Rewarding Learning

ADVANCED
General Certificate of Education
2018

Centre Number

Candidate Number

Mathematics

Assessment Unit C4
assessing
Module C4:
Core Mathematics 4

AMC41

[AMC41]
WEDNESDAY 6 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eight** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an H.B. pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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24AMC4102

1 A curve is defined by the parametric equations

$$x = 2t \qquad\qquad y = 4t^2 + t$$

Find the gradient of the curve when $t = 4$ [5]

2 The line L_1 has vector equation

$$\mathbf{r}_1 = \begin{pmatrix} 6 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

The line L_2 passes through the points $(2, 3, -1)$ and $(4, -1, 1)$.

(i) Find the vector equation of L_2 [3]

(ii) Show that L_1 and L_2 are perpendicular. [4]

3 The functions f and g are defined by

$$f(x) = \sqrt{2x + 5} \qquad \text{for } x \in \mathbb{R},\ x \geqslant -2.5$$

$$g(x) = \frac{1}{4x + 1} \qquad \text{for } x \in \mathbb{R},\ x \neq -0.25$$

(i) State the range of f. [1]

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(ii) Find the inverse function $f^{-1}(x)$ stating the domain of this function. [4]

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(iii) Solve the equation $fg(x) = 3$ [4]

[Turn over

4 **(i)** On the axes below sketch the graph of $y = \sec x$ for $0 \leqslant x \leqslant 2\pi$ [3]

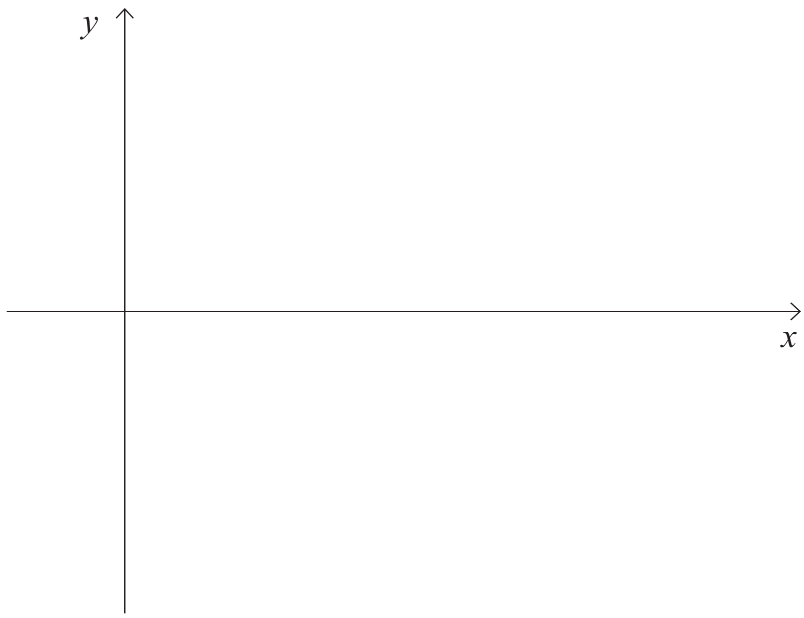

Fig. 1 below shows a cable reel.

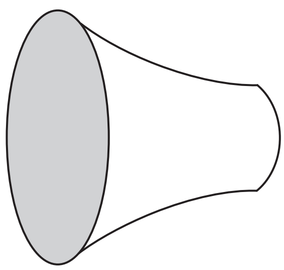

Fig. 1

The volume of this reel can be modelled by rotating the area bounded by the graph of $y = \sec x$, the x-axis and the ordinates $x = 2$ and $x = 3$ through 2π radians about the x-axis.

(ii) Find the volume of this reel. [5]

5 Solve the equation

$$\sin 2x = \tan x \qquad \text{for } 0° \leqslant x \leqslant 360°$$ [9]

6 A curve has the equation

$$y\mathrm{e}^{-2x} = 2x + y^2$$

(i) Show that the gradient function of this curve is given by

$$\frac{2 + 2y\mathrm{e}^{-2x}}{\mathrm{e}^{-2x} - 2y}$$

[7]

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The point P (0,1) lies on this curve.

(ii) Find the equation of the normal to this curve at the point P.
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

7 Newton's law of cooling states that the rate at which the temperature of a liquid is falling is proportional to the difference between the temperature of the liquid and the temperature of its surroundings at that instant.
A mug of hot coffee is placed in a room which has a constant temperature of 20°C.
After t minutes the coffee has cooled to θ°C.

The rate at which the coffee is cooling can be modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20) \qquad \text{where } k \text{ is a constant.}$$

At time $t = 0$, the coffee has a temperature of 100°C.
At $t = 5$, the coffee has a temperature of 68°C.

(i) Show that

$$\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}$$ [9]

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(ii) Find the temperature of the coffee at $t = 10$ [1]

8 **(a)** Find

$$\int x \cot^2 x \, dx$$ [6]

(b) (i) Write in partial fractions

$$\frac{2}{x(2x-1)}$$

[2]

(ii) Use the substitution $x = u^2$, where u is positive, to show that

$$\int_1^9 \frac{1}{x(2\sqrt{x} - 1)}\,dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be found. [8]

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For Examiner's use only	
Question Number	Marks
1	
2	
3	
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Total Marks	

Examiner Number