A-level
FURTHER MATHEMATICS
7367/3D

Paper 3 Discrete

Mark scheme

June 2025

Version: 1.0 Final

256A7367/3D/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles the 2nd answer	2.2a	B1	3
	Question total		**1**	

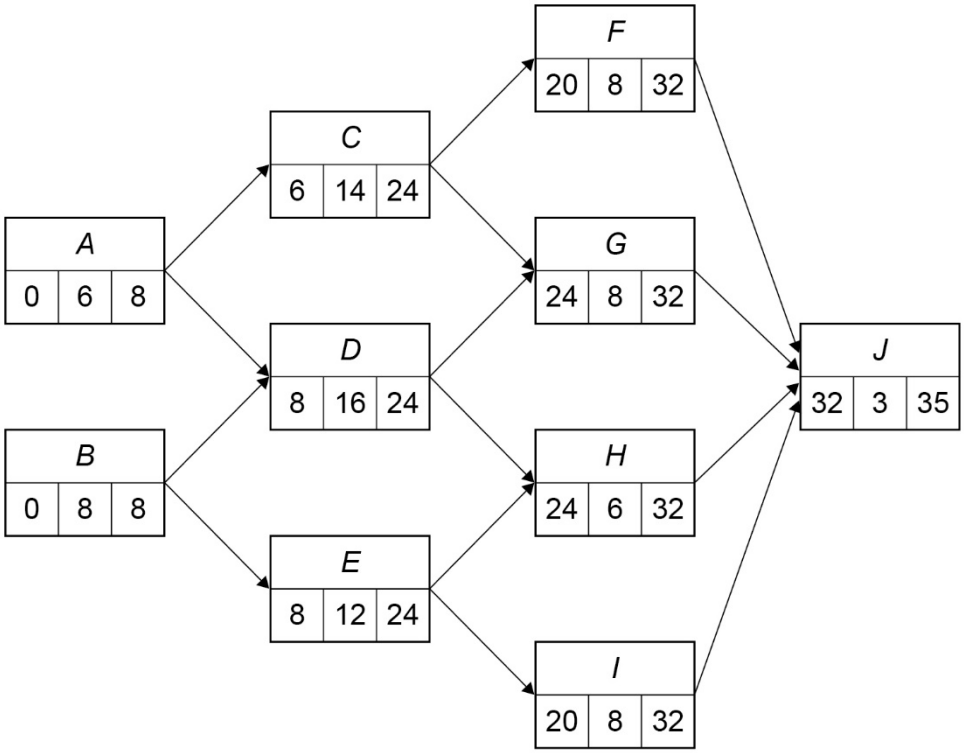
Q	Marking instructions	AO	Marks	Typical solution
2	Ticks the 3rd box	2.5	B1	$P = 25,\ x = 0,\ y = 5$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks the bottom right box	1.1b	B1	$*$ | e | a | b | c e | e | a | b | c a | a | e | c | b b | b | c | e | a c | c | b | a | e
	Question total		**1**	

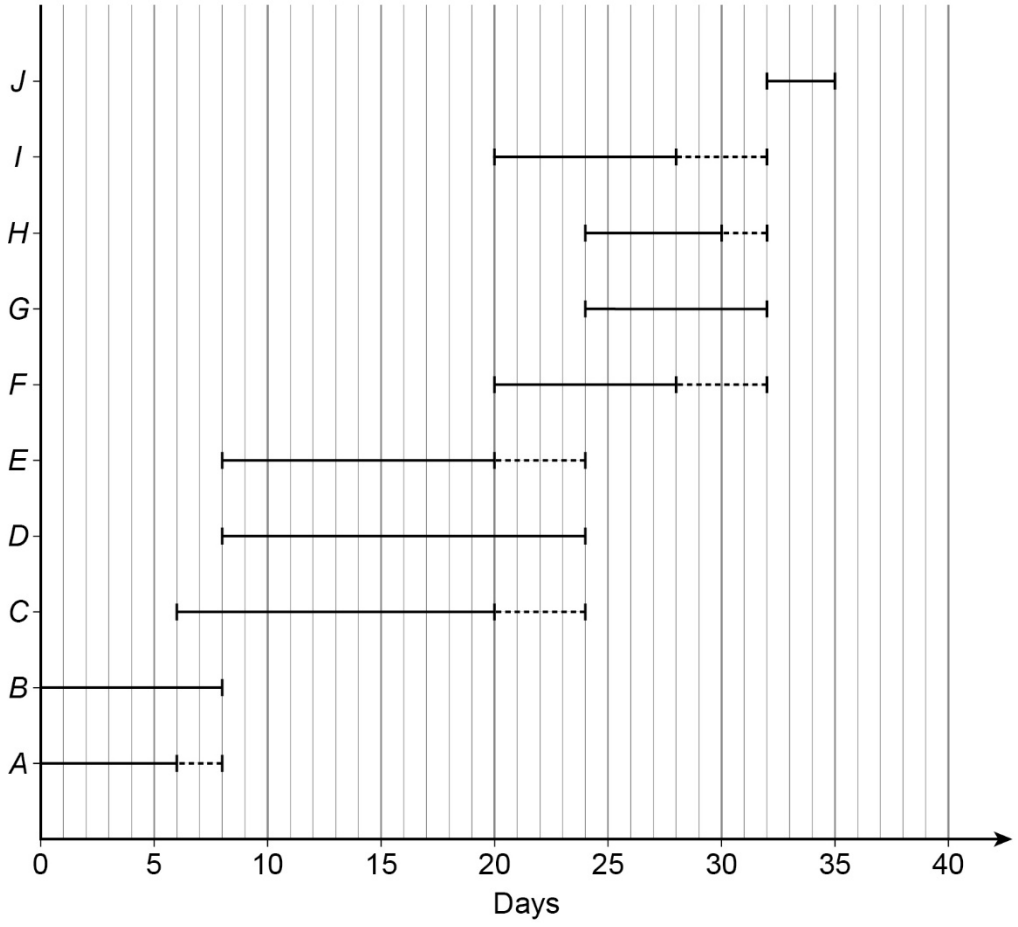
Q	Marking instructions	AO	Marks	Typical solution
4(a)	Identifies at least 2 correct arcs.	3.3	M1	Using Prim's algorithm starting from *B* *B*–*U*: 2.7 *N*–*U*: 1.4 *H*–*N*: 1.3 *L*–*N*: 3.4 $2.7 + 1.4 + 1.3 + 3.4 = 8.8$ miles
	Identifies the 4 correct arcs of the minimum spanning tree.	3.4	A1	
	Obtains 8.8 miles. Must have units.	3.2a	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	States that the connection between Hoole and Upton is not in their minimum spanning tree, provided their minimum spanning tree has at least 3 correct arcs or States that Hoole and Upton are not directly connected, provided their minimum spanning tree has at least 3 correct arcs	3.5a	E1F	The connection between Hoole and Upton is not in the minimum spanning tree. Hence, the minimum length of sewer pipes required is unaffected.
	Explains that the minimum length of sewer pipes does not change, provided their minimum spanning tree has at least 3 correct arcs	2.4	E1	
	Subtotal		**2**	

	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	Finds the correct earliest start time for each activity.	1.1b	B1	See below.
	Finds the correct latest finish time for each activity.	1.1b	B1	
				
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Determines the correct float for activity *F* Condone omission of units.	1.1b	B1	4 days
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Translates the information in the activity network into a diagram showing and labelling all 10 activities.	3.1a	M1	See below.
	All 4 critical activities shown correctly, either on individual rows or all on a single row.	1.1b	A1	
	Draws a fully correct diagram, including all floats.	1.1b	A1	
				
	Subtotal		**3**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains C and H for the nodes that connect to the supersource.	1.1b	B1	Nodes connected to supersource $= C$ and H Nodes connected to supersink $= D$ and G
	Obtains D and G for the nodes that connect to the supersink.	1.1b	B1	
	Subtotal		**2**	

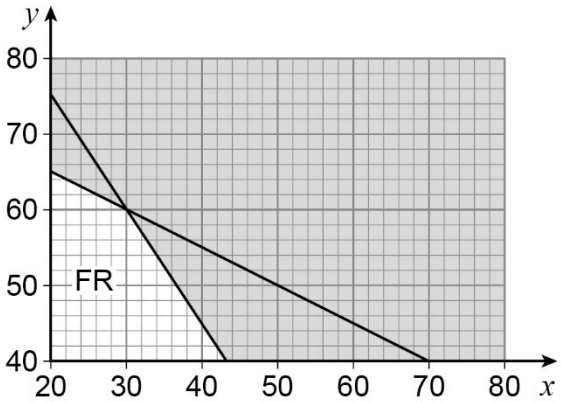
Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains 79 litres per second Condone missing of units.	1.1b	B1	$7 + 19 + 14 + 13 + 17 + 9$ $= 79$ litres per second
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(i)	Compares Andrew's flow of 79 with their value of the cut in **part (b)**.	2.4	M1	Andrew's flow of 79 has the same value as the cut $\{A,B,C,E,F,H\}\{D,G,I,J\}$ Hence, by the maximum flow-minimum cut theorem, the maximum flow through the network is 79 litres per second, meaning Andrew's claim is correct.
	Completes a reasoned argument with explicit reference to the maximum flow-minimum cut theorem, and concludes that Andrew's claim is correct.	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(c)(ii)	Gives a valid explanation with reference to the flows out of the sources and flows into the sinks not changing or Gives a valid explanation with reference to the value of the minimum cut not changing	2.4	E1	Adding a supersource and a supersink to a network does not change the minimum cut of the network, and so does not change the maximum flow through the network.
	Subtotal		**1**	

	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(i)	Draws the line $x + 2y = 150$ on the graph correctly.	1.1b	B1	See below.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)(ii)	Identifies their correct feasible region on the graph.	1.1b	B1F	See below.
				
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Draws an objective line or considers at least two of the vertices of their feasible region PI by correct answer	1.1a	M1	Using the objective line $y = -\frac{2}{3}x + c$ the optimal vertex is $(30, 60)$
	Finds the optimal vertex and deduces the correct number of malt loaves and rye loaves.	1.1b	A1	Hence, the bakery should make 30 malt loaves and 60 rye loaves each day.
	Subtotal		**2**	

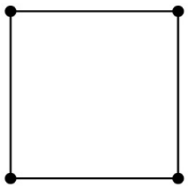
Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	Finds their maximum total profit for their number of malt loaves and rye loaves. Condone missing units.	3.2a	B1F	$P = 2 \times 30 + 3 \times 60$ $P = 240$ Hence, maximum total profit is £240
	Subtotal		**1**	

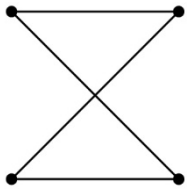
Q	Marking instructions	AO	Marks	Typical solution
7(b)(iii)	Infers a reason why the calculated maximum total profit may not be accurate.	2.2b	B1	A maximum total profit of £240 assumes that all loaves that are made each day are also sold. If all loaves are not sold then the profit will be less than £240
	Subtotal		**1**	

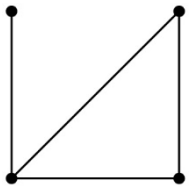
	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(i)	Recalls the formula $v-e+f=2$ for connected planar graphs.	1.2	B1	$v-e+f=2$ $n^2-(2n)+(3n-4)=2$ $n^2+n-6=0$ $(n-2)(n+3)=0$ Hence, $n=2,-3$ As n cannot be negative as the number of edges $2n$ cannot be negative, we can reject $n=-3$ Hence, $n=2$
	Translates the information about G into a three-term quadratic equation in n	3.1a	M1	
	Completes a reasoned argument to show that $n=2$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)(ii)	Finds the correct number of vertices, edges and faces for G	3.2a	B1	Number of vertices = 4 Number of edges = 4 Number of faces = 2
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)(i)	Draws a correct graph for *G* when *G* is also Eulerian.	1.1b	B1	
	Subtotal		**1**	

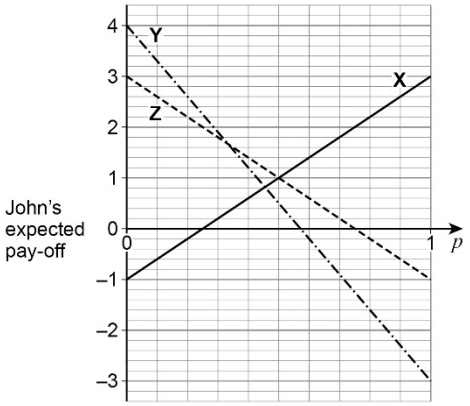
Q	Marking instructions	AO	Marks	Typical solution
8(b)(ii)	Redraws *G* as a bipartite graph or States that the vertices of *G* form two distinct sets or States *G* is isomorphic to $K_{2,2}$	1.1b	M1	*G* can be redrawn as  which shows that *G* is isomorphic to $K_{2,2}$
	Concludes that Alexie's claim is valid.	2.3	A1	Hence, Alexie's claim is valid.
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(i)	Draws a correct graph for *G* when *G* is also semi-Eulerian.	1.1b	B1	
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)(ii)	States that *G* contains a cycle or States that *G* has 4 vertices and 4 edges or States that *G* has 2 faces	1.1b	M1	*G* contains a cycle but a tree cannot contain a cycle, and so *G* cannot be a tree. Hence, Zackary's claim is not valid.
	Explains why *G* is not a tree and concludes that Zackary's claim is not valid.	2.3	A1	
	Subtotal		**2**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(i)	Deduces that strategy **B** dominates strategy **C**, so John should not play strategy **C**	2.2a	B1	John should never play strategy **C** since it is dominated by strategy **B**, as $-1 > -2$, $4 > 1$ and $3 > 1$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)(ii)	Introduces and defines a probability variable.	3.3	B1	Let John play strategy **A** with probability p and strategy **B** with probability $1-p$ If Elizabeth plays: **X**: expected gain for John $= 3p-(1-p) = 4p-1$ **Y**: expected gain for John $= -3p+4(1-p) = -7p+4$ **Z**: expected gain for John $= -p+3(1-p) = -4p+3$  $4p-1 = -7p+4$ $11p = 5$ $p = 5/11$ John should play strategy **A** with probability 5/11, strategy **B** with probability 6/11 and strategy **C** with probability 0
	Uses the probability variable to find at least one correct expected gain for John.	3.4	M1	
	Finds all three correct expected gains for John.	1.1b	A1	
	Constructs a graph with at least one vertical axis and plots at least two expected gains correctly. PI by correct value for probability variable	3.1a	M1	
	Identifies the correct optimal vertex from their graph and finds their value of the probability variable.	1.1b	A1F	
	Finds their correct optimal mixed strategy for John Condone omission of probability for playing strategy **C**	3.2a	A1F	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)	Obtains at least one correct inequality. Condone strict inequalities or incorrect orientation of inequalities.	3.1a	M1	$6p_1 + 3p_2 + 7p_3 \geq v$ $2p_1 + 4p_2 + p_3 \geq v$ $4p_1 + 3p_2 + 8p_3 \geq v$
	Obtains all three correct inequalities.	1.1b	A1	
	Subtotal		**2**	

	Question total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Shows that a general element of *G* combined with (0, 1) in either left-multiplication or right-multiplication remains unchanged.	1.1a	M1	$(a,b)\lozenge(0,1)$ $=(a\times 1+b\times 0, b\times 1-a\times 0)$ $=(a,b)$
	Completes a reasoned mathematical proof, including both left-multiplication and right-multiplication and a concluding statement that (0, 1) is the identity element of *G*	2.1	R1	$(0,1)\lozenge(a,b)$ $=(0\times b+1\times a, 1\times b-0\times a)$ $=(a,b)$ As $(a,b)\lozenge(0,1)=(a,b)$ and $(0,1)\lozenge(a,b)=(a,b)$ then $(0,1)$ is the identity element of *G*
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Sets up the condition for the inverse by considering a combination of the form $(a,b)\lozenge(c,d)=(0,1)$ or $(c,d)\lozenge(a,b)=(0,1)$ where (c,d) is the inverse of (a,b)	1.1a	M1	$(a,b)\lozenge(c,d)=(0,1)$ $(a\times d+b\times c, b\times d-a\times c)=(0,1)$ $ad+bc=0 \quad bd-ac=1$ $c=-\frac{ad}{b} \Rightarrow bd+\frac{a^2d}{b}=1$
	Forms two equations and eliminates an unknown (such as c or d) to find an equation for the other unknown (such as d or c)	1.1a	M1	$d=\frac{b}{a^2+b^2} \Rightarrow c=-\frac{a}{a^2+b^2}$ Hence the inverse of (a,b) is
	Deduces the correct inverse of (a,b) in terms of a and b	2.2a	A1	$\left(-\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$
	Subtotal		**3**	

	Question total		**5**	

	Question Paper total		**50**	