Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Mathematics
Statistics S4 Paper 6686_01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark
  ⌙

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1		
	To test $H_0 : \mu = 200$, $H_1 : \mu > 200$	B1
	Test statistic $t = \dfrac{202-200}{\sqrt{\frac{3.6}{10}}} = \dfrac{10}{3}$ or 3.3333…	M1A1
	Critical value(s): $t_9 = (\pm)2.821$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	
	Significant evidence that the **mean weight** of the **packets** of **almonds** is more than **200 g**	A1ft (5) **Total 5**
	Notes	
	1st B1 Both hypotheses with μ. 1st M1 Allow $\pm \dfrac{202-200}{\frac{s}{\sqrt{10}}}$ 1st A1 awrt 3.33 2nd B1 allow *p* value of awrt 0.00438 in place of critical value. CV must follow from H_1, sign must match *t*-value or be $\pm$ 2nd A1ft ft *t*-value if awarded B marks. Need correct conclusion in context containing the words **mean weight**, **almonds or packets** and **200g**	

Question Number	Scheme	Marks
2(a)	$\bar{x} = \frac{468}{9} = 52 \quad s^2 = \frac{9}{8}\left(\frac{24560}{9} - 52^2\right) = 28$	M1A1
	i) $t_8 = 2.306$	B1
	$95\%\ \text{CI} = 52 \pm 2.306 \times \frac{\sqrt{"28"}}{\sqrt{9}}$	M1
	$= (47.93\ldots, 56.06\ldots)$	A1
	ii) 95% CI is given by $\frac{8\times 28}{17.535} < \sigma^2 < \frac{8\times 28}{2.180}$	M1 B1
	$12.77 < \sigma^2 < 102.75$	
	$3.57 < \sigma < 10.14$	M1d A1
		(9)
(b)	$38\times 1.2 = 45.6$ or 26% or 1.26	B1
	45.6 is **below the CI** for *Fruity* therefore there is evidence that the **mean** for *Zesty* is more than **20% higher** than his Fruity	M1
	5.5 **is in the CI** therefore there is no evidence that the **standard deviation** is **less** than 5.5	M1
	He should not change to *Zesty*	A1cso
		(4)
	Notes	**Total 13**
(a)	M1 attempting s or s^2 A1 28 only	
(i)	B1 CV awrt 2.306 M1 $\bar{x} \pm t$-value $\times \frac{\sqrt{\text{their Var}}}{\sqrt{9}}$ A1 awrt 47.9 and awrt 56.1	
(ii)	1st M1 for $\frac{8\ (\text{their } s)^2}{\chi^2}$ B1 awrt 17.535 & awrt 2.18 M1d Dept on previous M mark. Rearranging leading to interval for σ- must square root A1 awrt 3.57 and 10.1	
(b)	B1 45.6 seen or awrt 26% or awrt 1.26 M1 correct reason for mean ft their CI M1 correct reason for sd ft their CI A1cso correct conclusion. Not ft on incorrect intervals	

Question Number	Scheme	Marks
3. (a)	Need assumption that the underlying distribution of the **difference** in reaction times is **normally distributed**.	B1 (1)
(b)	<table><tr><td>Student</td><td>*A*</td><td>*B*</td><td>*C*</td><td>*D*</td><td>*E*</td><td>*F*</td><td>*G*</td><td>*H*</td></tr><tr><td>Difference *Start - end*</td><td>0.8</td><td>1.1</td><td>-0.1</td><td>1.1</td><td>0.7</td><td>2.8</td><td>1.3</td><td>0.8</td></tr></table>	M1
	$\bar{d} = \frac{8.5}{8} = (\pm)1.0625$	M1
	$s^2 = \frac{8}{7}\left(\frac{13.73}{8} - 1.0625^2\right) = 0.67125$	M1
	or	
	$s^2 = \frac{1}{7}\left(13.73 - \frac{8.5^2}{8}\right) = 0.67125$	
	Test stat	
	$t = \frac{"1.0625" - m}{\sqrt{\frac{"0.67125"}{8}}}$	M1
	Critical value, $t_7(2.5\%) = \pm 2.365$ $t_7(0.005\%) = \pm 3.499$	B1
	$\frac{1.0625 - m}{\sqrt{\frac{0.67125}{8}}} = \pm 2.365$ $\frac{1.0625 - m}{\sqrt{\frac{0.67125}{8}}} = \pm 3.499$	M1d A1ft
	$0.049 < m < 0.377$ and $1.748 < m < 2.076$	A1 A1 (9)
	Notes	**Total 10**
(a)	B1 for a comment that mentions "differences" and "normal" distribution	
(b)	M1 attempting differences	
	M1 attempt to find $\bar{d} = \frac{\sum "\text{their } d"}{8}$	
	M1 attempting s or s^2 $\frac{1}{7}\left(\sum "\text{their } d^2" - \frac{\left(\sum "\text{their } d"\right)^2}{8}\right)$ $s = 0.8192\ldots$	
	M1 for attempting the correct test statistic $\frac{\bar{d} - m}{s/\sqrt{8}}$, allow any letter	
	B1 Both critical values correct (ignore sign)	
	M1d dependent on previous M being awarded. Having a pair of equations with the same sign and one of each CV. Ft their test statistic and CV	
	A1**ft ft their CV** four equations, may be implied by both ranges correct	
	A1 awrt $0.049 < m <$ awrt 0.377 allow $\leq$ instead of $<$	
	A1 awrt $1.75 < m <$ awrt 2.08 allow $\leq$ instead of $<$	
	NB if test stat the wrong way round remove one of the A marks awarded at the end	

Question Number	Scheme	Marks
4(a)	$H_0 : \sigma_G^2 = \sigma_T^2$ against $H_1 : \sigma_G^2 > \sigma_T^2$	B1
	Test stat, $F_{4,5} = \frac{0.66^2}{0.31^2} = 4.53$ $\left(\frac{1}{F_{4,5}} = \frac{0.31^2}{0.66^2} = 0.221\right)$	M1A1
	Critical value, $F_{4,5} = 5.19\,(0.1927)$	B1
	Not in critical region, therefore no evidence to reject H_0	
	No evidence of difference in **standard deviation (allow variance)**	A1cso
		(5)
(b)	$s_p^2 = \frac{5\times 0.31^2 + 4\times 0.66^2}{5+4}$	M1
	$s_p^2 = 0.24698...$ or $s_p = 0.49697...$ awrt 0.247 or 0.497	A1
	$H_0 : \mu_G = \mu_T + 4$ $H_1 : \mu_G > \mu_T + 4$	B1
	critical value CR: $t_9(0.05) > \pm 1.833$	B1
	$t = \pm\frac{10.12-5.27-4}{\sqrt{0.24698\left(\frac{1}{5}+\frac{1}{6}\right)}} = \pm 2.8245\ldots$ or p =awrt 0.0099549 awrt 2.82, 2.825	M1 A1
	There is evidence to reject H_0 μ_G is greater than $\mu_T + 4$. The suppliers claim is supported.	A1
		(7)
(c)	$\frac{\bar{X}_G - \bar{X}_T - 4}{\sqrt{0.24698\left(\frac{1}{5}+\frac{1}{6}\right)}} > 1.833$	M1
	$\bar{X}_G - \bar{X}_T > 1.833\times\sqrt{0.24698\left(\frac{1}{5}+\frac{1}{6}\right)} + 4$	
	$\bar{X}_G - \bar{X}_T > 4.55$	A1
		(2)
(d)	No change to standard deviation	B1
	$\bar{X}_G - \bar{X}_T = 4.35$	M1
	Previously they would have changed to *Goglue*, now they will remain with *Tackfast*	A1
	or they will no longer change, or they would have changed but now they will not oe	
		(3)
	Notes	**Total 17**
(a)	B1 both hypotheses. allow $H_0 : \sigma_T = \sigma_G$ against $H_1 : \sigma_G > \sigma_T$. Must use σ or σ^2 and make clear which is H_0 and which is H_1. Do not allow in words M1 allow 0.31 and 0.66 rather than 0.31^2 and 0.66^2 if they write the formula down B1 correct CV for their F or a correct comparison if use p Final A1: – All previous marks must be awarded. Variances are the same or var are not different	
(b)	M1 Allow use of 0.31 and 0.66. May be seen in part(a) B1 both hypotheses using μ. Do not allow $\geq$ sign instead of >. May use different letters eg A and B but they must be defined. B1 correct CV but must match t-value or a correct comparison if use p M1 use of correct formula with their s_p – condone missing 4 M1 use of correct formula with their s_p. (which must have been attempted) A1 A correct statement or longhand of suppliers claim with the word **force** and **mean** and is **more than 4 greater** oe Do not allow contradicting statements.	
(c)	M1 correct LHS with 1.833(or their CV used in (b)) NB subst in 4.55 for is M0	

Question Number	Scheme	Marks
5.(a)	0.05 or 5%	B1
		(1)
(b)	Let the CR be $\overline{X} > k$ $\mathrm{P}\left(\overline{X} > k \mid \overline{X} \text{ is } \mathrm{N}\left(150, \frac{16}{n}\right)\right) = 0.05$	
	$\therefore \frac{\overline{k} - 150}{\frac{4}{\sqrt{n}}} = 1.6449$	M1B1A1
	$\overline{k} = 150 + 1.6449 \times \frac{4}{\sqrt{n}}$	
	$\therefore \frac{\overline{k} - 152}{\frac{4}{\sqrt{n}}} = -1.2816$	M1B1A1
	$\overline{k} = 152 - 1.2816 \times \frac{4}{\sqrt{n}}$	
	$150 + 1.6449 \times \frac{4}{\sqrt{n}} < 152 - 1.2816 \times \frac{4}{\sqrt{n}}$ or $\frac{150 + \frac{6.5796}{\sqrt{n}} - 152}{\frac{4}{\sqrt{n}}} = -1.2816$	M1dd
	$\left[\sqrt{n}\right] > 5.853$	A1
	$[n >] 34.25$	M1
	$n = 35$	A1cso
		(10)
	Notes	Total 11
(b)	M1 $\therefore \frac{\overline{k} - 150}{4/\sqrt{n}} = z\text{-value}$, $\lvert z \rvert > 1.5$ B1 awrt $\pm$ 1.6449 A1 correct equation = awrt1.65/1.64 M1 $\therefore \frac{\overline{k} - 152}{4/\sqrt{n}} = z\text{-value}$, $1 < \lvert z \rvert < 1.5$ B1 $\pm$ awrt 1.2816 A1 correct equation = awrt $-$ 1.28 M1dd dependent on both previous M marks being awarded. forming an equation and solving leading to $n = \ldots$ or $\sqrt{n} = \ldots$ A1 awrt 5.85 M1 for squaring A1cso 35 only	

Question Number	Scheme	Marks
6.(a)	$\mathrm{E}(X^N) = \int_0^{2\theta} \frac{x^{N+1}}{2\theta^2} \mathrm{d}x$	M1
	$= \left[\frac{x^{N+2}}{2(N+2)\theta^2} \right]_0^{2\theta}$	A1
	$= \frac{(2\theta)^{N+2}}{2(N+2)\theta^2}$	
	$= \frac{2^{N+1}}{N+2}\theta^N$ (*)	A1cso
		(3)
(b)	$\mathrm{E}(X) = \frac{4\theta}{3}$	B1
	$\mathrm{Var}(X) = 2\theta^2 - \left(\text{"} \frac{4\theta}{3} \text{"} \right)^2 \quad = \frac{2\theta^2}{9}$	M1A1 (3)
(c)	$q = \frac{3}{4}$	B1
	$\mathrm{Var}(S_1) = \frac{9}{16} \times \text{"} \frac{2\theta^2}{9n} \text{"}$	M1
	$= \frac{\theta^2}{8n}$	A1cso
	as $n \to \infty$ $\mathrm{Var}(S) \to 0 \therefore$ s[ince it is unbiased] it is a consistent estimator	
		(3)
(d)	$\mathrm{E}(S_2) = a \times \text{"} \frac{4\theta}{3} \text{"} + b \times \frac{\theta}{3}$	M1
	$a \times \frac{4\theta}{3} + b \times \frac{\theta}{3} = \theta$ or $4a + b = 3$	A1
	$\mathrm{Var}(S_2) = a^2 \times \text{"} \frac{2\theta^2}{9} \text{"} + b^2 \times \frac{\theta^2}{27}$	M1
	$\mathrm{Var}(S_2) = a^2 \times \frac{2\theta^2}{9} + (3-4a)^2 \times \frac{\theta^2}{27}$ or $\mathrm{Var}(S_2) = \left(\frac{3-b}{4} \right)^2 \times \frac{2\theta^2}{9} + b^2 \times \frac{\theta^2}{27}$	M1
	$\frac{\mathrm{dVar}(S_2)}{\mathrm{d}a} = \frac{4a\theta^2}{9} - \frac{8(3-4a)\theta^2}{27}$ or $\frac{-(3-b)\theta^2}{36} + \frac{2b\theta^2}{27}$	M1
	$\frac{4a\theta^2}{9} - \frac{8(3-4a)\theta^2}{27} = 0$ or $\frac{-(3-b)\theta^2}{36} + \frac{2b\theta^2}{27} = 0$	M1
	$\frac{44a}{27} = \frac{24}{27}$ or $\frac{11}{108}b = \frac{1}{12}$	
	$a = \frac{6}{11}, \quad b = \frac{9}{11}$	A1 (7)

Question Number	Scheme	Marks
(e)	$\text{Var}(S_2) = \left(\frac{"6"}{11}\right)^2 \times \frac{2\theta^2}{9} + \left(\frac{"9"}{11}\right)^2 \times \frac{\theta^2}{27}$	M1
	$\text{Var}(S_2) = \frac{\theta^2}{11}$	(1)
(f)	S_1 is the better estimator when $\frac{\theta^2}{8n} < \frac{\theta^2}{11} \Rightarrow n > \frac{11}{8}$	M1
	S_2 is the better estimator when $n < \frac{11}{8}$	
	Therefore S_1 is the better estimator since $n \geq 2$	A1cso
		(2)
	Notes	**Total 19**
(a)	M1 attempting to integrate $\frac{x^{N+1}}{2\theta^2}$, $x^{N+1} \to x^{N+2}$ condone missing limits A1 correct integration A1 fully correct solution – must see substitution of 2θ	
(b)	B1 must have E(X) = M1 allow their E(X) if one has been given otherwise must be correct in here A1 must be using part(a), do not allow if integrated from scratch.	
(c)	M1 for $\frac{9}{16} \times \frac{\text{their Var}(X)}{n}$ A1 cso and for as $n \to \infty$ $\text{Var}(S) \to 0 \therefore$ since it is unbiased it is a consistent estimator	
(d)	M1for $a \times \text{their E}(X) + b \times \frac{\theta}{3}$ A1a correct equation with no θ M1 $a^2 \times \text{their Var}(X) + b^2 \times \frac{\theta^2}{27}$ M1 subst in for a or b M1 differentiating with respect to a or b M1 putting dVar/da = 0 and solving leading to a = … or b = … A1 allow awrt 0.545 and awrt 0.818	
(e)	M1 subst a and b in to find Var(S_2)	
(f)	M1 for reason $\frac{\theta^2}{8n} < \frac{\theta^2}{11} \Rightarrow n > \frac{11}{8}$ or $\text{Var}(S_1) \leq \frac{\theta^2}{16} < \frac{\theta^2}{11}$ A1cso correct selection	

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