oak trees per square)	B1	2.5
		(1)	
(e)	No pooling needed so degrees of freedom is 6 – 2 = 4	B1	1.1b
	Critical value is 9.488 (accept 9.49)	B1	1.1a
	Not significant so Poisson (or Simone's) model is suitable	B1	2.2b
		(3)	
(f)	Poisson model has better fit so suggests that oak trees occur at random Or binomial suggests deliberately planted or cultivated	B1	2.2b
	Therefore the forest is likely to be wild not cultivated	B1	3.5a
		(2)	
		(19 marks)	

	Notes
(a)	M1 for choosing binomial A1 for B(6, p) can be in words and allow B(6, 0.55)
(b)	1st M1 for pooling last 2 classes (E_i = 13.08 but accept 13.1) 2nd M1 for at least 3 correct values or expressions. Either row to at least 2 sf 1st A1 for awrt 8.31 (8.31 gets 3/3) [NB no pooling gives awrt 16.8458.. and implies M0M1A0] 1st B1 for 3 degrees of freedom 2nd B1ft for critical value of 7.815 (e.g. $\nu = 4$ use 9.488) 3rd M1 for a correct conclusion (non-contextual ignore any contradictory contextual comments for this mark) based on their cv and their test statistic This mark can be implied by a fully correct solution ending with correct contextual conclusion 2nd A1 for correct conclusion in context with **all other marks scored**
(c)	1st M1 for selecting a correct model Po(3.3) [Allow Po(awrt 3.3)] 2nd M1 for use of the model with an expression or correct value for s or t 1st A1 for one correct 2nd A1 for both correct (allow awrt 2dp)
(d)	B1 for correct hypotheses must mention Poisson: use of Po(3.3) is B0
(e)	1st B1 for correct degrees of freedom $\nu = 4$ only 2nd B1 for selecting correct critical value (9.488 only) 3rd B1 for not significant conclusion based on 8.749 vs their cv (condone use of Po(3.3) here)
(f)	1st B1 for choosing Poisson as better or stating Poisson implies wild or bino'l implies cultivated 2nd B1 (dep on rejecting bin and accepting Poisson) for clearly stating woodland is wild If the tests give the same results then 2nd B0 automatically

Qu	Scheme	Marks	AO
5(a)	$H_0 : \lambda = 2.5$ (or $\mu = 7.5$) $H_1 : \lambda \neq 2.5$ (or $\mu \neq 7.5$)	B1	2.5
	[X = no. of accidents in a 3-month period] $X \sim \text{Po}(7.5)$	M1	3.3
	$P(X \leqslant 2) = 0.0203$ (calc: 0.020256…) { or $P(X \leqslant 3) = 0.0591$}		
	$P(X \leqslant 13) = 0.9784$ so $P(X \geqslant 14) = 0.0216$ (calc: 0.0215646…)	M1	3.4
	{or $P(X \geqslant 15) = 0.0103$}		
	Giving Critical region of: $\boldsymbol{X \leqslant 2}$	A1	1.1b
	$\boldsymbol{X \geqslant 14}$	A1	1.1b
		(5)	
(b)	[0.0203 +0.0216] =awrt **<u>0.0419</u>** <u>or</u> (calc: 0.041821366… awrt **<u>0.0418</u>**)	B1ft	1.2
		(1)	
(c)	[Let M = no of 3-month periods with a significant result]		
	$M \sim B(8, \text{“}0.0419\text{”})$	M1	3.3
	$[P(M \geqslant 2)] = 1 - P(M \leqslant 1)$	M1	1.1b
	[= 1 – 0.9584…]		
	=0.04153…(calc: 0.041394…) **[<u>0.04139~ 0.04154]</u>**	A1cso	1.1b
		(3)	
(d)	$Y \sim \text{Po}(6.3)$	M1	3.3
	P(Type II error) = $P(3 \leqslant Y \leqslant 13)$ or $P(Y \leqslant 13) - P(Y \leqslant 2)$	M1	3.4
	[= 0.9945147… – 0.049846…]		
	= 0.9446… awrt **<u>0.945</u>**	A1	1.1b
		(3)	
		(12 marks)	

	Notes
(a)	B1 for both hypotheses in terms of λ or μ (either way around) 1st M1 for selecting the correct Po model. Sight or use of Po(7.5) may be implied by 2nd M1 2nd M1 for using the correct model to find one of these probs with correct label (2sf or better) 1st A1 for one end correct 2nd A1 for a fully correct CR Allow any letter, even CR $\leqslant 2$ or set notation but **not** $P(X \leqslant 2)$ Can have $X < 3$ and $X > 13$ etc
(b)	B1ft for awrt 0.0419 <u>or</u> awrt 0.0418 <u>or</u> ft addition of their **two** probs provided both are 0 < prob < 0.025 (awrt 3sf)
(c)	1st M1 for selecting a correct binomial model, ft their answer to part (b) 2nd M1 for a correct probability statement of $1 - P(M \leqslant 1)$ **dep on** <u>a</u> binomial selected A1cso for answer in range [0.04139, 0.04154] **dep on** use of B(8, “0.0419”) or better
(d)	1st M1 for selecting a Po(6.3) model 2nd M1 for a correct probability statement using their Poisson model and their CR in (a) which may have just one tail. A1 for awrt 0.945

Qu	Scheme	Marks	AO
6 (a)	$G(1) = 1 \Rightarrow k\ln 2 = 1$ so $k = \underline{\underline{\dfrac{1}{\ln 2}}}$	B1	2.1
		(1)	
(b)	$\left\{G(t) = \dfrac{1}{\ln 2}[\ln 2 - \ln(2-t)]\right\} \Rightarrow G'(t) = \dfrac{1}{\ln 2}\left[\dfrac{1}{2-t}\right]$ or $\dfrac{1}{\ln 2}(2-t)^{-1}$	M1 A1	2.1 1.1b
	$[E(X) =]\ G'(1) = \dfrac{1}{\ln 2}$	A1	1.1b
	$G''(t) = \dfrac{1}{\ln 2}\times\left[\dfrac{1}{(2-t)^2}\right]$	M1 A1	2.1 1.1b
	$\text{Var}(X) = G''(1) + G'(1) - [G'(1)]^2 = \dfrac{1}{\ln 2} + \dfrac{1}{\ln 2} - \left(\dfrac{1}{\ln 2}\right)^2$	M1	2.1
	$= \underline{\dfrac{1}{\ln 2}\left(2 - \dfrac{1}{\ln 2}\right)}$	A1	1.1b
		(7)	
(c)	$P(X = 3)$ = coefficient of t^3 by Maclaurin need $G'''(0)$	M1	3.1a
	$G'''(t) = \dfrac{1}{\ln 2}\dfrac{2}{(2-t)^3}$	A1ft	1.1b
	$P(X = 3) = \dfrac{G'''(0)}{3!}$	M1	3.2a
	$= \dfrac{\frac{1}{4\ln 2}}{6} = \dfrac{1}{24\ln 2} = 0.0601122\ldots$ awrt $\underline{\mathbf{0.0601}}$	A1	1.1b
		(4)	
		(12 marks)	

	Notes
(a)	B1 for finding k (must be exact)
(b)	1st M1 for an attempt to differentiate $G(t)$ e.g. $A(2-t)^{-1}$ (o.e.)
	1st A1 for a correct first derivative (condone k or use of $\frac{1}{\ln 2} =$ awrt 1.44)
	2nd A1 for correct $E(X)$ or $G'(1)$ (allow awrt 1.44 calc: 1.442695…but not k) seen anywhere
	2nd M1 for attempting second derivative (ft their $G'(t)$)
	3rd A1 for a correct 2nd derivative (condone k or use of $\frac{1}{\ln 2} =$ awrt 1.44)
	3rd M1 for a correct method for Var(X) (some substitution into the correct formula)
	4th A1 for $\dfrac{1}{\ln 2}\left(2-\dfrac{1}{\ln 2}\right)$ o.e. but must simplify i.e. collect like terms
	[Mark final answer – penalise incorrect log work etc]
	NB 0.8040211.. is A0 unless exact answer seen
(c)	1st M1 for a suitable strategy to solve the problem (finding link with Maclaurin) Need mention of coefficient of t^3 **and** [$G'''(t)$ or $G'''(0)$](condone $G'''(1)$)
	1st A1ft for 3rd derivative , ft their 2nd derivative in (b) (provided $G''(t)$ not const) Correct $G'''(t)$ or $G'''(0)$ scores 1st M1 1st A1ft
	2nd M1 for translating Maclaurin to probability (a correct expression)
	2nd A1 for $\frac{1}{24\ln 2}$ or awrt 0.0601

ALT	**Log series** 1st M1 attempt to write G(t) in suitable form as far as: $k[\ln 2 - \ln(2[1-\frac{t}{2}])]$
	1st A1 reaching $-k\ln(1-\frac{t}{2})$ 2nd M1 use of $-\ln(1-x)$ series (<u>some</u> correct substitution) NB $\mathrm{G}(t) = \frac{1}{\ln 2}\left(\frac{t}{2}+\frac{t^2}{8}+\frac{t^3}{24}+...\right)$

Qu	Scheme	Marks	AO
7(a)(i)	[B~Geo($\frac{1}{3}$)] P(B = 4) = $\left(\frac{2}{3}\right)^3 \times \frac{1}{3}$	M1	3.3
	$= \underline{\frac{8}{81}}$	A1	1.1b
(ii)	P($B \leqslant 5$) = 1 – P(B > 5) <u>or</u> $1-\left(\frac{2}{3}\right)^5$	M1	2.1
	$= \underline{\frac{211}{243}}$	A1 (4)	1.1b
(b)	$\mathrm{E}(B^2) = \mathrm{Var}(B) + [\mathrm{E}(B)]^2$	M1	2.1
	From formula booklet: E(B) = $\frac{1}{\frac{1}{3}}$ = 3 and Var(B) = $\frac{1-\frac{1}{3}}{\left(\frac{1}{3}\right)^2}$ = 6	B1	1.1b
	So E(B^2) = 6 + 9 = **<u>15</u>**	A1 (3)	1.1b
(c)	[Let R = no. of the spin when it first lands on red] $X = R$~Geo($\frac{2}{3}$)	M1	3.3
	Require E(e^X) = $\sum_{x=1}^{\infty} \mathrm{e}^x \left(\frac{1}{3}\right)^{x-1} \frac{2}{3}$	M1	3.1a
	$= \frac{2\mathrm{e}}{3}\sum_{x=1}^{\infty}\left(\frac{\mathrm{e}}{3}\right)^{x-1}$	M1	2.1
	$= \frac{2\mathrm{e}}{3} \times \frac{1}{1-\frac{\mathrm{e}}{3}}$ <u>or</u> $\frac{2\mathrm{e}}{3-\mathrm{e}}$	A1	1.1b
	E(e^X) = 19.297… {> 15 = E(B^2)} so Tamara should **choose red** since it has the greater expected score	A1 (5)	2.2a
		(12 marks)	

	Notes
(a)(i)	M1 for selecting the correct model i.e. Geo(p) (May be implied by a correct expression) A1 for $\frac{8}{81}$ (= 0.098765… accept awrt 0.0988)
(ii)	M1 for a suitable strategy to use the geometric model to find a correct expression A1 for $\frac{211}{243}$ (= 0.868312…accept awrt 0.868)
(b)	M1 for a suitable strategy to find E(B^2) [allow $\mathrm{G}''(1)+\mathrm{G}'(1)$] B1 for use of the correct formulae to find E(B) = 3 <u>and</u> Var(B) = 6 <u>or</u> $\mathrm{G}''(1) = 12$ A1 for 15
SC	**Formula for E(B^2)** Allow M1B1A0 for $\mathrm{E}(B^2) = \frac{2-p}{p^2}$ (o.e.)

Qu7	Notes
(c)	1st M1 for choosing a suitable geometric model (sight of Geo($\frac{2}{3}$) or at least 3 correct probabilities) 2nd M1 for realising the need for appropriate expected value and using E(g(X)) [Need sum and f(x)] NB simply finding $e^{E(X)} = e^{1.5}$ = awrt 4.48 is M0 and probably no more marks. 3rd M1 for a suitable strategy to turn the expression into a sum that can be found 1st A1 for correct use of sum to infinity of geometric series 2nd A1 for interpreting the outcome of the calculations in terms of a solution to the problem must choose red and see the awrt 19.3 (and allow ft of their E(B^2) < 19)

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