Rewarding Learning

ADVANCED
General Certificate of Education
2023

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AMT11]

TUESDAY 6 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1 **(i)** $A_{\text{sect}} = \frac{1}{2} r^2 \theta$		
$= \frac{1}{2}(7.2)^2(1.2)$	M1	
$= 31.104 \text{ cm}^2$	W1	
$= 31.1 \text{ cm}^2$ (3sf)		
(ii) $x^2 = 7.2^2 + 7.2^2 - 2(7.2)(7.2) \cos 1.2$	M1W1	
$x^2 = 66.1107\ldots$		
$x = 8.13$ cm (3sf)	W1	
(iii) $s = r\,\theta$		
$= 7.2(1.2)$	M1	
$= 8.64$	W1	
Perimeter = 8.64 + 8.13		
=16.8 cm (3sf)	MW1	8

2 **(a)** $|4x - 1| > 5$

AVAILABLE MARKS

$4x - 1 > 5$ M1W1

$4x > 6$

$x > \frac{6}{4}$ W1

$x > \frac{3}{2}$

or

$4x - 1 < -5$

$4x < -4$

$x < -1$ W1

(b) **(i)** $(1 + qx)^p = 1 - 6x + 24x^2 + \ldots$

$(1 + qx)^p = 1 + pqx + \frac{p(p-1)}{2!}(qx)^2 + \ldots$ M1W1

$-6 = pq$ MW1

$\frac{q^2(p^2 - p)}{2} = 24$ MW1

$p = -3$ M1W1

$q = 2$ W1

(ii) Valid for

$|2x| < 1$

$|x| < \frac{1}{2}$ MW1 12

3 **(i)** 4 strips = 5 ordinates,
$h = 0.25$ MW1

MW2

x	y
0	0
0.25	0.062986
0.5	0.265165
0.75	0.670739
1	1.414214

$A_R = \frac{0.25}{2}[0 + 1.414214 + 2(0.062986 + 0.265165 + 0.670739)]$ M1

$A_R = 0.426$ square units (3sf) W1

(ii) $y = x^2\sqrt{x^3 + 1}$

$y^2 = x^4(x^3 + 1)$ M1

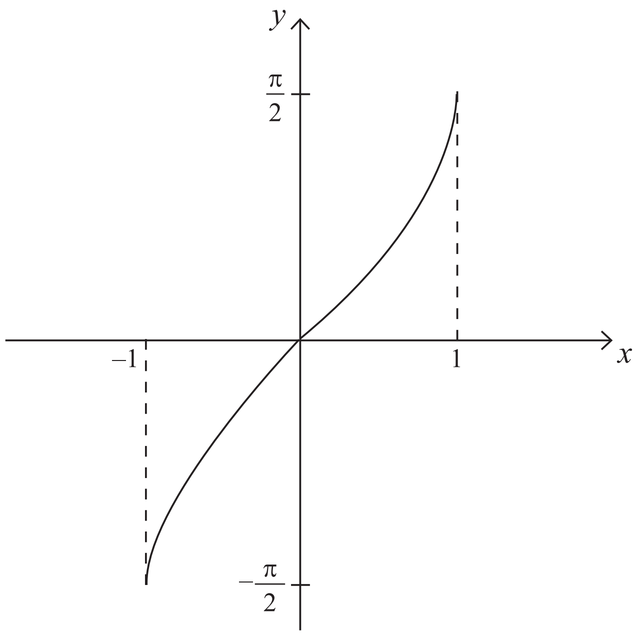
$y^2 = x^7 + x^4$ W1

$V = \pi \int_0^1 (x^7 + x^4)\,dx$ M1 M1

$V = \pi\left[\frac{1}{8}x^8 + \frac{1}{5}x^5\right]_0^1$ MW1

$V = \frac{13\pi}{40}$ cubic units W1 11

4 **(i)** M1 MW1 MW1



(ii) $-1 \leqslant x \leqslant 1, -\frac{\pi}{2} \leqslant f(x) \leqslant \frac{\pi}{2}$ MW2

(iii) $y = 3 + 2f(x)$

Stretch parallel to y-axis sf 2 M1 MW1
followed by
Translation of +3 parallel to the y-axis. MW1 8

AVAILABLE MARKS

5 **(i)** $\dfrac{25x}{(x-2)(x+3)^2} \equiv \dfrac{A}{(x-2)} + \dfrac{B}{(x+3)} + \dfrac{C}{(x+3)^2}$ M1

$25x \equiv A(x+3)^2 + B(x-2)(x+3) + C(x-2)$ MW1

When $x = -3$ M1

$-75 = 0A + 0B - 5C$

$C = 15$ W1

When $x = 2$

$50 = (5)^2 A + 0B + 0C$

$A = 2$ W1

Equate x^2 coefficients M1

$0 = A + B$

$B = -2$ W1

$\dfrac{25x}{(x-2)(x+3)^2} \equiv \dfrac{2}{(x-2)} - \dfrac{2}{(x+3)} + \dfrac{15}{(x+3)^2}$

(ii) $\displaystyle\int_0^1 \frac{2}{(x-2)} - \frac{2}{(x+3)} + 15(x+3)^{-2}\,dx$ M1

$= [2\ln|x-2| - 2\ln|x+3| - 15(x+3)^{-1}]_0^1$ MW3

$= \left[2\ln|-1| - 2\ln|4| - \dfrac{15}{4}\right] - [2\ln|-2| - 2\ln|3| - 5]$ M1

$= 2\ln\left(\dfrac{3}{8}\right) + \dfrac{5}{4}$ W1 13

				AVAILABLE MARKS
6	(i)	$S_n = \frac{n}{2}[2a + (n - 1)d]$		
		$3250 = \frac{n}{2}[200 + (n - 1)50]$	M1W1	
		$6500 = 200n + 50n^2 - 50n$		
		$0 = n^2 + 3n - 130$	W1	
		$0 = (n - 10)(n + 13)$		
		$n = 10$ or $n = -13$ (not possible)	W1	
		10 questions in total.	MW1	
	(ii)	JOHN		
		$2325 = \frac{6}{2}[2a + 5d]$	M1	
		$2325 = 6a + 15d$	W1	
		MARTHA		
		$825 = \frac{3}{2}[2a + 2d]$		
		$825 = 3a + 3d$	MW1	
		Suitable solution of 2 simultaneous equations	M1	
		$a = 200$, $d = 75$		
		First prize £200 then £75 increase for each subsequent question correct.	W2	11

		AVAILABLE MARKS
7 **(a)** $\cos(A + B) \equiv \cos A \ \cos B - \sin A \sin B$		
$\cos(A + A) \equiv \cos^2 A - \sin^2 A$	MW1	
$\cos^2 A \equiv 1 - \sin^2 A$	M1	
$\cos 2A \equiv 1 - \sin^2 A - \sin^2 A$ $\equiv 1 - 2 \sin^2 A$ as required	MW1	
(b) $\sin A = \dfrac{2}{\sqrt{7}}$ and A is acute		
$\tan A = \dfrac{2}{\sqrt{3}}$	MW1	
$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$	M1	
$= \dfrac{2\left(\dfrac{2}{\sqrt{3}}\right)}{1-\left(\dfrac{2}{\sqrt{3}}\right)^2}$		
$= -4\sqrt{3}$	W1	
(c) **(i)** $2 \cos 2x + 5 \sin 2x = R(\cos 2x \cos \alpha + \sin 2x \sin \alpha)$	MW1	
$\Rightarrow R \cos \alpha = 2$ and $R \sin \alpha = 5$	MW1	
$\Rightarrow \tan \alpha = \dfrac{5}{2}$	M1	
$\alpha = 68.2°$ (3sf)	W1	
Also $R = \sqrt{2^2 + 5^2}$	M1	
$\Rightarrow R = \sqrt{29}$	W1	
(ii) $\cos(2x - 68.1986) = \dfrac{3}{\sqrt{29}}$	M1	
related acute angle = 56.145…	MW1	
$2x - 68.1986 = \pm\ 56.145\ldots$		
$x = 6.03°, 62.2°$	W2	16

AVAILABLE MARKS

8 **(a)** $3y^2 + 2x - \frac{3x}{y} = 5$

$6y\frac{dy}{dx} + 2 - \left[\frac{\left(3y - 3x\frac{dy}{dx}\right)}{y^2}\right] = 0$ MW1 M1 W1
M1 W2

$6y^3\frac{dy}{dx} + 2y^2 - 3y + 3x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^3 + 3x) = 3y - 2y^2$ M1

$\frac{dy}{dx} = \frac{3y - 2y^2}{6y^3 + 3x}$ W1

(b) **(i)** $y = 3xe^{2x}$

$\frac{dy}{dx} = 3e^{2x} + 6xe^{2x}$ M1 W2

$= 3e^{2x}(1 + 2x)$

$\frac{dy}{dx} = 3e^{2x}(1 + 2x)$

When $x = -2$

$\frac{dy}{dx} = 3e^{-4}(1 - 4)$ M1

$\frac{dy}{dx} = -\frac{9}{e^4}$ W1

When $x = -2$,
$y = 3(-2)e^{-4}$

$y = -\frac{6}{e^4}$ MW1

Therefore
$y = mx + c$

$-\frac{6}{e^4} = -\frac{9}{e^4}(-2) + c$ M1

$c = -\frac{24}{e^4}$ W1

$y = -\frac{9}{e^4}x - \frac{24}{e^4}$

$e^4 y = -9x - 24$ MW1

AVAILABLE MARKS

(ii) Turning point when $\frac{dy}{dx} = 0$ M1

$0 = 3e^{2x}(1 + 2x)$

$3e^{2x} = 0$

no solution

$1 + 2x = 0$

$x = -\frac{1}{2}$ W1

When $x = -\frac{1}{2}$ (–0.5), $y = -\frac{3}{2e}$ (–0.552) MW1

Nature of turning points

$\frac{d^2y}{dx^2} = 6(1 + 2x)e^{2x} + 3e^{2x}(2)$ M1W1

$= 6e^{2x}(1 + 2x) + 6e^{2x}$

When $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} > 0$ MW1

Therefore minimum point at $\left(-\frac{1}{2}, -\frac{3}{2e}\right)$ M1

24

		AVAILABLE MARKS
9 **(a)** **(i)** Intersection $f(x) - g(x) = 0$		
$e^x - (x + 4) = 0$		
$e^x - x - 4 = 0$	MW1	
When $x = 1$		
$e - 5 = -2.2817$	MW1	
When $x = 2$		
$e^2 - 6 = 1.3890$	MW1	
Change of sign between $x = 1$ and $x = 2$ and continuous function indicates a root between these x values.	M1	
(ii) $h(x) = e^x - x - 4$	M1	
$h'(x) = e^x - 1$	M1 W1	
$x_{n+1} = x_n - \dfrac{h(x_n)}{h'(x_n)}$		
$x_1 = 1.5 - \dfrac{e^{1.5} - 1.5 - 4}{e^{1.5} - 1} = 1.792476$	M1W1	
$x_2 = 1.75$ (3sf)	MW1	
(b) $y = \cot t, \quad x = \operatorname{cosec}^2 t$		
$\dfrac{dy}{dt} = -\operatorname{cosec}^2 t$	M1 MW1	
$\dfrac{dx}{dt} = -2\operatorname{cosec} t \operatorname{cosec} t \cot t$	MW2	
$\dfrac{dy}{dx} = \dfrac{-\operatorname{cosec}^2 t}{-2\operatorname{cosec}^2 t \cot t}$	M1 W1	
$\dfrac{dy}{dx} = \dfrac{1}{2}\tan t$	MW1	17

				AVAILABLE MARKS
10	(i)	$\dfrac{dD}{dt} \propto D^2$	M1	
		$\dfrac{dD}{dt} = kD^2$	W1	
	(ii)	$\dfrac{dD}{dt} = kD^2$		
		$\displaystyle\int \frac{dD}{D^2} = \int k\ dt$	M1 W1	
		$-\dfrac{1}{D} = kt + c$	M1W2	
		$t = 0, D = 3, c = -\dfrac{1}{3}$	M1 W1	
		$t = 4, D = 1.2, k = -\dfrac{1}{8}$	MW1	
		$D = 0.5$		
		$-2 = -\dfrac{1}{8}t - \dfrac{1}{3}$	M1	
		$t = 13\frac{1}{3}$ minutes	W1	12
11	(a)	$\displaystyle\int_0^{\frac{\pi}{6}} (\sin x + \cos x)^2\ dx$		
		$= \displaystyle\int_0^{\frac{\pi}{6}} (\sin^2 x + 2 \sin x \cos x + \cos^2 x)\ dx$	MW1	
		$= \displaystyle\int_0^{\frac{\pi}{6}} (1 + \sin 2x)\ dx$	M1M1	
		$= \left[x - \dfrac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$	MW1 MW1	
		$= \left(\dfrac{\pi}{6} - \dfrac{1}{2}\ \cos\left(\dfrac{\pi}{3}\right)\right) - \left(0 - \dfrac{1}{2}\cos 0\right)$	M1	
		$= \dfrac{\pi}{6} + \dfrac{1}{4}$	W1	
	(b)	$\displaystyle\int_1^3 g(x) - f(x)\ dx$		
		$= \displaystyle\int_1^3 (x + 1 - x \ln x)\ dx$	MW1	
		$= \left[\dfrac{1}{2}x^2 + x - \left\{\dfrac{1}{2}x^2 \ln x - \displaystyle\int \frac{1}{2}x^2\left(\frac{1}{x}\right)dx\right\}\right]_1^3$	MW1 M2W2 MW2	
		$= \left[\dfrac{1}{2}x^2 + x - \dfrac{1}{2}x^2 \ln x + \dfrac{1}{4}x^2\ \right]_1^3$	MW1	
		$= \left[\dfrac{3}{4}(3)^2 + 3 - \dfrac{1}{2}(3)^2 \ln 3\right] - \left[\dfrac{3}{4} + 1 - \dfrac{1}{2}\ln 1\right]$	M1	
		$= \left(8 - \dfrac{9}{2}\ln 3\right)$ square units	W1	18
			Total	**150**