GCE A LEVEL MARKING SCHEME

SUMMER 2022

A LEVEL (NEW)
FURTHER MATHEMATICS
UNIT 6 FURTHER MECHANICS B
1305U60-1

INTRODUCTION

This marking scheme was used by WJEC for the 2022 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

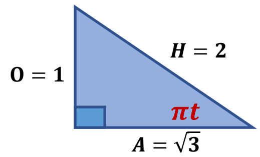
WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL FURTHER MATHEMATICS

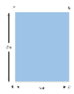
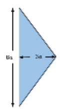
UNIT 6 FURTHER MECHANICS B

SUMMER 2022 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	$a = v\frac{\mathrm{d}v}{\mathrm{d}x}$	M1	Used
	$\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{96}{(4x+9)^2}$	B1	
	$a = \frac{24}{4x+9} \times -24(4x+9)^{-2} \times 4$	A1	cao, isw
	$a = -\frac{2304}{(4x+9)^3}$	**[3]**	
(b)	(i) $-\frac{4}{3} = -\frac{2304}{(4x+9)^3}$	M1	FT their a from part (a)
	$4x + 9 = \sqrt[3]{1728}$	m1	Only FT $ax + b = \sqrt[3]{c}$ from the form $-\frac{4}{3} = \frac{k}{(4x+9)^3}$
	$x = \frac{3}{4}$	A1	cao
	(ii) $v = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{24}{4x+9}$		
	$\int (4x+9)\mathrm{d}x = 24\int \mathrm{d}t$	M1	Separation of variables
	$2x^2 + 9x = 24t\ (+C)$	A1	All correct
	When $t = 0, x = -2$ $(\Rightarrow\ C = -10)$	m1	Use of initial conditions
	$t = \frac{1}{24}(2x^2 + 9x + 10)$ or $t = \frac{1}{12}x^2 + \frac{3}{8}x + \frac{5}{12}$	A1	Correct expression only $(t =)$
	Substitute x from (i) into expression for t above $T = \frac{1}{24}\left(2\left(\frac{3}{4}\right)^2 + 9\left(\frac{3}{4}\right) + 10\right)$	M1	Sub. their x into their t expression involving x and t
	$T = \frac{143}{192} = 0\cdot74(4791\ldots)$	A1 **[9]**	FT their x if used in the correct expression only
	Total for Question 1	**12**	

Q2	Solution	Mark	Notes
(a)	(i) $x = \sin(\pi t) + \sqrt{3}\cos(\pi t)$.		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = v = \pi\cos(\pi t) - \sqrt{3}\,\pi\sin(\pi t)$	B1	$\dot{x}, v = \cdots$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\pi^2\sin(\pi t) - \sqrt{3}\,\pi^2\cos(\pi t)$	M1	$\ddot{x}, \dot{v}, a = \cdots$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\pi^2 x$	A1	Convincing
	$\therefore$ motion is SHM (with $\omega = \pi$)		
	Value of x at the centre of motion $= 0$	B1	
	(ii) Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$ (s)	B1	Convincing
	Amplitude, $a =$ value of x when $v = 0$ $\pi\cos(\pi t) - \sqrt{3}\,\pi\sin(\pi t) = 0$	M1	FT their v
	$\tan(\pi t) = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3}\right)$		
	$\sin(\pi t) = \frac{1}{2}$ or $\cos(\pi t) = \frac{\sqrt{3}}{2}$ **OR** $x\|_{t=\frac{1}{6}}$	m1	Either trig. ratio **OR** sub. $t = \frac{1}{6}$ into x
	$a = \left(\frac{1}{2}\right) + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$		
	$a = 2$ (m)	A1	cao 
		[8]	
(b)	Q has same period as $P \Rightarrow \omega = \pi$ amplitude is a		Condone repeated use of a
	$v^2 = \omega^2(a^2 - x^2)$, $\omega = \pi$, $x = \pm 2\sqrt{3}$, $v = \pm 2\pi$	M1	FT their $\omega = k\pi$
	$(2\pi)^2 = \pi^2\left(a^2 - \left(2\sqrt{3}\right)^2\right)$,	A1	Correct equation
	$a = 4$ (m)	A1	cao
		[3]	
(c)	$x = \pm 4\sin(\pi t)$	M1	Allow $\pm a\cos(\pi t)$, a from part (b)
	$\sin(\pi t) + \sqrt{3}\cos(\pi t) = \pm 4\sin(\pi t)$	m1	RHS $= \pm a\cos(\pi t)$
	$\tan(\pi t) = \frac{\sqrt{3}}{3}$ or $\tan(\pi t) = -\frac{\sqrt{3}}{5}$	A1	
	$t = \frac{1}{6} = 0\cdot16(66\ldots)$ or $t = 0\cdot89(385\ldots)$	A1	cao
		[4]	
	Total for Question 2	**15**	

<table>
<tr><th>Q3</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>(a)</td><td>$(\bar{y} =)\quad 4a$</td><td>B1

[1]</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Shape</th><th>Area/mass</th><th>Distance from AE</th></tr>
<tr><td></td><td>$8a \times 6a$
$(= 48a^2)$</td><td>$3a$</td></tr>
<tr><td></td><td>$\frac{8a \times 3a}{2}$
$(12a^2)$</td><td>$6a + \frac{1}{3}(3a)\ (= 7a)$</td></tr>
<tr><td></td><td>$\frac{\pi(3a)^2}{2}$
$\left(= \frac{9\pi a^2}{2}\right)$</td><td>$\frac{4(3a)}{3\pi}\left(= \frac{4a}{\pi}\right)$</td></tr>
<tr><td>Lamina</td><td>$a^2\left(60 - \frac{9\pi}{2}\right)$</td><td>$\bar{x}$</td></tr>
</table>

Moments about AE

$a^2\left(60 - \frac{9\pi}{2}\right)\bar{x} = (48a^2)(3a) + (12a^2)(7a) - \left(\frac{9\pi a^2}{2}\right)\left(\frac{4a}{\pi}\right)$

$\left(\frac{120 - 9\pi}{2}\right)\bar{x} = 144a + 84a - 18a$

$\bar{x} = \frac{140}{40 - 3\pi}a$
</td><td>B3

B1

M1

A1

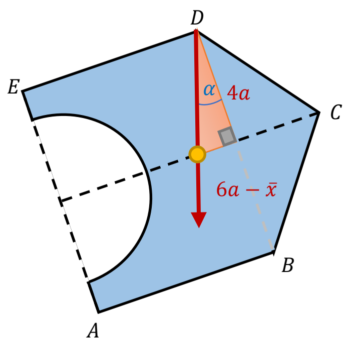
A1

[7]</td><td>Candidates may legitimately include a ρ term for mass per unit area

B3 6
B2 any 4 or 5,
B1 any 2 or 3 correct

Allow $-\frac{\pi(3a)^2}{2}$ or $-\frac{4(3a)}{3\pi}$

Masses and moments consistent
All terms, allow one sign error

FT Correct for their table, provided semicircle is subtracted in lamina area and moment
$\bar{x} = \frac{420}{120 - 9\pi}a$
Convincing</td></tr>
<tr><td>(c)</td><td>(i)


If hanging in equilibrium, vertical passes through centre of mass.

$\alpha = \tan^{-1}\left(\frac{6a - \bar{x}}{4a}\right)$ OR $\alpha = \tan^{-1}\left(\frac{4a}{6a - \bar{x}}\right)$

$\alpha = 90 - 70 \cdot 44(07\ldots)^{\circ}$

$\alpha = 19 \cdot 55(92\ldots)^{\circ}$
</td><td>M1

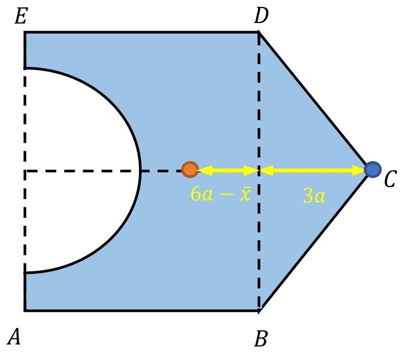
A1

A1</td><td>Correct triangle identified
Condone missing a's

Note that
$6a - \bar{x} = \left(\frac{100 - 18\pi}{40 - 3\pi}\right)a$
$= (1 \cdot 4211\ldots)a$

cso, accept answers rounding to
$\theta = 19^{\circ}$ or 20°</td></tr>
</table>

(ii)



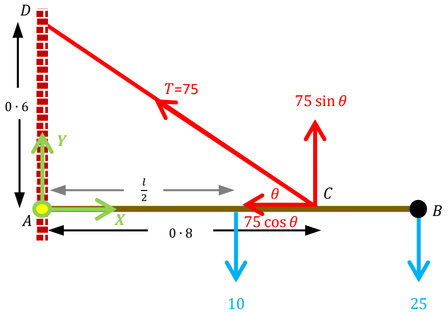
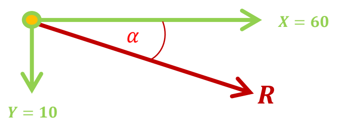
	Mark	Notes
Moments about BD	M1	Condone missing a's
$M \times \left(6 - \dfrac{140}{40 - 3\pi}\right) a = kM \times 3a$	A1	$M \times (6 - \bar{x})a = kM \times 3a$
$k = \frac{1}{3}\left(6 - \dfrac{140}{40 - 3\pi}\right)$		
$k = \frac{1}{3}(1 \cdot 42 \ldots)$		
$k = 0 \cdot 47(37 \ldots) \qquad = \frac{1}{3}\left(\frac{100 - 18\pi}{40 - 3\pi}\right)$	A1 **[6]**	cso, accept answers rounding to $k = 0 \cdot 47$

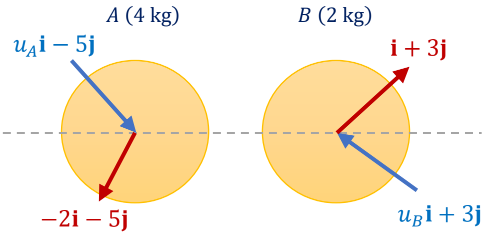
<u>Alternative Solution</u>

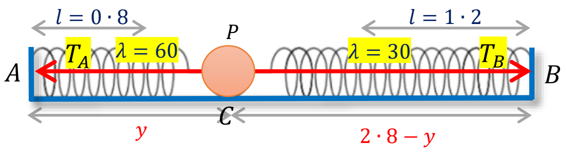
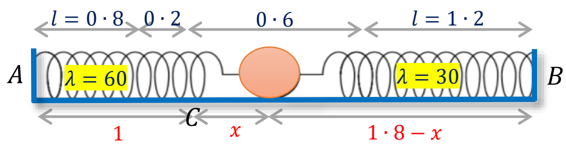
Shape	Area/mass	Distance from AE	Distance from BD
Lamina	M	$\bar{x}$	$6a - \bar{x}$
Particle	kM	$9a$	$3a$
New Lamina	$(k + 1)M$	$6a$	0

	Mark	Notes
Moments about AE	M1	Condone missing a's
$(k + 1)M \times 6a = M \times \bar{x} + kM \times 9a$	A1	
$k = \frac{1}{3}\left(6 - \dfrac{140}{40 - 3\pi}\right)$		
$k = \frac{1}{3}(1 \cdot 42 \ldots)$		
$k = 0 \cdot 47(37 \ldots) \qquad = \frac{1}{3}\left(\frac{100 - 18\pi}{40 - 3\pi}\right)$	A1	cso, accept answers rounding to $k = 0 \cdot 47$

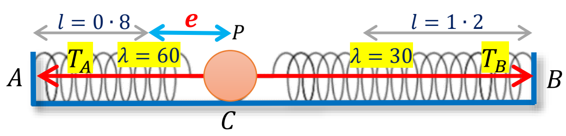
Total for Question 3 **14**

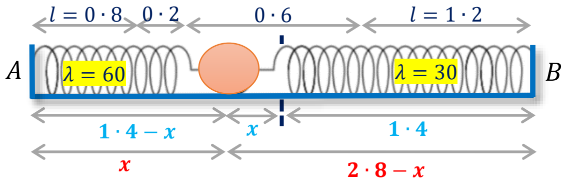
Q4	Solution	Mark	Notes
(a)	 		length of rod $AB = l$ $\sin\theta = 0\cdot6$ $\cos\theta = 0\cdot8$
	Moments about A	M1	Dim. correct equation with 3 terms
	$75\sin\theta \times 0\cdot8 = 10 \times \frac{l}{2} + 25 \times l$	A1 A1	-1 each error
	$l = 1\cdot2 \quad \text{(m)}$	A1	cao
		[4]	
(b)	Resolve vertically $Y + 75\sin\theta = 10 + 25$ $Y = -10 \quad \text{(N)}$ *Y* pointing downwards $(75\sin\theta = Y + 10 + 25)$ $Y = 10$	M1 A1	Dim. correct equation, no extra/missing forces
	Resolve horizontally $X = 75\cos\theta$ $X = 60 \quad \text{(N)}$	M1 A1	Dim. correct equation, no extra forces
	$R = \sqrt{60^2 + 10^2}$	m1	Provided both M's awarded, FT their X and Y
	$R = 10\sqrt{37} = 60\cdot82(76\ldots)$ (N)	A1	cao
	 		
	$\tan\alpha = \frac{10}{60}$	m1	Provided both M's awarded, FT their X and Y
	$\alpha = 9\cdot46(23\ldots)°$ **below the horizontal**	A1	cao
		[8]	
	Total for Question 4	12	

Q5	Solution	Mark	Notes
(a)	 		Before collision After collision $e = \frac{2}{5}$
	Con. of momentum (along line of centres)	M1	Attempted. Allow 1 sign error. $4(u_A\mathbf{i} - 5\mathbf{j}) + 2(u_B\mathbf{i} + 3\mathbf{j}) = 4(-2\mathbf{i} - 5\mathbf{j}) + 2(\mathbf{i} + 3\mathbf{j})$
	$4u_A + 2u_B = 4(-2) + 2(1)$	A1	All correct, oe
	$(2u_A + u_B = -3)$ $\quad 4u_A\mathbf{i} + 2u_B\mathbf{i} = -6\mathbf{i}$		Condone **i**'s, i.e.
	Restitution (along line of centres)	M1	Attempted. Allow 1 sign error.
	$(1) - (-2) = -\frac{2}{5}(u_B - u_A)$	A1	All correct, condone **i**'s,
	$(2u_A - 2u_B = 15)$ $\quad 4u_A\mathbf{i} + 2u_B\mathbf{i} = -6\mathbf{i}$		$\frac{2}{5} = -\frac{1--2}{u_B - u_A} = \frac{1--2}{u_A - u_B}$
	Solving equations	m1	One variable eliminated
	$u_A = \frac{3}{2}$ $\quad u_B = -6$		
	Velocities before collision		
	Sphere $A = \frac{3}{2}\mathbf{i} - 5\mathbf{j}$ $\quad (\text{ms}^{-1})$	A1	cao
	Sphere $B = -6\mathbf{i} + 3\mathbf{j}$ $\quad (\text{ms}^{-1})$	A1 **[7]**	cao
(b)	Wall is parallel to vector **i** since impulse only has a **j** component	B1 **[1]**	Parallel to vector **i** since … • No **i** component • No momentum in **i** direction • Perpendicular to wall
(c)	Impulse, $\mathbf{I}$ = change in momentum $32\mathbf{j} = 4\mathbf{v} - 4(-2\mathbf{i} - 5\mathbf{j})$	M1	Used, $32\mathbf{j} = -4\mathbf{v} + 4(-2\mathbf{i} - 5\mathbf{j})$ $32 = 4v - 4(-5)$ Condone **j**'s on the above
	$\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$	A1	
	speed $= \sqrt{2^2 + 3^2}$ $= \sqrt{13}$ $\quad (\text{ms}^{-1})$ $\quad$ or $\quad = 3 \cdot 60(55 \ldots)$	B1 **[3]**	FT their $\sqrt{13}$ derived from $\mathbf{v} = -2\mathbf{i} + a\mathbf{j},\ a \neq 0$
(d)	Loss in KE $= \frac{1}{2}(4)(2^2 + 5^2) - \frac{1}{2}(4)\left(\sqrt{13}^2\right)$ OR Loss in KE $= \frac{1}{2}(4)(5^2) - \frac{1}{2}(4)(3^2)$	M1	Difference in KE, any order At least one v^2 correct
	Loss in KE $= 32$ $\quad$ (J)	A1 **[2]**	FT provided loss (in KE) >0
	Total for Question 5	13	

Q6	Solution	Mark	Notes
			$AB = 2\cdot 8$ m
(a)	Let $AC = y$		Use of Hooke's Law
	$T_A = \frac{60(y-0\cdot 8)}{0\cdot 8}$ $(= 75y - 60)$	M1	$\frac{60\,dist}{0\cdot 8}$ or $\frac{30\,dist}{1\cdot 2}$ Any algebraic extension/distance
	$T_B = \frac{30(2\cdot 8 - 1\cdot 2 - y)}{1\cdot 2}$ $(= 40 - 25y)$	A1	T_B or T_A correct
	In equilibrium, $T_A = T_B$	m1	
	$\frac{60(y-0\cdot 8)}{0\cdot 8} = \frac{30(2\cdot 8 - 1\cdot 2 - y)}{1\cdot 2}$		
	$75y - 60 = 40 - 25y$		
	$y = 1$ (m)	A1	Convincing
		[4]	
(b)			$AB = 2\cdot 8$ m
	(i) Let x denote the displacement of P from C		
	$T_A = \frac{60(0\cdot 2 + x)}{0\cdot 8}$ $(= 15 + 75x)$	B1	either term, oe
	$T_B = \frac{30(0\cdot 6 - x)}{1\cdot 2}$ $(= 15 - 25x)$		
	Apply N2L to P,	M1	Dim. correct. T_B, T_A opposing
	$T_B - T_A = 4\frac{d^2x}{dt^2}$	A1	
	$\frac{30(0\cdot 6 - x)}{1\cdot 2} - \frac{60(0\cdot 2 + x)}{0\cdot 8} = 4\frac{d^2x}{dt^2}$		
	$-100x = 4\frac{d^2x}{dt^2}$		
	$\frac{d^2x}{dt^2} = -25x$	A1	Allow for any defined x, e.g. $\frac{d^2x}{dt^2} = -25(x-1)$
	∴ SHM with $\omega = 5$ (with centre at C)	B1	Must come from $\ddot{x} - \omega^2 x$
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{5}$	B1	FT ω

	(ii) Amplitude, $a = 1 \cdot 4 - 1 = 0 \cdot 4$ (m) Using $x = \pm a\cos\omega t$ with $a = 0 \cdot 4$, $\omega = 5$ $-0 \cdot 2 = 0 \cdot 4\cos 5t$ $t = \frac{2\pi}{15} = 0 \cdot 418(879\ldots)$ (s)	B1 M1 A1 A1 **[10]**	 Allow $x = \pm a\sin(\omega t)$ FT a and ω FT for $-0 \cdot 2 = a\cos\omega t$ cao
	Total for Question 6	**14**	

Q6	Alternative Solution	Mark	Notes
			$AB = 2 \cdot 8$ m
(a)	Let e = extension in AP $T_A = \frac{60}{0 \cdot 8}e \quad (= 75e)$ $T_B = \frac{30(0 \cdot 8 - e)}{1 \cdot 2} \quad (= 20 - 25e)$ In equilibrium, $T_A = T_B$ $\frac{60}{0 \cdot 8}e = \frac{30(0 \cdot 8 - e)}{1 \cdot 2}$ $75e = 20 - 25e \quad \Rightarrow \quad e = 0 \cdot 2$ $AC = 0 \cdot 8 + 0 \cdot 2 = 1$ (m)	 M1 A1 m1 A1 **[4]**	Use of Hooke's Law $\frac{60\,dist}{0 \cdot 8}$ or $\frac{30\,dist}{1 \cdot 2}$ Any algebraic distance/extension T_B or T_A correct Convincing

Q6	Alternative Solution	Mark	Notes
(b)			$AB = 2\cdot8\ \mathrm{m}$
	(i) Let x denote the displacement of P from • the midpoint of AB • A		
	$T_A = \frac{60(1\cdot4-0\cdot8-x)}{0\cdot8}$ $T_A = \frac{60(x-0\cdot8)}{0\cdot8}$		$T_A = 45 - 75x$ or $75x - 60$
		B1	either term, oe
	$T_B = \frac{30(1\cdot4-1\cdot2+x)}{1\cdot2}$ $T_B = \frac{30(2\cdot8-1\cdot2-x)}{1\cdot2}$		$T_B = 5 + 25x$ or $40 - 25x$
	Apply N2L to P,	M1	Dim. correct. T_B, T_A opposing
	$4\frac{d^2x}{dt^2} = \begin{cases} T_A - T_B \\ T_B - T_A \end{cases}$	A1	
	$4\frac{d^2x}{dt^2} = \begin{cases} \frac{60(1\cdot4-0\cdot8-x)}{0\cdot8} - \frac{30(1\cdot4-1\cdot2+x)}{1\cdot2} \\ \frac{30(2\cdot8-1\cdot2-x)}{1\cdot2} - \frac{60(x-0\cdot8)}{0\cdot8} \end{cases}$		
	$4\frac{d^2x}{dt^2} = \begin{cases} 40 - 100x \\ 100 - 100x \end{cases}$		
	$\frac{d^2x}{dt^2} = \begin{cases} -25(x - 0\cdot4) \\ -25(x - 1) \end{cases}$	A1	
	∴ SHM with $\omega = 5$ (with centre at $x = 0\cdot4$, i.e. C) (with centre at $x = 1$, i.e. C)	B1	
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{5}$	B1	FT ω
	(ii) Amplitude, $a = 1\cdot4 - 1 = 0\cdot4$ (m)	B1	
	Using $x - 0\cdot4 = \pm a\cos\omega t$ with $a = 0\cdot4$, $\omega = 5$	M1	Allow $x = \pm a\sin(\omega t)$ FT a and ω
	$0\cdot6 - 0\cdot4 = -0\cdot4\cos 5t$	A1	FT RHS with $x = 1\cdot4 - 0\cdot8$
	$t = \frac{2\pi}{15} = 0\cdot418(879\ldots)$ (s)	A1	cao
	OR		
	Using $x - 1 = \pm a\cos\omega t$ with $a = 0\cdot4$ $\omega = 5$	(M1)	
	$0\cdot8 = 1 + 0\cdot4\cos 5t$	(A1)	
	$-0\cdot2 = 0\cdot4\cos 5t$		
	$t = \frac{2\pi}{15} = 0\cdot418(879\ldots)$ (s)	(A1)	
		[10]	
	Total for Question 6	14	