Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 3C Further Mechanics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

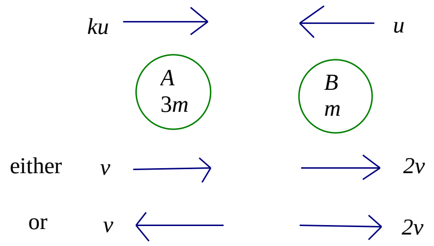
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

M(A)	Taking moments about A.
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS, LHS	Right hand side, left hand side.

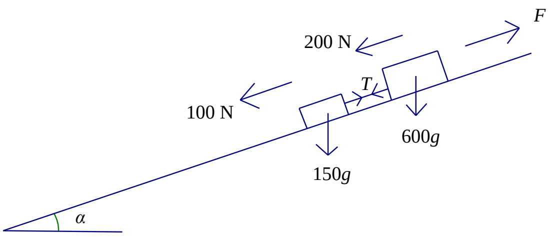
Question	Scheme	Marks	AOs
1a			
	Note that if they start with their 2v to the left this creates an impossible situation (the particles need to pass through each other). The maximum score is M1M1M1.		
	Impulse received by *B*:	M1	3.4
	$\frac{3}{2}mu = m\left(2v - \left(-u\right)\right)$	A1	1.1b
	$v = \frac{u}{4}$	A1	1.1b
		(3)	

1b	Use of CLM or Impulse-momentum for one option for *A*:	M1	3.4
	$3kmu - mu = 2mv + 3mv\left(= \frac{5mu}{4}\right)$ or $3m(v - ku) = -\frac{3mu}{2}$ $\left(3mu\left(\frac{1}{4}+\frac{1}{2}\right) = 3mku\right)$	A1ft	1.1b
	$k = \frac{3}{4}$	A1	1.1b
	Form a second equation in *k* $\left(3mku - mu = 2mv - 3mv\left(= -\frac{mu}{4}\right) \text{ or } 3m(v+ku) = \frac{3mu}{2}\right)$	M1	3.1a
	$k = \frac{1}{4}$	A1	1.1b
		(5)	
			(T0tal 8 Marks)

Notes

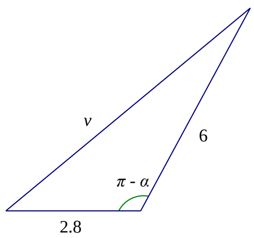
(a)M1	Form impulse-momentum equation for *B* (or *A*). May be expressed as either $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ or $\mathbf{I} + m\mathbf{u} = m\mathbf{v}$. Dimensionally correct. Must be considering difference in velocities Must have a correct combination of mass and velocity: pairing velocity of one with the mass of the other scores M0 Allow for subtraction the wrong way round or impulse in the wrong direction. Assuming that you have not seen an incorrect formula stated, allow for $2v + u$ without overt evidence of subtraction. Allow if the common factor of *m* is not seen
A1	Correct unsimplified equation for *B* (or *A*). Allow without *m*
A1	Correct answer only
(b) M1	Correct method to form an equation in *k*. Must be dimensionally correct Condone sign errors in CLM. Allows marks for CLM equation here if seen in (a) and used correctly to find *k* here. Rules for impulse-momentum as above. M1 is available if they have not reversed the direction of the impulse. An equation which allows for the change in direction by using **u** – **v** can score full marks. Could be working with either option for the direction of motion of *A*
A1ft	Correct unsimplified equation in *u*, *v* or their *v*
A1	One correct solution Be aware that a sign error in the impulse-momentum equation for *A* can lead to a fortuitous answer. A fortuitous answer scores A0 (FYI the incorrect answers are $\frac{-7}{4}$ and $\frac{1}{4}$)

M1	Correct method to form a second equation in k (reversing the direction of motion of A)
A1	Second correct solution

Question	Scheme	Marks	AOs
2.			
	Equation of motion for the system or for the van	M1	3.3
	$F-(100+200)-(150+600)g\sin\alpha=(150+600)a$ or $F-200-T-600g\sin\alpha=600a$	A1 A1	1.1b 1.1b
	Equation of motion for the trailer	M1	3.1b
	$T-100-150g\sin\alpha=150a$	A1	1.1b
	Use of $F=\dfrac{12000}{9}$	M1	3.4
	Solve for T	M1	1.1b
	$T=307\,(310)\,(\text{N})$	A1	2.2a
			(Total 8 Marks)

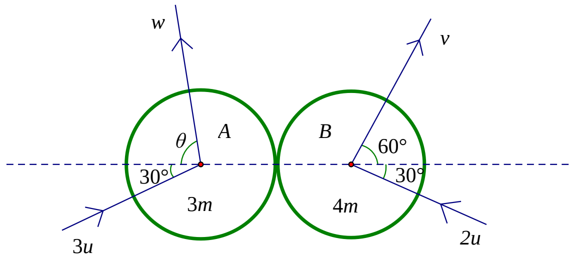
Notes

M1	Need all terms and no extras (the inclusion of + T -T is not an error). Dimensionally correct. Condone sign errors and sin/cos confusion Must have non-zero acceleration and include the driving force
A1 A1	Unsimplified equation in F or their F (and T if relevant) with at most one error Correct unsimplified equation in F or their F (and T if relevant)
M1	Need all terms. Dimensionally correct. Condone sign errors and sin/cos confusion Or a second equation of motion involving the driving force.
A1	Correct unsimplified equation (in T and / or F or their F if relevant)
M1	**Use** of $P=Fv$ seen or implied.
M1	**Complete** method to find T $\left(\text{FYI}: a=0.72(4)\right)$
A1	Tension correct to 3 sf or 2 sf A fractional answer $\left(\dfrac{920}{3}\right)$ is not acceptable because this result follows the use of g = 9.8

Question	Scheme	Marks	AOs
3	Impulse momentum equation(s)	M1	3.1a
	$\begin{pmatrix} 3\times\cos\alpha \\ 3\times\sin\alpha \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_x - 2.8 \\ v_y \end{pmatrix}$ $\left(v_x = \frac{32}{5}, \quad v_y = \frac{24}{5} \right)$	A1 A1	1.1b 1.1b
	$v = \frac{1}{5}\sqrt{32^2 + 24^2}$	M1	1.1b
	$= 8\left(\text{m s}^{-1}\right)$	A1	1.1b
	Alternative working parallel and perpendicular to the impulse: $\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} v_1 - 2.8\times\cos\alpha \\ v_2 \pm 2.8\times\sin\alpha \end{pmatrix}$ $v_1 = 7.68, v_2 = \pm 2.24$ $v = \sqrt{7.68^2 + 2.24^2} = 8\left(\text{m s}^{-1}\right)$		
		(5)	
3alt			
	Using cosine rule:	M1	
	$v^2 = 2.8^2 + 6^2 - 2\times 2.8\times 6\cos(\pi - \alpha)$	A1 A1	
	Solve for v	M1	
	$v = 8\left(\text{m s}^{-1}\right)$	A1	
		(5)	
			(Total 5 marks)

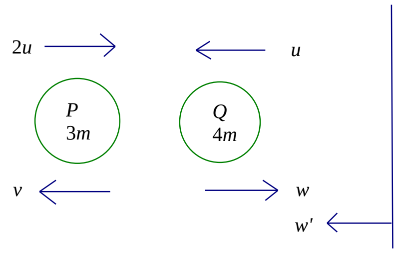
Notes	
M1	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ in two dimensions. (i.e. resolving used) Dimensionally correct. Allow for a combined equation in vector format or for just one component. Condone sin/cos confusion. Allow if m seen but not substituted.
A1 A1	Equation for one component correct unsimplified Equations for both components correct unsimplified Allow A1A1 for a correct unsimplified vector equation Allow A marks if in terms of m and α
M1	Correct use of Pythagoras for their components to obtain the numerical value of the speed This may be seen or implied: an alert candidate might spot the 3, 4, 5 triangle.

A1	Correct only
Alt	
M1	Correct use of cosine rule in a dimensionally correct triangle. The lengths of the sides must be consistent, i.e. v, 2.8 and 6 or $\frac{1}{2}v$, 1.4 and 3 and it must be a correct vector triangle (vectors combined correctly)
A1 A1	Unsimpified equation with at most one error Correct unsimplified equation
M1	Substitute for trig. and solve for *v*
A1	Correct only

Question	Scheme	Marks	AOs
4			
	Parallel to line of centres:	M1	3.1b
	$9mu\cos 30° - 8mu\cos 30° = 4mv\cos 60° - 3mw\cos\theta$ $(u\cos 30° = 2v - 3w\cos\theta)$ $(u\cos 30° = 2v - 3w_x)$	A1	1.1b
	$\updownarrow A: \ (w_y =) w\sin\theta = 3u\sin 30° \left(= \frac{3u}{2}\right)$ $\updownarrow B: \ v\sin 60° = 2u\sin 30° (= u)$ $\left(v = \frac{2u}{\sqrt{3}}\right)$	B1 B1	3.4 3.4
	$(w_x =) w\cos\theta = \frac{1}{3}(2v - u\cos 30°) = \frac{5u\sqrt{3}}{18}$ $(w_y =) w\sin\theta = \frac{3u}{2}$ $\left(\Rightarrow \tan\theta = \frac{9\sqrt{3}}{5}, \quad \theta = 72.2°\right)$	M1	1.1b
	Direction deflected by $77.8°$ ($78°$ or better)	A1	2.2a
	Magnitude of impulse	M1	3.1b
	$= 4m\left(v\cos 60° - (-2u\cos 30°)\right)$	A1	1.1b
	$= 4m\left(\frac{1}{2} \times \frac{u}{\sin 60°} - \left(-2u\frac{\sqrt{3}}{2}\right)\right) = \frac{16\sqrt{3}}{3}mu$	A1	2.2a
	OR: magnitude $= 3m(3u\cos 30° + w\cos\theta)$ $= 3m\left(+\frac{5u\sqrt{3}}{18} - \left(-\frac{3u\sqrt{3}}{2}\right)\right) = \frac{16\sqrt{3}}{3}mu$		
		(9)	
		(Total 9 Marks)	

M1	Use of CLM parallel to the line of centres. Need all 4 terms. Dimensionally correct. Condone sign errors and sin/cos confusion.

A1	Correct unsimplified equation. Allow e.g. w_x in place of $w\cos\theta$ and v_x in place of $v\cos 60°$. Allow if they have divided through by a common factor e.g. m
	NB there is no mark for the correct use of the impact law because the candidates are not required to find the coefficient of restitution. They might however find it as part of an alternative method. In this case, the M marks below are for a complete correct method to achieve the required result. Ignore work to find e if it is not used.
B1 B1	No change perpendicular to line of centres for one sphere. Allow e.g. w_y in place of $w\sin\theta$. Check the diagrams – the vertical components are often shown there. No change perpendicular to line of centres for both spheres
M1	Use scalar product or solve simultaneous equations to find θ for a relevant angle using their w_x They need to get as far as θ = a numerical value for a relevant angle
A1	$78°$ or better
M1	Use of $I = mv - mu$ in direction of line of centres. Condone subtraction in either order Allow M1 if they think that they have subtracted but they have not actually taken account of the change of direction. Allow M1 if they go direct to the correct expression with a + without telling you that they have taken account of the change in direction Allow M1 if they go straight to an unsimplified expression in surds using values already found earlier.
A1	Correct unsimplified expression. Allow the negative of this
A1	Any equivalent simplified form. Must be positive. Condone if they change sign at the very end without explaining why. Accept $9.2(376...)mu$ (2 sf or better)
	NB You might see candidates using the right angle and matrix multiplication to rotate the initial velocity of *B* to find the correct components of the velocity of *B* after impact.

Question	Scheme	Marks	AOs
5a			
	Using CLM:	M1	3.4
	$6mu-4mu=-3mv+4mw \quad (2u=-3v+4w)$	A1	1.1b
	Use of impact law	M1	3.1a
	$w+v=e\times 3u$	A1	1.1b
	Complete method to find w	M1	2.1
	$\begin{cases} 3w+3v=9eu \\ -3v+4w=2u \end{cases} \Rightarrow 7w=9eu+2u, \quad w=\frac{u}{7}(9e+2) \quad *$	A1*	2.2a
		(6)	
5b	$w'=\frac{1}{2}\times\frac{u}{7}(9e+2) \quad \left(=\frac{u}{14}(9e+2)\right)$	B1	1.1b
	$v=\frac{u}{7}(12e-2)$	B1	1.1b
	For a second collision: $w'>v$	M1	3.3
	$9e+2>2(12e-2), \quad 0<e<\frac{2}{5}$	A1	1.1b
		(4)	
			(Total 10 marks)

Notes	
(a) M1	Use of CLM. Need all terms. Must be dimensionally correct. Condone sign errors. Accept consistent cancelling of m
A1	Correct unsimplified equation for CLM. They can have v in either direction
M1	Correct use of the impact law (used the right way round) Condone sign errors in finding speed of approach and speed of separation.
A1	Correct unsimplified equation. Signs consistent with equation for CLM.
M1	Complete method to find w e.g. by forming simultaneous equations using CLM and Impact Law and solving. This requires both of the preceding M marks
A1*	Obtain **given answer** from correct working. Accept with 2 + 9e in place of 9e + 2

	Check that the answer does follow from the working.
(b) B1	Speed of Q after impact with the wall. Any equivalent form. Correct speed can be implied by a correct negative velocity.
B1	Speed of P after impact with Q. Accept $\pm$. Any equivalent form in u and e (seen or implied)
M1	Form correct inequality using their v and w'. A correct inequality has P and Q both moving away from the wall
A1	Correct interval only. Accept unsimplified fraction. Need both ends of the interval. Must be strict inequality at both ends.

Question	Scheme	Marks	AOs
6(a)	GPE lost by B – GPE gained by A	M1	3.4
	$=4\times g\times 3-2\times g\sin\theta\times 3$	A1	1.1b
	$=82\,(82.3)\,(\text{J})$	A1	1.1b
		(3)	
6(b)	Total KE gained $=\frac{1}{2}\times 6\times 4.5^2\ (=60.75)\,(\text{J})$	B1	3.1b
	Max friction $\mu 2g\cos\theta\ (=\mu\times 2\times 9.8\times\cos\theta=15.68\mu)$	B1	3.1b
	Work done against friction $=3\times F_{\max}\ (=47.04\mu)$	B1ft	3.4
	Work-energy equation: their GPE lost = their KE gained + their WD against friction	M1	3.4
	$82.32=60.75+47.04\mu$	A1	1.1b
	$\mu=0.459\,(0.46)$	A1	1.1b
		(6)	
6(c)	Work-energy equation for A:	M1	3.4
	$\frac{1}{2}\times 2\times 4.5^2=2g\sin\theta\times d+2g\cos\theta\times\mu d$ $\left(=19.6\times\frac{3}{5}\times d+19.6\times\frac{4}{5}\times\mu d\right)$	A1ft A1ft	1.1b 1.1b
	$d=1.07\,(1.1)$	A1	1.1b
		(4)	
			(Total 13 marks)

Notes

(a)M1	Expression for change in GPE. Must be dimensionally correct and resolved terms where necessary. Allow subtraction either way round
A1	Correct unsimplified expression for the change in PE (before substitution for $\sin\theta$) Allow subtraction either way round
A1	2 sf or 3 sf. Accept 8.4g or $\frac{42g}{5}$ ISW Must be positive but condone a sign change at the end without explanation
(b) B1	Gain in KE for the system (not just for one block)

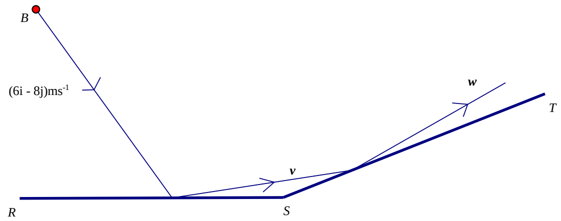
B1	Correct unsimplified expression for $F_{\max}$ seen or implied
B1ft	Correct expression for work done: follow their $F_{\max}$ This is dependent on them having found an expression for $F_{\max}$
M1	Complete method using work-energy to form an equation in μ. Require all terms (needs to consider the KE and GPE of both blocks). Dimensionally correct. Condone sign errors.
A1	Correct unsimplified equation in μ
A1	3 sf or 2 sf only
	NB: It is possible to find the value of μ by finding the tension in the string and forming a work-energy equation for particle *B*, but in this case the first B1 is for KE of *B* and correct tension (25.7(N)) B1 for $F_{\max}$ B1ft is for work done by the tension in the string and against friction M1 for 3 x 25.7 = 20.25 + 35.28 + 3 x 15.68μ O.E.
(c)M1	All terms required. Dimensionally correct. Condone sign errors and sin / cos confusion. If the equation uses $d+3$ in place of *d* in the PE term it is correct if it also includes a term for the initial PE. If the equation uses $d+3$ in place of *d* in the term for work done then it scores M0.
A1 A1	Unsimplified equation in *d* and μ with at most one error Correct unsimplified equation in *d* and μ The ft is on their μ if they have substituted a value.
A1	3 sf or 2 sf only

Question	Scheme	Marks	AOs
7(a)	EPE at $A=\dfrac{\lambda a^2}{2a}$ or EPE at $B=\dfrac{\lambda(2a)^2}{2a}$	M1	2.1
	Form work-energy equation:	M1	3.3
	$\dfrac{\lambda a^2}{2a}+mg\times 3a=\dfrac{\lambda(2a)^2}{2a}$ $\left(\dfrac{\lambda a}{2}+3mga=2\lambda a\right)$	A1 A1	1.1b 1.1b
	$3mg=\dfrac{3\lambda}{2} \Rightarrow \lambda=2mg$ *	A1*	2.2a
		(5)	
7(b)	Extension at equilibrium:	M1	2.1
	$\dfrac{2mgx}{a}=mg \Rightarrow x=\dfrac{a}{2}$ *	A1*	1.1b
	Alternative for the first M1A1:		
	Use the work-energy equation to obtain $\dfrac{\mathrm{d}V^2}{\mathrm{d}x}$ and set the derivative equal to zero	M1	
	$\dfrac{1}{a}\times 2x-1=0\Rightarrow x=\dfrac{a}{2}$	A1	
	Use work-energy equation to find max speed:	M1	3.4
	$\dfrac{2mgx^2}{2a}+mg\times(2a-x)+\dfrac{1}{2}mV^2=\dfrac{2mg(2a)^2}{2a}$ $\left(\dfrac{ag}{4}+\dfrac{3ag}{2}+\dfrac{1}{2}V^2=4ag\right)$	A1 A1	1.1b 1.1b
	$V=3\sqrt{\dfrac{ag}{2}}$	A1	2.2a
		(6)	
7(c)	e.g. for B1 Need to include the GPE of the spring The extension of the spring at equilibrium will be different The spring will have KE You would need to include the KE of the spring in the energy equation You would need to include the GPE of the spring in the energy equation The GPE of the system changes It would take work to raise the spring so the package would have less KE If the spring has mass then GPE of the spring would need to be included	B1	3.5b

		(1)	
(Total 12 Marks)			

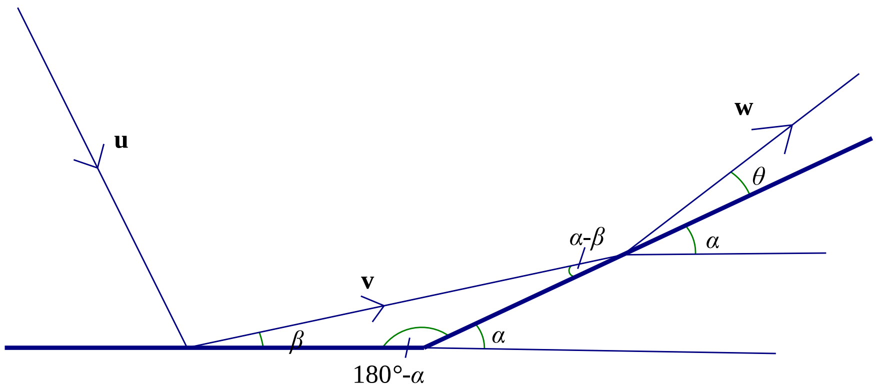
Notes	
(a) M1	Correct method for EPE seen or implied Need something of the form $\frac{1}{2}kx^2$ where $k = \frac{\lambda}{a}$ Must be using the formula for EPE correctly at least once
M1	Require all terms. Dimensionally correct. Condone their EPE. Condone sign errors
A1 A1	Unsimplified equation with at most one error. A repeated error in EPE formula is one error Correct unsimplified equation.
A1*	Obtain **given answer** from correct working
(b) M1	Use correct method for tension to find the extension at equilibrium. Need to see the formula for tension **used**. **Allow verification** with an appropriate conclusion If they use SHM they must use $F = ma$ to prove that P is moving with SHM, otherwise 0/2.
A1*	Correct answer from correct work Allow verification **with an appropriate conclusion**
Alt:M1	Or an equivalent method for finding the turning point of a quadratic
Alt:A1*	Correct answer from correct work
M1	Use given x to form work-energy equation. Need all terms, and dimensionally correct. Condone sign errors. Accept with values of λ and x not substituted
A1 A1	Unsimplified equation with at most one error. Need given λ and given x substituted at some point. A repeated error in the formula for EPE is one error. Correct unsimplified equation with given λ and given x substituted at some point
A1	Use correct method for tension to find the extension at equilibrium. Any equivalent form. $2.1\sqrt{ag}$ or better
(c) B1	Any valid response. B0 if answer includes an additional incorrect factor. Must be specific e.g. not just “the GPE changes”, but the GPE of the system changes is OK. Must relate to an effect on the energy equation E.g. for B0 The extension changes *AB* will increase The tension/energy/GPE/work done etc would increase The KE/GPE/EPE/acceleration/extension/velocity changes The mass of the spring would drag down and the EPE would change The EPE/KE/GPE etc would be variable

	There would be tension in the spring as well It has weight The velocity would decrease as energy is converted

Question	Scheme	Marks	AOs
8a			
	$\mathbf{v} = 6\mathbf{i} + ...$	B1	3.4
	$.....8e\mathbf{j}$	B1	3.4
	impact with $ST \Rightarrow \frac{8e}{6} < \frac{1}{2}$, $0 < e < \frac{3}{8}$	B1	3.1b
		(3)	
8b	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	Component parallel to ST: $(6\mathbf{i}+2\mathbf{j})\cdot\lambda(2\mathbf{i}+\mathbf{j})$	M1	3.1b
	$= \left((6\mathbf{i}+2\mathbf{j})\cdot\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) = \right)\frac{1}{\sqrt{5}}(12+2)$	A1	1.1b
	Component perpendicular to ST: $\pm\left(\frac{1}{2}(6\mathbf{i}+2\mathbf{j})\cdot\gamma(-\mathbf{i}+2\mathbf{j})\right)$	M1	3.4
	$= \frac{1}{2\sqrt{5}}(-6+4)$	A1	1.1b
	$\mathbf{w} = \frac{14}{\sqrt{5}}\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}) + \frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}(-\mathbf{i}+2\mathbf{j})$	M1	3.1b
	$\mathbf{w} = \left(\frac{28}{5} - \frac{1}{5}\right)\mathbf{i} + \left(\frac{14}{5} + \frac{2}{5}\right)\mathbf{j} = \left(\frac{27}{5}\mathbf{i} + \frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ $(\mathrm{m\ s^{-1}})$	A1	2.2a
		(7)	
8b alt 1	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	$\mathbf{w} = a\mathbf{i}+b\mathbf{j} \Rightarrow (6\mathbf{i}+2\mathbf{j}).(2\mathbf{i}+\mathbf{j}) = (a\mathbf{i}+b\mathbf{j}).(2\mathbf{i}+\mathbf{j})$	M1	
	$14 = 2a + b$	A1	
	$\pm\frac{1}{2}(6\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j}) = (a\mathbf{i}+b\mathbf{j}).(-\mathbf{i}+2\mathbf{j})$	M1	
	$2b - a = \pm 1$	A1	
	Solve simultaneous equations for a and b	M1	
	$\mathbf{w} = \left(\frac{27}{5}\mathbf{i} + \frac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ $(\mathrm{m\ s^{-1}})$	A1	
		(7)	
8balt 2	Perpendicular to ST: direction $\pm\mu(-\mathbf{i}+2\mathbf{j})$	B1	1.2
	$\mathbf{v} = 6\mathbf{i}+2\mathbf{j} = p(2\mathbf{i}+\mathbf{j}) + q(-\mathbf{i}+2\mathbf{j})$	M1	3.1b

	$6=2p-q$, $2=p+2q$ $\left(p=\dfrac{14}{5},\quad q=\dfrac{-2}{5}\right)$	A1	1.1b
	Component perpendicular to ST $\pm\dfrac{1}{2}\times q(-\mathbf{i}+2\mathbf{j})$	M1	3.4
	$\pm\dfrac{1}{2}\times q(-\mathbf{i}+2\mathbf{j})$	A1	1.1b
	Solve for p and q to obtain velocity $\mathbf{w}=\dfrac{14}{5}(2\mathbf{i}+\mathbf{j})+\dfrac{1}{2}\times\dfrac{2}{5}(-\mathbf{i}+2\mathbf{j})$	M1	3.1b
	$\mathbf{w}=\left(\dfrac{27}{5}\mathbf{i}+\dfrac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	2.2a
		(7)	

8balt 3



	$\alpha-\beta=8.1...°$	B1	
	Component of $\mathbf{w}$ parallel to ST is $\lvert\mathbf{v}\rvert\cos(\alpha-\beta)$	M1	
	$=\sqrt{40}\cos(\alpha-\beta)\left(=\sqrt{40}\times\dfrac{7}{\sqrt{50}}=6.26..\right)$	A1	
	Component of $\mathbf{w}$ perpendicular to ST is $\dfrac{1}{2}\lvert\mathbf{v}\rvert\sin(\alpha-\beta)$	M1	
	$\pm\dfrac{1}{2}\times\sqrt{40}\sin(\alpha-\beta)\left(=\dfrac{\sqrt{40}}{2}\times\dfrac{1}{\sqrt{50}}=0.447...\right)$	A1	
	$\mathbf{w}=\lvert\mathbf{w}\rvert\cos(\alpha+\theta)\mathbf{i}+\lvert\mathbf{w}\rvert\sin(\alpha+\theta)\mathbf{j}$	M1	
	$\mathbf{w}=\left(\dfrac{27}{5}\mathbf{i}+\dfrac{16}{5}\mathbf{j}\right)$ or $(5.4\mathbf{i}+3.2\mathbf{j})$ (m s^{-1})	A1	
		(7)	
			(Total 10 marks)

8a	
B1	Component parallel to the wall unchanged. Could be on a diagram or implied if they use $e\tan\alpha=\tan\beta$
B1	Use of impact law perpendicular to the wall. Could be on a diagram or implied if they use $e\tan\alpha=\tan\beta$

B1	Use the direction to determine the range for e. (could come via $e\tan\alpha = \tan\beta < 1/2$)
8b	
B1	Correct vector perpendicular to ST seen or implied μ can have any scalar value
M1	Use scalar product to find component of **v** parallel to ST. λ can have any scalar value
A1	Correct unsimplified expression for the magnitude
M1	Use scalar product and impact law perpendicular to ST to find magnitude of component perpendicular to the wall. For their perpendicular vector Must clearly be using $e = \frac{1}{2}$. γ can have any scalar value.
A1	Correct unsimplified expression for the perpendicular component. Allow $\pm$
M1	Combine the magnitudes and directions to obtain the velocity. The perpendicular should now be in the correct direction.
A1	Correct simplified velocity.
8b alt	
B1	Correct vector perpendicular to ST seen or implied. μ can have any scalar value
M1	Correct method for component parallel to ST
A1	Correct equation in a and b
M1	Correct method for component perpendicular to ST Allow $\pm$ For their perpendicular vector
A1	Correct equation in a and b
M1	Solve for a and b to obtain velocity. Using the correct direction for the perpendicular component
A1	Correct simplified answer.
8b alt2	
B1	Correct vector perpendicular to ST seen or implied. μ can have any scalar value
M1	Split **v** into components parallel and perpendicular to ST
A1	Two equations in p and q
M1	Use the impact law perpendicular to ST For their perpendicular vector
A1	Correct unsimpified perpendicular component. With q or their q

M1	Solve for p and q to obtain velocity Using the correct direction for the perpendicular component
A1	Correct simplified total.
8balt3	
B1	Seen or implied. $\sin(\alpha-\beta)=\frac{1}{\sqrt{50}}, \cos(\alpha-\beta)=\frac{7}{\sqrt{50}},$ $\tan(\alpha-\beta)=\frac{1}{7}$
M1	Correct use of their $\lvert\mathbf{v}\rvert$ and their $\alpha-\beta$
A1	Correct unsimplified
M1	Correct use of $\frac{1}{2}$, their $\lvert\mathbf{v}\rvert$ and their $\alpha-\beta$
A1	Correct unsimplified
M1	Use of Pythagoras and correct method for $\theta+\alpha$. $\cos(\alpha+\theta)=\frac{27}{\sqrt{5}\sqrt{197}}, \sin(\alpha+\theta)=\frac{16}{\sqrt{5}\sqrt{197}}$ $\alpha+\theta=30.65°$
A1	Correct simplified total.