GCE A LEVEL

1300U30-1

Z22-1300U30-1

TUESDAY, 7 JUNE 2022 – AFTERNOON

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

1300U301
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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid. You may use a pencil for graphs and diagrams only.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, for example, | 0 | 1 | . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

0 1 Solve the equation

$$6\sec^2 x - 8 = \tan x$$

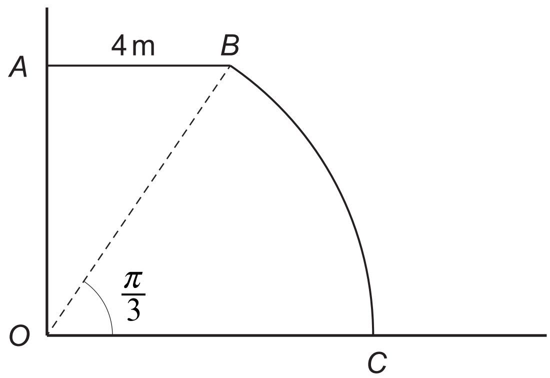
for $0° \leqslant x \leqslant 360°$. [6]

0 2 Differentiate the following functions with respect to x.

a) $x^3 \ln(5x)$ [3]

b) $(x + \cos 3x)^4$ [2]

0 3 The diagram below shows a plan of the patio Eric wants to build.



The walls *OA* and *OC* are perpendicular. The straight line *AB* is of length 4 m and is perpendicular to *OA*. The shape *OBC* is a sector of a circle with centre *O* and radius *OC*.

The angle *BOC* is $\frac{\pi}{3}$ radians. Calculate the area of the patio *OABC*. Give your answer correct to 2 decimal places. [4]

0 4 The sum to infinity of a geometric series with first term a and common ratio r is 120. The sum to infinity of another geometric series with first term a and common ratio $4r^2$ is $112\frac{1}{2}$. Find the possible values of r and the corresponding values of a. [6]

0 5 The function $f(x)$ is defined by

$$f(x) = \frac{6x+4}{(x-1)(x+1)(2x+3)}\ .$$

a) Express $f(x)$ in terms of partial fractions. [4]

b) Find $\int \frac{3x+2}{(x-1)(x+1)(2x+3)}\,\mathrm{d}x$, giving your answer in the form $a\ln|g(x)|$, where a is a real number and $g(x)$ is a function of x. [5]

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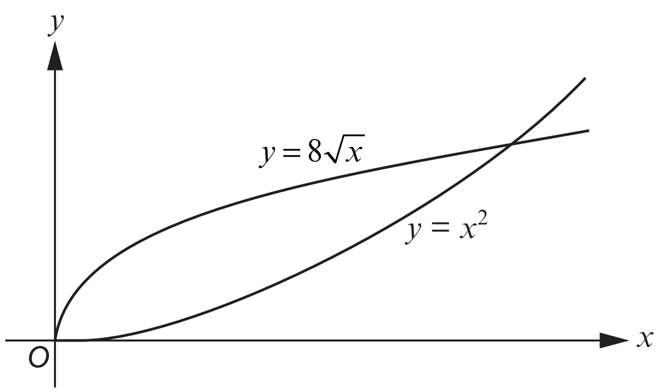
0 6 Geraint opens a savings account. He deposits £10 in the first month. In each subsequent month, the amount he deposits is 20 pence greater than the amount he deposited in the previous month.

a) Find the amount that Geraint deposits into the savings account in the 12th month. [2]

b) Determine the number of months it takes for the total amount in the savings account to reach £954. [3]

0 7 The diagram below shows a sketch of the curves $y = x^2$ and $y = 8\sqrt{x}$.



Find the area of the region bounded by the two curves. [7]

TURN OVER

0 8 Find the first three terms in the binomial expansion of $\dfrac{2-x}{\sqrt{1+3x}}$ in ascending powers of x. State the range of values of x for which the expansion is valid.

By writing $x=\dfrac{1}{22}$ in your expansion, find an approximate value for $\sqrt{22}$ in the form $\dfrac{a}{b}$, where a, b are integers whose values are to be found. [8]

0 9 For each of the following sequences, find the first 5 terms, u_1 to u_5. Describe the behaviour of each sequence.

a) $u_n=\sin\left(\dfrac{n\pi}{2}\right)$ [2]

b) $u_6=33,\ u_n=2u_{n-1}-1$ [3]

1 0 Solve the equation

$$\frac{6x^5-17x^4-5x^3+6x^2}{(3x+2)}=0\,.$$ [5]

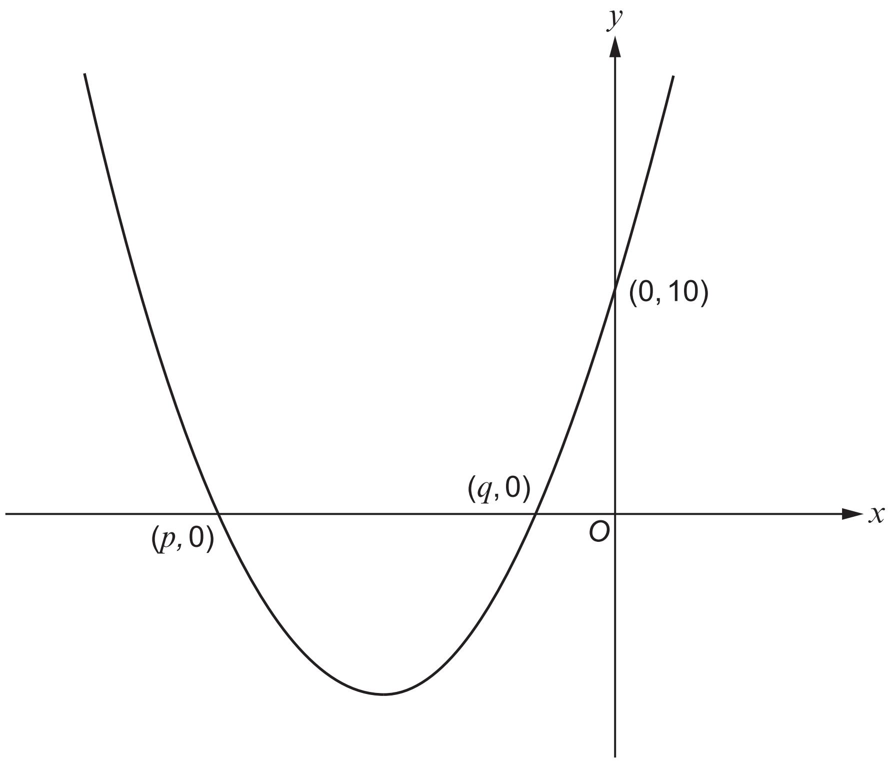
1 1 **a)** Express $9\cos x+40\sin x$ in the form $R\cos(x-\alpha)$, where R and α are constants with $R>0$ and $0°<\alpha<90°$. [3]

b) Find the maximum possible value of $\dfrac{12}{9\cos x+40\sin x+47}$. [2]

12 The diagram below shows a sketch of the graph of $y = f(x)$, where

$$f(x) = 2x^2 + 12x + 10.$$

The graph intersects the x-axis at the points $(p, 0)$, $(q, 0)$ and the y-axis at the point (0, 10).



a) Write down the value of $ff(p)$. [1]

b) Determine the values of p and q. [2]

c) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b, c are constants whose values are to be found. Write down the coordinates of the minimum point. [3]

d) Explain why $f^{-1}(x)$ does not exist. [1]

e) The function $g(x)$ is defined as

$$g(x) = f(x) \quad \text{for} \quad -3 \leqslant x < \infty.$$

i) Find an expression for $g^{-1}(x)$. [4]

ii) Sketch the graph of $y = g^{-1}(x)$, indicating the coordinates of the points where the graph intersects the x-axis and the y-axis. [2]

TURN OVER

1 3 A function is defined by $f(x) = 2x^3 + 3x - 5$.

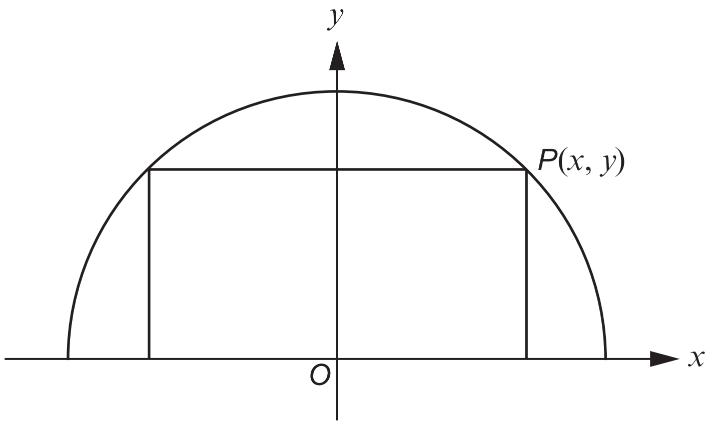
a) Prove that the graph of $f(x)$ does not have a stationary point. [2]

b) Show that the graph of $f(x)$ does have a point of inflection and find the coordinates of the point of inflection. [4]

c) Sketch the graph of $f(x)$. [1]

1 4 Evaluate the integral $\int_0^{\pi} x^2 \sin x \, \mathrm{d}x$. [6]

1 5 A rectangle is inscribed in a semicircle with centre *O* and radius 4. The point $P(x, y)$ is the vertex of the rectangle in the first quadrant as shown in the diagram.



a) Express the area A of the rectangle as a function of x. [2]

b) Show that the maximum value of A occurs when $y = x$. [7]

16 The parametric equations of the curve *C* are

$$x = 3 - 4t + t^2, \qquad y = (4 - t)^2.$$

a) Find the coordinates of the points where *C* meets the y-axis. [3]

b) Show that the x-axis is a tangent to the curve *C*. [5]

17 **a)** Prove that

$$\cos(\alpha - \beta) + \sin(\alpha + \beta) \equiv (\cos\alpha + \sin\alpha)(\cos\beta + \sin\beta).$$ [2]

b) **i)** Hence show that $\dfrac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta}$ can be expressed as $\cos\theta + \sin\theta$.

ii) Explain why $\dfrac{\cos 3\theta + \sin 5\theta}{\cos 4\theta + \sin 4\theta} \neq \cos\theta + \sin\theta$ when $\theta = \dfrac{3\pi}{16}$. [3]

18 **a)** Use a suitable substitution to find

$$\int \frac{x^2}{(x+3)^4}\,\mathrm{d}x.$$ [5]

b) Hence evaluate $\displaystyle\int_0^1 \frac{x^2}{(x+3)^4}\,\mathrm{d}x$. [2]

END OF PAPER