AS
FURTHER MATHEMATICS
7366/1

Paper 1

Mark scheme

June 2025

Version: 1.0 Final

256A7366/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the 3rd answer.	1.1b	B1	$7 - \mathrm{i}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles the 1st answer.	1.1b	B1	$3 - 5\mathrm{i}$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks the 3rd box.	2.2a	B1	$x = -1,\ x = 2,\ y = 1$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the 2nd box.	1.1b	B1	$1 - 2x^2$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains $-\dfrac{11}{3}$	1.1b	B1	$\alpha\beta\gamma\delta = -\dfrac{11}{3}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains $\dfrac{5}{3}$	1.1b	B1	$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \dfrac{5}{3}$
	Subtotal		**1**	

	Question total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Correct method for the modulus. or Correct method for the argument.	1.1a	M1	$z^2 = 36(\cos 2.4 + \mathrm{i}\sin 2.4)$
	Obtains $36(\cos 2.4 + \mathrm{i}\sin 2.4)$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Correct method for the modulus. or Correct method for the argument.	1.1a	M1	$\frac{z}{w} = 3(\cos 0.8 + \mathrm{i}\sin 0.8)$
	Obtains $3(\cos 0.8 + \mathrm{i}\sin 0.8)$	1.1b	A1	
	Subtotal		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Identifies the error, eg 3rd line.	2.3	B1	The error is in the 3rd line of working. Geraldine uses an incorrect definition of tanh. She should replace $\tanh y$ with $\dfrac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$
	Explains that $\tanh y$ should be $\dfrac{e^{y}-e^{-y}}{e^{y}+e^{-y}}$	2.4	E1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Substitutes $-\dfrac{24}{25}$ and uses the power law of logs. **PI** by $-\ln 7$	1.1a	M1	$\tanh^{-1}\left(-\dfrac{24}{25}\right) = \dfrac{1}{2}\ln\left(\dfrac{1-\dfrac{24}{25}}{1+\dfrac{24}{25}}\right)$ $= \dfrac{1}{2}\ln\left(\dfrac{1}{49}\right)$ $= \ln\sqrt{\dfrac{1}{49}}$ $= \ln\left(\dfrac{1}{7}\right)$
	Obtains $\ln\left(\dfrac{1}{7}\right)$	2.1	A1	
	Subtotal		**2**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Completes a rigorous argument to prove the required result. Must include the LHS, at least one intermediate stage, and the RHS. Accept the abbreviation LHS.	2.1	R1	$\frac{1}{r^2}-\frac{1}{(r+1)^2}=\frac{(r+1)^2-r^2}{r^2(r+1)^2}$ $=\frac{r^2+2r+1-r^2}{r^2(r+1)^2}$ $=\frac{2r+1}{r^2(r+1)^2}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Writes at least two pairs of subtracting fractions.	1.1a	M1	$\sum_{r=1}^{n}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=1}^{n}\left(\frac{1}{r^2}-\frac{1}{(r+1)^2}\right)$ $=\frac{1}{1^2}-\cancel{\frac{1}{2^2}}$ $+\cancel{\frac{1}{2^2}}-\cancel{\frac{1}{3^2}}$ $+$ .... $+\cancel{\frac{1}{(n-1)^2}}-\cancel{\frac{1}{n^2}}$ $+\cancel{\frac{1}{n^2}}-\frac{1}{(n+1)^2}$ $=1-\frac{1}{(n+1)^2}$ $=\frac{(n+1)^2-1}{(n+1)^2}$ $=\frac{n^2+2n}{(n+1)^2}$
	Writes the nth pair of fractions and at least one pair of cancelling fractions.	1.1a	M1	
	Completes a reasoned argument using the method of differences to reach the required result. This mark is only available if at least the first two pairs of fractions and the nth pair are shown. Accept $\frac{1n^2+2n}{(n+1)^2}$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Substitutes $r = 2c$ **or** $r = c - 1$ into their $\dfrac{an^2 + bn}{(n+1)^2}$	1.1a	M1	$\displaystyle\sum_{r=c}^{2c} \frac{2r+1}{r^2(r+1)^2} = \sum_{r=1}^{2c} \frac{2r+1}{r^2(r+1)^2} - \sum_{r=1}^{c-1} \frac{2r+1}{r^2(r+1)^2}$ $= \dfrac{(2c)^2 + 2(2c)}{(2c+1)^2} - \dfrac{(c-1)^2 + 2(c-1)}{c^2}$ $= \dfrac{4c^3(c+1) - (2c+1)^2(c^2 - 2c + 1 + 2c - 2)}{c^2(2c+1)^2}$
	Substitutes $r = 2c$ **and** $r = c - 1$ into their $\dfrac{an^2 + bn}{(n+1)^2}$ and subtracts.	3.1a	M1	$= \dfrac{4c^3(c+1) - (2c+1)^2(c^2 - 1)}{c^2(2c+1)^2}$ $= \dfrac{(c+1)\left(4c^3 - (4c^2 + 4c + 1)(c-1)\right)}{c^2(2c+1)^2}$
	Obtains $\dfrac{(3c+1)(c+1)}{c^2(2c+1)^2}$	2.1	R1	$= \dfrac{(c+1)\left(4c^3 - (4c^3 - 3c - 1)\right)}{c^2(2c+1)^2}$ $= \dfrac{(3c+1)(c+1)}{c^2(2c+1)^2}$
	Subtotal		**3**	

	Question total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9	Multiplies LHS and RHS by $(3x-9)^2$ or Subtracts one side from the other and correctly combines the fractions. or Cross multiplies and solves for x **PI** by obtaining at least one correct region or a correct quadratic factor or both $\frac{2}{3}$ and 4	3.1a	M1	$(2x+1)(3x-9) \geq (x-1)(3x-9)^2$ but $x \neq 3$ $0 \geq (3x-9)\left((x-1)(3x-9)-(2x+1)\right)$ $(3x-9)(3x^2-14x+8) \leq 0$ $(3x-9)(3x-2)(x-4) \leq 0$ Critical values are: $\frac{2}{3}$, 3 , 4 $x \leq \frac{2}{3}$, $3 < x \leq 4$
	Obtains a correct quadratic factor eg $3x^2-14x+8$ **PI** by obtaining $\frac{2}{3}$ and 4 or States $x \neq 3$ **PI** by $x > 3$	1.1a	M1	
	Obtains at least one correct region. Condone $3 \leq x \leq 4$	1.1a	M1	
	Obtains $x \leq \frac{2}{3}$, $3 < x \leq 4$	1.1b	A1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Writes down $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$	1.2	B1	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Obtains $\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$	1.1b	B1	$\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Finds a correct direction vector between A and their A' Do not award this mark if their 'direction' vector is used as a position vector.	2.2a	M1	$\mathbf{r} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$
	Writes a correct vector equation using accurate notation.	2.5	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)(i)	Writes down (2, 5, –2) Condone the position vector.	3.1a	B1	(2, 5, –2)
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)(ii)	Obtains $\sqrt{5}$	3.1a	B1	Shortest distance between l and B is $= \sqrt{(2-3)^2 + (5-7)^2 + (0)^2}$ $= \sqrt{5}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
10(d)(iii)	Correct method for the area of triangle ABA' for their part **(d)(ii)** **PI** by AWRT 8.9	3.1a	M1	Area $= \frac{1}{2}\times\left(4-(-4)\right)\times\sqrt{5}$ $= 4\sqrt{5}$
	Obtains $4\sqrt{5}$ Accept AWRT 8.94	2.2a	A1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Obtains at least three correct elements. May be unsimplified.	1.1a	M1	$\begin{bmatrix} \cos 150° & -\sin 150° \\ \sin 150° & \cos 150° \end{bmatrix}$ $= \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$
	Obtains $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$ Accept $\frac{1}{2}\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{bmatrix}$ or $-\frac{1}{2}\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
11(b)	Obtains at least three correct elements. May be unsimplified, eg $\begin{bmatrix} \frac{3}{4} - \frac{1}{4} & \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} & -\frac{1}{4} + \frac{3}{4} \end{bmatrix}$	1.1a	M1	$\mathbf{A}^2 = \begin{bmatrix} \cos 300° & -\sin 300° \\ \sin 300° & \cos 300° \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
	Obtains $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ Accept $\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
11(c)	Attempts a correct calculation for the rotation angle eg 150×5 oe **PI** by 750 or 30 or $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	3.1a	M1	750° – 720° = 30° Anticlockwise rotation of 30° about the origin
	Obtains the correct transformation. Accept anticlockwise rotation of 750° about the origin oe Accept omission of ‘anticlockwise’.	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
11(d)	Considers multiples of 150 and/or 360 eg list of three multiples eg $150n$ eg $360m$	3.1a	M1	LCM of 150 and 360 $= 2^3 \times 3^2 \times 5^2$ $= 1800$ $n = 1800 \div 150 = 12$
	Finds a common multiple of 150 and 360 eg 3600 or Writes an expression for n in terms of another integer, eg $n = \frac{360m}{150}$ where $m \in \mathbb{N}$ **PI** by an n-value which is a multiple of 12	1.1b	A1	
	Obtains 12	2.2a	A1	
	Subtotal		**3**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
12	Rewrites the given equation in exponential form.	1.1a	M1	$5\sinh x - 3\cosh x = 6$ $5\left(\frac{e^x - e^{-x}}{2}\right) - 3\left(\frac{e^x + e^{-x}}{2}\right) = 6$ $5e^x - 5e^{-x} - 3e^x - 3e^{-x} = 12$ $2e^x - 8e^{-x} = 12$ $e^x - 6 - 4e^{-x} = 0$ $e^{2x} - 6e^x - 4 = 0$ $(e^x - 3)^2 = 13$ $e^x = 3 \pm \sqrt{13}$ but $e^x > 0$ $x = \ln\left(3 + \sqrt{13}\right)$
	Obtains a correct 3-term exponential equation. **PI** by a correct value for e^x	1.1b	A1	
	Solves their 3-term quadratic equation in e^x to find a value for e^x May be unsimplified.	3.1a	M1	
	Obtains $3 + \sqrt{13}$ or $3 - \sqrt{13}$	1.1b	A1	
	Gives a correct reason for ignoring their negative value of e^x	2.4	E1F	
	Obtains $\ln\left(3 + \sqrt{13}\right)$ only.	1.1b	A1	
	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Writes a correct expression for V in terms of x Condone missing $\mathrm{d}x$	1.2	M1	$V = \pi\int_1^4 \left(\frac{m}{x}\right)^2 \mathrm{d}x$ $= \pi m^2\left[-\frac{1}{x}\right]_1^4$ $= \frac{3\pi m^2}{4}$
	Obtains $\frac{3}{4}m^2\pi$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Writes a correct expression for the stretched volume in terms of m (and x) eg $\pi\int_1^4 \left(\frac{5m}{x}\right)^2 \mathrm{d}x$ or Writes their part **(a)** $\times 5^2$	3.1a	M1	$\frac{3\pi m^2}{4} \times 5^2 = 6\pi$ $m^2 = \frac{8}{25}$ $m > 0$ so $m = \frac{2}{5}\sqrt{2}$
	Forms a correct equation in terms of m only or Forms the equation their part **(a)** $\times 5^n = 6\pi$ where $n = 1$, 2 or 3 **PI** by $m^2 = \frac{8}{5}$ or $m^2 = \frac{8}{125}$ oe	1.1a	M1	
	Obtains $\frac{2}{5}\sqrt{2}$ oe eg $\frac{\sqrt{8}}{5}$ Negative root does not have to be considered.	1.1b	A1	
	Subtotal		**3**	

	Question total		**5**	

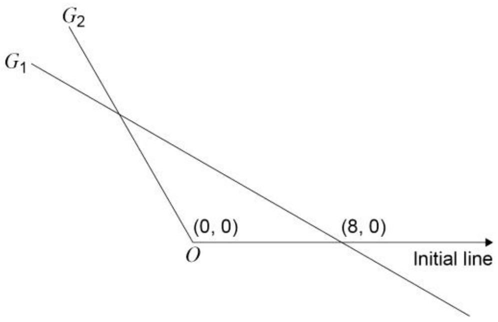
Q	Marking instructions	AO	Marks	Typical solution
14(a)(i)	Obtains $\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$	1.1b	B1	$\lvert\mathbf{A}\rvert = 4-3 = 1$ $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$
	Obtains $\begin{bmatrix} -1 & -2 \\ 1.5 & 2.5 \end{bmatrix}$ or $\frac{1}{2}\begin{bmatrix} -2 & -4 \\ 3 & 5 \end{bmatrix}$ or $-\frac{1}{2}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$	1.1b	B1	$\lvert\mathbf{B}\rvert = -10+12 = 2$ $\mathbf{B}^{-1} = \frac{1}{2}\begin{bmatrix} -2 & -4 \\ 3 & 5 \end{bmatrix}$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)(ii)	Obtains at least three correct elements of **AB** May be unsimplified.	1.1a	M1	$\mathbf{AB} = \begin{bmatrix} 4 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ -3 & -2 \end{bmatrix}$ $= \begin{bmatrix} 29 & 22 \\ -8 & -6 \end{bmatrix}$ $(\mathbf{AB})^{-1} = \frac{1}{2}\begin{bmatrix} -6 & -22 \\ 8 & 29 \end{bmatrix}$
	Correctly verifies that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ Condone missing conclusion.	2.2a	R1	$\mathbf{B}^{-1}\mathbf{A}^{-1} = \frac{1}{2}\begin{bmatrix} -2 & -4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$ $= \frac{1}{2}\begin{bmatrix} -6 & -22 \\ 8 & 29 \end{bmatrix}$ so $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	Writes $(\mathbf{M}^{-1})^1 = \mathbf{M}^{-1}$ and $(\mathbf{M}^1)^{-1} = \mathbf{M}^{-1}$	2.2a	B1	When $n = 1$: $(\mathbf{M}^{-1})^1 = \mathbf{M}^{-1}$ and $(\mathbf{M}^1)^{-1} = \mathbf{M}^{-1}$ so, the rule is true for $n = 1$ Assume the rule is true for $n = k$ $(\mathbf{M}^{-1})^k = (\mathbf{M}^k)^{-1}$ $(\mathbf{M}^{-1})^k\,\mathbf{M}^{-1} = (\mathbf{M}^k)^{-1}\,\mathbf{M}^{-1}$ $(\mathbf{M}^{-1})^{k+1} = (\mathbf{M}\mathbf{M}^k)^{-1}$ $= (\mathbf{M}^{k+1})^{-1}$ so, the rule is also true for $n = k + 1$ So, by induction, $(\mathbf{M}^{-1})^n = (\mathbf{M}^n)^{-1}$ is true for all $n \in \mathbb{N}$
	Assumes $(\mathbf{M}^{-1})^k = (\mathbf{M}^k)^{-1}$ and considers $(\mathbf{M}^{-1})^k\,\mathbf{M}^{-1}$ or $\mathbf{M}^{-1}(\mathbf{M}^{-1})^k$	2.4	M1	
	Completes working to show $(\mathbf{M}^k)^{-1}\mathbf{M}^{-1}$ or $\mathbf{M}^{-1}(\mathbf{M}^{-1})^k$ is equivalent to $(\mathbf{M}^{k+1})^{-1}$	2.2a	A1	
	Completes a reasoned argument by stating that the rule is true for $n = 1$ **and** if the rule is true for $n = k$ then it is also true for $n = k + 1$ **and** concludes that (by induction) $(\mathbf{M}^{-1})^n = (\mathbf{M}^n)^{-1}$ is true for all $n \in \mathbb{N}$ This mark is dependent on all previous marks. The algebra must use an alternative letter to n Condone reference to ‘rule’/‘statement’/‘it’ in the concluding statement.	2.1	R1	
	Subtotal		**4**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Rearranges the equation in terms of $r\sin\theta$ and $r\cos\theta$	3.1a	M1	$r = \dfrac{8}{\cos\theta + \sqrt{3}\sin\theta}$ $r\cos\theta + \sqrt{3}r\sin\theta = 8$ $x + \sqrt{3}y = 8$ $y = -\dfrac{1}{\sqrt{3}}x + \dfrac{8}{\sqrt{3}}$
	Replaces $r\cos\theta$ with x and $r\sin\theta$ with y	1.2	M1	
	Obtains $y = -\dfrac{1}{\sqrt{3}}x + \dfrac{8}{\sqrt{3}}$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Draws a straight line through the 2nd, 1st and 4th quadrants.	1.1b	B1	When $\theta = 0$ then $r = \dfrac{8}{\cos 0 + \sqrt{3}\sin 0} = 8$ 
	Gives the initial line intersection as (8, 0) or Writes 8 next to the initial line intersection.	1.1b	B1	
	Draws a straight line from O into the 2nd quadrant. Must intersect G_1 if drawn.	1.1b	B1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)	Substitutes $\theta = \frac{2\pi}{3}$ into the RHS of the G_1 polar equation.	3.1a	M1	$r = \dfrac{8}{\cos\frac{2\pi}{3} + \sqrt{3}\sin\frac{2\pi}{3}}$ $= 8$ Intersection point = $\left(8, \frac{2\pi}{3}\right)$
	Obtains 8	1.1b	A1	
	Obtains $\left(8, \frac{2\pi}{3}\right)$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(d)	Shows a full method for the area of the triangle. Accept $\frac{1}{2} \times 8 \times 8 \times \sin 120$	3.1a	M1	area = $\frac{1}{2} \times 8 \times 8 \times \sin\left(\frac{2\pi}{3}\right)$ $= 16\sqrt{3}$
	Obtains $16\sqrt{3}$ Accept AWRT 27.7 FT their 8's to at least 3sf if approximated	1.1b	A1F	
	Subtotal		**2**	

	Question total		**11**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	Completes the square to obtain $(w+2)^2 = -5$ or Substitutes $a = 1$, $b = 4$, $c = 9$ into the quadratic formula. or Calculates the sum and product of the roots. or Substitutes at least one of the given roots into the equation. or Expands $\left(w-\left(-2+\mathrm{i}\sqrt{5}\right)\right)\left(w-\left(-2-\mathrm{i}\sqrt{5}\right)\right)$ or Rearranges the equation $w=-2\pm\mathrm{i}\sqrt{5}$ to obtain $(w+2)^2 = -5$	1.1a	M1	$w^2 + 4w + 4 = -5$ $(w+2)^2 = -5$ $w + 2 = \pm\sqrt{-5}$ $w = -2 \pm \mathrm{i}\sqrt{5}$
	Completes a reasoned argument to prove the required result. Must include a conclusion which could be $w^2+4w+9=0$ or $w=-2\pm\mathrm{i}\sqrt{5}$	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)	Explains why the centre lies on the real axis. Condone an incomplete explanation, eg the real axis is the perpendicular bisector of the chord. Accept an algebraic proof that the imaginary part is 0	2.4	E1	Only points on the real axis are equidistant from $-2+\mathrm{i}\sqrt{5}$ and $-2-\mathrm{i}\sqrt{5}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(c)(i)	Forms a correct equation in terms of the centre of *C*	3.1a	M1	Let centre of *C* be x where $x \in \mathbb{R}$ $\sqrt{(x+2)^2+\sqrt{5}^2} = \sqrt{(7-x)^2+\sqrt{14}^2}$ $x^2+4x+9 = x^2-14x+63$ $18x = 54$ $x = 3$
	Obtains 3 Accept $3+0\mathrm{i}$ Condone $(3, 0)$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
16(c)(ii)	Uses their centre to form an expression for the radius of *C*	3.1a	M1	radius $= \sqrt{(7-3)^2+\sqrt{14}^2}$ $= \sqrt{30}$ Locus of z is $\lvert z-3\rvert = \sqrt{30}$
	Obtains a correct radius (excluding $\sqrt{5}$ and $\sqrt{14}$) for their centre. FT their real centre to any one of the given points.	1.1b	A1F	
	Obtains $\lvert z-3\rvert = \sqrt{30}$ FT their real centre and real radius. Note: M1A0A1 can be awarded.	3.2a	A1F	
	Subtotal		**3**	

	Question total		**8**	

	Question Paper total		**80**	