GCE AS/A Level

0981/01

S17-0981-01

MATHEMATICS – M2
Mechanics

TUESDAY, 20 JUNE 2017 – AFTERNOON

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The position vector of a particle *P* at time t seconds is given by

$$\mathbf{r} = t\sin t\,\mathbf{i} + t\cos t\,\mathbf{j}.$$

(a) (i) Find the velocity vector of *P* and an expression for the speed of *P* at time t seconds in its simplest form.

(ii) Given that the mass of *P* is 3 kg, write down the momentum vector of *P* at time t seconds. [6]

(b) At time $t = \frac{\pi}{6}$, the vector $b\mathbf{i} + \sqrt{3}\mathbf{j}$ is perpendicular to $\mathbf{r}$. Find the value of b. [5]

2. A particle *P*, of mass 0·8 kg, moves along the x-axis so that its velocity at time t seconds is v ms^{-1}, where $v = 4t^3 - 6t + 7$. Given that the displacement of *P* is 5 m from the origin when $t = 0$, find

(a) the displacement of *P* from the origin when $t = 2$, [5]

(b) the force acting on *P* when $t = 3$. [4]

3. A vehicle of mass 3000 kg has an engine that is capable of producing power up to 12 000 W. The vehicle moves up a slope inclined at an angle α to the horizontal, where $\sin\alpha = 0{\cdot}1$. The resistance to motion experienced by the vehicle is constant at 460 N.

(a) Find the maximum acceleration of the vehicle when its velocity is 3 ms^{-1}. [4]

(b) The vehicle now travels at a velocity of v ms^{-1} against an additional braking force of $10v$ N. The other resistance to motion remains constant at 460 N. Determine the maximum value of v. Give your answer correct to 2 decimal places. [5]

4. *A* and *B* are points a distance 18 m apart on horizontal ground. An object *P* is projected from *A* towards *B* with velocity 15 ms^{-1} at an angle of 60° to the horizontal. Simultaneously, another object *Q* is projected from *B* towards *A* with velocity v ms^{-1} at an angle of 30° to the horizontal. The objects collide.

(a) Find the value of v. [5]

(b) Show that the time from projection to collision is 0·6 seconds. [3]

(c) Determine the speed of the object *P* just before collision. [4]

5. A vehicle of mass 4000 kg is moving up a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{20}$. At time $t = 0$ s, the speed of the vehicle is 2 ms^{-1}. At time $t = 8$ s, the vehicle has travelled 30 m up the hill from its initial position and its speed is 5 ms^{-1}. The vehicle's engine is working at a constant rate of 43 000 W. Find the total work done against the resistive forces during this 8 second period. [8]

6. A particle *P*, of mass 5 kg, is attached to one end of a light inextensible string of length 0·8 m. The other end of the string is attached to a fixed point *O*. Initially, the particle *P* is held at rest with the string *OP* taut and inclined at an angle of 60º to the downward vertical through *O*. The particle *P* is then projected with speed u ms^{-1} in a downward direction perpendicular to the string, so that *P* starts to describe a vertical circle with centre *O*. When the string *OP* is inclined at an angle θ to the downward vertical, the speed of *P* is v ms^{-1}.

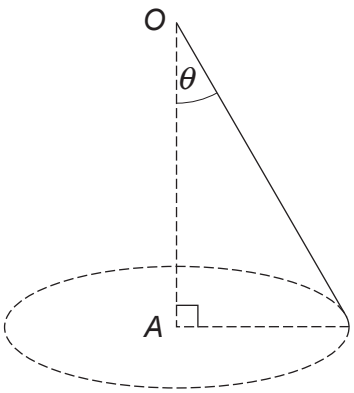
(a) Find, in terms of u and θ, an expression for v^2. [4]

(b) Find, in terms of u and θ, an expression for the tension in the string when *OP* makes an angle θ with the downward vertical. [4]

(c) Determine the least value of u so that the particle describes complete circles. [2]

(d) Suppose that the string is replaced by a light rod. Determine the least value of u so that the particle describes complete circles. [2]

7. A particle of mass 2 kg is suspended from a fixed point *O* by means of an elastic string of natural length 3 m and modulus of elasticity λ N. The particle describes a horizontal circle with constant angular speed ω rad s^{-1}, with the string being of constant length l m, where $l > 3$. The centre of the circle *A* is vertically below *O* and the angle between the string and the downward vertical is θ.



(a) Show that $\cos\theta = \dfrac{g}{l\omega^2}$. [6]

(b) Given that the tension in the string is $20g$ N and $\omega^2 = 3g$,

(i) find the value of $\cos\theta$,

(ii) show that $l = \dfrac{10}{3}$,

(iii) calculate the value of λ,

(iv) find the elastic energy in the string. [8]

END OF PAPER

