GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP2
0978-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

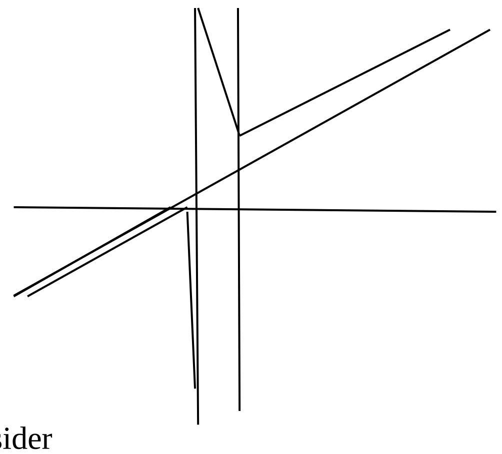
WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

FP2 – June 2017 - Mark Scheme

Ques	Solution	Mark	Notes
1	Consider $f(-x) = \sec(-x) + (-x)\tan(-x)$ $= \sec x + x\tan x \quad (= f(x))$ Therefore f is even.	M1 A1 A1	M0 if particular value used This line must be seen
2	$\int_0^2 \frac{2x^2+5}{x^2+4}\,dx = \int_0^2 \frac{2x^2+8}{x^2+4}dx - \int_0^2 \frac{3}{x^2+4}\,dx$ $= [2x]_0^2 - \frac{3}{2}\left[\tan^{-1}\frac{x}{2}\right]_0^2$ $= 4 - \frac{3}{8}\pi$	M1A1 A1B1 A1	Award the B1 for a correct integration of $\frac{k}{x^2+4}$
3	$-8i = 8(\cos 270^\circ + i\sin 270^\circ)$ Root1 = $2(\cos 90^\circ + i\sin 90^\circ)$ $= 2i$ Root2 = $2(\cos 210^\circ + i\sin 210^\circ)$ $= -\sqrt{3} - i$ Root3 = $2(\cos 330^\circ + i\sin 330^\circ)$ $= \sqrt{3} - i$	B1B1 M1M1 A1 M1 A1 A1	B1 modulus, B1 argument M1for $\sqrt[3]{\text{mod}}$, M1 for arg/3 Special case – B1 for spotting 2i
4(a) (b)	Using deMoivre's Theorem, $z^n + z^{-n} = \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$ $= \cos n\theta + i\sin n\theta + \cos(n\theta) - i\sin(n\theta)$ $= 2\cos n\theta$ $z^n - z^{-n} = \cos n\theta + i\sin n\theta - \cos(-n\theta) - i\sin(-n\theta)$ $= 2i\sin n\theta$ $(z + z^{-1})^5 = z^5 + 5z^3 + 10z + 10z^{-1} + 5z^{-3} + z^{-5}$ oe $= (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$ $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ $\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$	M1 A1 M1 A1 M1A1 A1 A1 A1	

Ques	Solution	Mark	Notes
(c)	$\int_0^{\pi/2}\cos^5\theta d\theta = \int_0^{\pi/2}\left(\frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta\right)d\theta$	M1	FT from (b)
	$= \left[\frac{1}{80}\sin 5\theta + \frac{5}{48}\sin 3\theta + \frac{5}{8}\sin\theta\right]_0^{\pi/2}$	A1	No A marks if no working
	$= \frac{1}{80} - \frac{5}{48} + \frac{5}{8}$	A1	
	$= \frac{8}{15}$	A1	Award FT mark only if answer less that 1
5	Rewrite the equation in the form $2\sin 2\theta \sin 3\theta = \sin 3\theta$	M1A1	Accept answers in degrees
	$\sin 3\theta(2\sin 2\theta - 1) = 0$	A1	
	Either $\sin 3\theta = 0$	M1	
	$3\theta = n\pi$ giving $\theta = \frac{n\pi}{3}$	A1	
	Or $\sin 2\theta = \frac{1}{2}$	M1	
	$2\theta = \left(2n + \frac{1}{2} \pm \frac{1}{3}\right)\pi$	A1	
	giving $\theta = \left(n + \frac{1}{4} \pm \frac{1}{6}\right)\pi$	A1	Accept equivalent forms
6(a)	Let $\frac{24x^2+31x+9}{(x+1)(2x+1)(3x+1)} = \frac{A}{x+1} + \frac{B}{2x+1} + \frac{C}{3x+1}$	M1	
	$= \frac{A(2x+1)(3x+1) + B(x+1)(3x+1) + C(x+1)(2x+1)}{(x+1)(2x+1)(3x+1)}$		
	$x = -1$ gives $A = 1$	A1	
	$x = -½$ gives $B = 2$	A1	
	$x = -1/3$ gives $C = 6$	A1	FT their A,B,C if possible
(b)(i)	$\int_0^2 f(x)\,dx = \int_0^2 \frac{1}{x+1}dx + \int_0^2 \frac{2}{2x+1}dx + \int_0^2 \frac{6}{3x+1}dx$	M1	Their answer should be $\ln(3^A 5^{B/2} 7^{C/3})$ but only FT if this gives $\ln N$
	$= [\ln(x+1)]_0^2 + [\ln(2x+1)]_0^2 + 2[\ln(3x+1)]_0^2$	A2	Award A1 for 2 correct integrals
	$(= \ln 3 + \ln 5 + 2\ln 7)$ $= \ln 735$ cao	A1	
(ii)	The integral cannot be evaluated because the interval of integration contains points at which the integrand is not defined.	B1	

7(a)	$\sqrt{(x-a)^2+y^2}=x+a$ $(x-a)^2+y^2=(x+a)^2$ $x^2-2ax+a^2+y^2=x^2+2ax+a^2$ $y^2=4ax$	**M1** **A1** **A1**	Convincing
(b)	EITHER $\frac{dx}{dt}=2at, \frac{dy}{dt}=2a$ $\frac{dy}{dx}=\frac{2a}{2at}=\frac{1}{t}$ OR $2y\frac{dy}{dx}=4a$ $\frac{dy}{dx}=\frac{2a}{y}=\frac{1}{t}$ Gradient of normal = $-t$ The equation of the normal is $y-2at=-t(x-at^2)$	**M1** **A1** **(M1)** **(A1)** **A1** **A1**	 $y = -tx + at^3 + 2at$
(c)	EITHER The normal intersects the parabola again where $2as-2at=-t(as^2-at^2)$ $=-at(s-t)(s+t)$ Cancelling $a(s-t)$ both sides because $s \neq t$, $2 = -t(s+t)$ $s=-\frac{2}{t}-t$ OR The normal intersects the parabola again where $2as = -ats^2 + at^3 + 2at$ $ts^2 + 2s - 2t - t^3 = 0$ Solving, $s=\frac{-2\pm\sqrt{4+8t^2+4t^4}}{2t}$ $= -\frac{2}{t}-t, t$ (Rejecting t), $s = -\frac{2}{t}-t$	**M1** **A1** **A1** **A1** **A1** **(M1)** **(A1)** **(M1)** **(A1)** **(A1)**	

8(a)(i)	$x = -1$	**B1**	
(ii)	$y = x + 3$	**B1**	
(b)	$f'(x) = 1 - \frac{1}{(x+1)^2}$	**B1**	
	Stationary points occur where $f'(x) = 0$	**M1**	
	$(x+1)^2 = 1$	**A1**	
	Giving (0,4) and (– 2,0) cao	**A1A1**	
(c)(i)	$f''(x) = \frac{2}{(x+1)^3}$	**B1**	
(ii)	$f''(0) = 2$ therefore (0,4) is a minimum	**B1**	B1 FT for deriv = $\frac{k}{(x+1)^3}$
	$f''(-2) = -2$ therefore (– 2,0) is a maximum	**B1**	
(d)		**G1** **G1** **G1**	G1 each branch, G1 asymptotes correctly positioned cao
(e)	Consider		
	$x + 3 + \frac{1}{x+1} = 5$	**M1**	
	$x^2 - x - 1 = 0$	**A1**	
	x = 1.618, – 0.618	**A1**	
	f^{-1}(S) = [– 0.618, 1.618]	**A1**	Accept $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$