GCE AS/A level

0974/01

MATHEMATICS – C2
Pure Mathematics

A.M. THURSDAY, 22 May 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_1^3 \log_{10}(3x-1)\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_1^3 \log_{10}(3x-1)^2\,\mathrm{d}x.$$ [1]

2. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\cos^2\theta + 1 = 4\sin^2\theta - 2\cos\theta.$$ [6]

(b) The angle α satisfies

$$\sin(\alpha + 40°) = \frac{1}{\sqrt{2}}$$

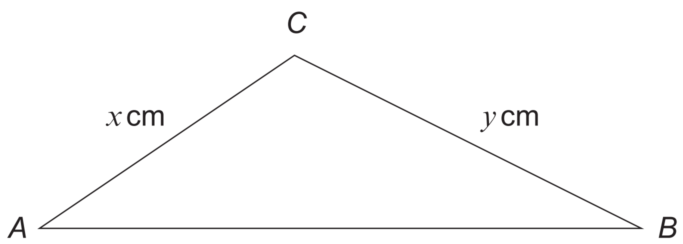
and $$\sin(\alpha - 35°) = \frac{\sqrt{3}}{2}.$$

Given that $0° < \alpha < 180°$, find the value of α. [3]

(c) Find all values of ϕ in the range $0° \leqslant \phi \leqslant 360°$ satisfying

$$\frac{7}{\cos\phi} - \frac{10}{\sin\phi} = 0.$$ [3]

3. The diagram below shows a sketch of the triangle *ABC* with $\sin A = \frac{4}{5}$, $\sin B = \frac{8}{17}$, $\cos C = -\frac{13}{85}$, $AC = x$ cm and $BC = y$ cm.



(a) Show that $y = 1{\cdot}7x$. [2]

(b) Given that $AB = 10{\cdot}5$ cm, **use the cosine rule** to find the exact value of x. [4]

4. *(a)* An arithmetic series has first term a and common difference d.
Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$ [3]

(b) The first term of an arithmetic series is 3 and the common difference is 2. The sum of the first n terms of the series is 360.
Write down an equation satisfied by n. Hence find the value of n. [3]

(c) The tenth term of another arithmetic series is seven times the third term. The sum of the eighth and ninth terms of the series is 80. Find the first term and common difference of this arithmetic series. [4]

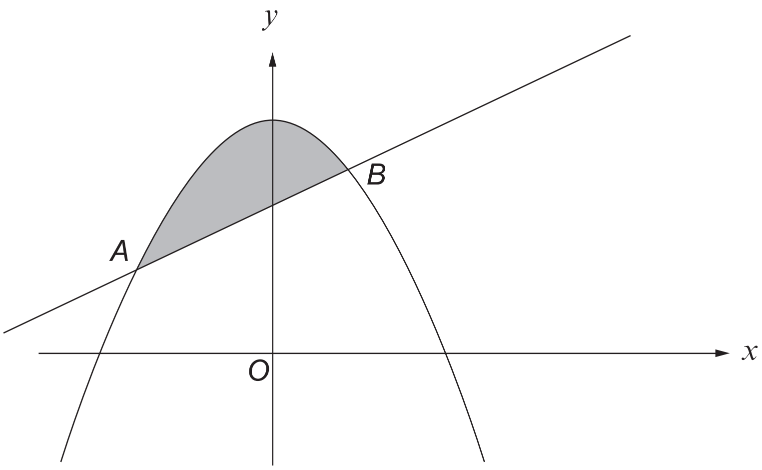
5. A geometric series has first term a and common ratio r. The sum of the second and third terms of the series is –216. The sum of the fifth and sixth terms of the series is 8.

(a) Prove that $r = -\frac{1}{3}$. [5]

(b) Find the sum to infinity of the series. [3]

6. *(a)* Find $\int \left(\frac{5}{x^{\frac{3}{4}}} - 7\sqrt{x} \right) \mathrm{d}x.$ [2]

(b)



The diagram shows a sketch of the curve $y = 16 - x^2$ and the line $y = x + 10$. The line and the curve intersect at the points *A* and *B*.

(i) Find the coordinates of *A* and *B*.

(ii) Find the area of the shaded region. [10]

TURN OVER

7. *(a)* Solve the equation

$$3^{\frac{5x}{4}-2} = 7.$$

Show your working and give your answer correct to three decimal places. [3]

(b) The positive numbers a and b are such that

$$\log_a b = 5.$$

(i) Express b as a power of a.

(ii) **Using your answer to part (i)**, evaluate $\log_b a$. [3]

8. *(a)* The circle C_1 has centre $A(-2, 9)$ and radius 5. The circle C_2 has centre $B(10, -7)$ and radius 15.

(i) Show that C_1 and C_2 touch, justifying your answer.

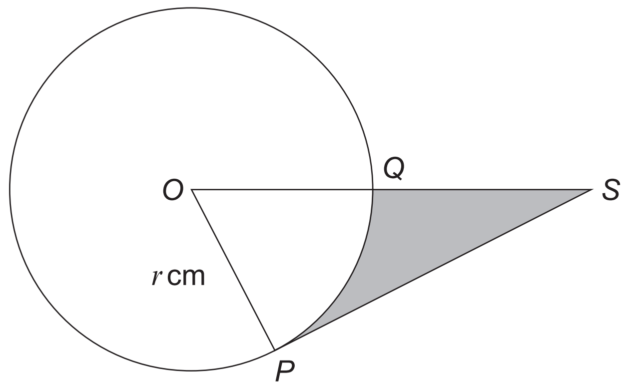
(ii) Given that the circles touch at the point $P(1, 5)$, find the equation of the common tangent at P. [7]

(b) Gareth, who has been asked by his teacher to investigate the properties of another circle C_3, claims that the equation of this circle C_3 is given by

$$x^2 + y^2 + 4x - 6y + 20 = 0.$$

Show that Gareth cannot possibly be correct. [3]

9.



The diagram shows a circle with centre O and radius r cm. The points P and Q are on the circle and $P\hat{O}Q = 0{\cdot}9$ radians. The tangent to the circle at P intersects the line OQ produced at the point S.

(a) Find an expression in terms of r for

(i) the area of sector POQ,
(ii) the length of PS,
(iii) the area of triangle POS. [3]

(b) Given that the area of the shaded region is $95{\cdot}22\,\text{cm}^2$, find the value of r. [3]

END OF PAPER