Rewarding Learning

ADVANCED
General Certificate of Education
2023

Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

TUESDAY 13 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

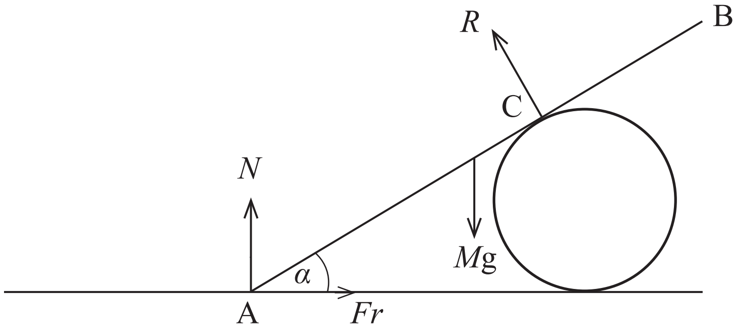
(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A: Mechanics

			AVAILABLE MARKS
1	**(i)** $(M + m) \times 0 = Mv + mu$	M2 W2	
	$Mv = -mu$		
	$v = -\dfrac{m}{M}u$		
	Speed $= \dfrac{m}{M}u \text{ ms}^{-1}$	W1	
	(ii) $I = Mv - Mu$		
	$I = M\dfrac{m}{M}(-u) - M \times 0$	M1	
	$= -mu$ Ns	W1	
	$Ft = I$		
	$F(0.002) = -mu$	M1	
	Force exerted is 500 *mu* Newtons opposite to the direction of the cannonball.	W2	10
2	**(i)** $\mathbf{s} = \mathbf{u}t + \dfrac{1}{2}\mathbf{a}t^2$	M1	
	Displacement from O is $(23\mathbf{i} + 49\mathbf{j})t + \dfrac{1}{2}(-g\mathbf{j})t^2 + 10\mathbf{j}$	W2 MW1	
	$= 23t\mathbf{i} + (10 + 49t - 4.9t^2)\mathbf{j}$	W1	
	(ii) $s_y = 49t - 4.9t^2 + 10$	MW1	
	$\dfrac{ds_y}{dt} = 49 - 9.8t$	M1 W1	
	$\dfrac{ds_y}{dt} = 49 - 9.8t = 0$	MW1	
	$t = 5$	W1	
	(iii) Air resistance is negligible or the ball is a particle.	MW1	11

3 (i)



MW3

(ii) M(A) $Mg\cos\alpha\,(3L) = R\,(4L)$ M2 W2

$$\cos\alpha = \frac{\frac{3}{5}Mg(4L)}{Mg\,(3L)} = \frac{4}{5}$$

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\alpha = 36.9° \text{ (3 sf)}$$ W1

(iii) Resolve horizontally $Fr = R\sin\alpha$ M1

$$= \frac{3}{5} \times \frac{3}{5} Mg$$

$$= \frac{9}{25} Mg$$ W1

Resolve vertically $R\cos\alpha + N = Mg$ M1 W1

$$N = Mg - \frac{3}{5} \times \frac{4}{5} Mg$$

$$= \frac{13}{25} Mg$$

$$Fr = \mu N$$

$$\frac{9}{25} Mg = \mu \frac{13}{25} Mg$$ M1W1

$$\mu = \frac{9}{13}$$ W1 15

AVAILABLE MARKS

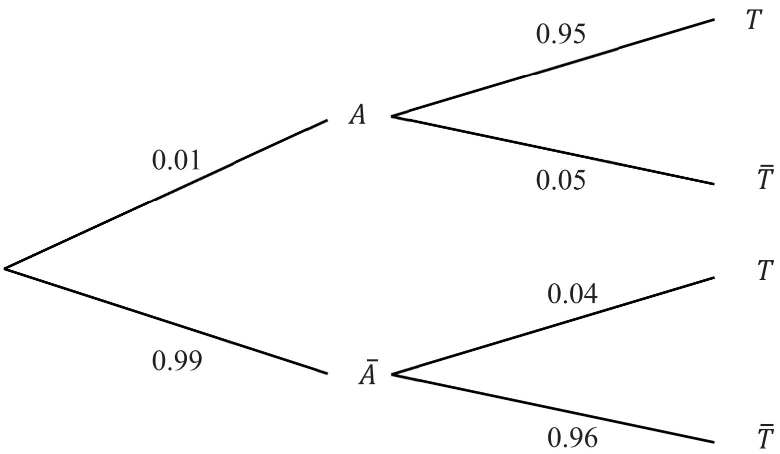
		AVAILABLE MARKS
4 **(i)** $x = \frac{1}{3}t^3 + \frac{3}{4}t^2 + 7t$		
When $t = 2$ $\quad x = \frac{1}{3}(2)^3 + \frac{3}{4}(2)^2 + 7(2)$	M1	
$= \frac{8}{3} + 3 + 14$		
$= \frac{59}{3}$		
Distance AB is $\frac{59}{3}$ m	W1	
(ii) $x = \frac{1}{3}t^3 + \frac{3}{4}t^2 + 7t$		
$v = t^2 + \frac{3}{2}t + 7$	M1 W2	
At time $t = 2$		
$v = 2^2 + \frac{3}{2}(2) + 7 = 14\ \text{ms}^{-1}$ in the direction AB	MW1	
(iii) $2 \leqslant t \leqslant 3 \qquad v = 14\sec^2\left(\frac{\pi t}{4} - \frac{\pi}{2}\right)$		
$x = 14 \times \frac{4}{\pi}\tan\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) + c$	M1 MW2	
At $t = 2$, $x = \frac{59}{3}$, hence $c = \frac{59}{3}$	MW1	
$t = 3$, $x = 14 \times \frac{4}{\pi}\tan\left(\frac{\pi}{4}\right) + \frac{59}{3}$	M1	
Distance AC is $\left(\frac{56}{\pi} + \frac{59}{3}\right)$ m	W1	
(iv) As t approaches 4, x and/or v tends to infinity, which is unrealistic.	MW2	14

Section B: Statistics

				AVAILABLE MARKS
5	**(i)**	One-tailed.	M1	
	(ii)	The p-value is the probability that the test statistic takes a value which is equal to or more extreme than the observed value when the null hypothesis is taken as true.	M2	
	(iii)	Since $0.00845 < 0.01$, we reject H_0 and conclude that there is sufficient evidence of a negative correlation between unemployment percentage and average income for the city.	MW3	
	(iv)	The p-value doubles to 0.0169 and $0.0169 > 0.01$ so the economist would not reject H_0 and he would conclude that there is insufficient evidence to suggest any correlation between the unemployment percentage and average income for the city.	MW2	8
6		$\Phi(z_1) = 0.75$	M1	
		$z_1 = 0.675$	MW1	
		$\dfrac{571.71 - \mu}{\sigma} = 0.675$	M1	
		$571.71 - \mu = 0.675\sigma$	W1	
		$\Phi(z_2) = 0.05$	M1	
		$z_2 = -1.645$	MW1	
		$\dfrac{558.95 - \mu}{\sigma} = -1.645$		
		$558.95 - \mu = -1.645\sigma$	MW1	
		$571.71 - \mu = 0.675\sigma$		
		$558.95 - \mu = -1.645\sigma$	M1	
		$12.76 = 2.32\sigma$		
		$\sigma = 5.5$ ml	MW1	
		$\mu = 568$ ml	MW1	10

AVAILABLE MARKS

7 **(i)** A = person is allergic to milk.
T = positive test result.



MW1

$P(T) = 0.01 \times 0.95 + 0.99 \times 0.04$ M1

$= 0.0491$ W1

(ii) $P(A|T) = \dfrac{0.01 \times 0.95}{0.01 \times 0.95 + 0.99 \times 0.04}$ M1 MW2

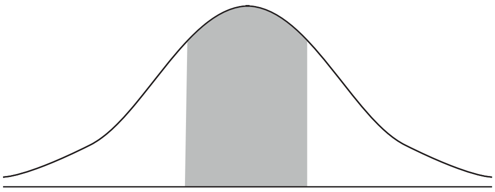
$= 0.193$ W1

(iii) The test is undependable as only about 20% of positive test results are accurate. MW2 9

8 (i) Solution 1 (using calculator)

AVAILABLE MARKS

$X \sim N(12, 3.5^2)$ MW1



MW1

Using calculator

$P(10 < X < 15) = 0.520$ M1 W1

Solution 2 (using z-scores)

$X \sim N(12, 3.5^2)$

$$P(10 < X < 15) = P\left(\frac{10-12}{3.5} < Z < \frac{15-12}{3.5}\right)$$ M1 MW1

$$= P(-0.571 < Z < 0.857)$$ W1

$$= [\Phi(0.571) - 0.5] + [\Phi(0.857) - 0.5]$$

$$= 0.520$$ W1

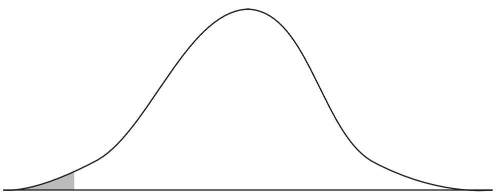
(ii) Solution 1 (using probability)

$H_0: \mu = 12$

$H_1: \mu < 12$ (one-tailed test) MW1

Reject H_0 if $P(\overline{X} < 10.8) < 0.05$ MW1

$\overline{X} \sim N\left(12, \frac{3.5^2}{40}\right)$ M1



MW1

$P(\overline{X} < 10.8) = 0.0151$ W1

Since $0.0151 < 0.05$, we reject H_0 and conclude that there is sufficient evidence at the 5% level of significance to suggest that the average time to answer calls at the switchboard has decreased. MW2

Solution 2 (using z-scores)

		AVAILABLE MARKS
$H_0: \mu = 12$ $H_1: \mu < 12$ (one-tailed test)	MW1	
Reject H_0 if $z < -1.645$	MW1	
$\overline{X} \sim N\left(12, \frac{3.5^2}{40}\right)$		
$z = \dfrac{10.8 - 12}{\frac{3.5}{\sqrt{40}}}$	M1 W1	
$= -2.168\ldots$	W1	
Since $-2.168 < -1.645$, we reject H_0 and conclude that there is sufficient evidence at the 5% level of significance to suggest that the average time to answer calls at the switchboard has decreased.	MW2	11
9 **(i)** If Daniel is just guessing, the probability of getting an answer correct is 0.2 If he is not guessing, but has revised, then the probability of getting an answer correct is greater than 0.2. This requires a one-tailed test.	MW1	
(ii) Let p be the probability of getting an answer correct.		
$H_0 : p = 0.2$		
$H_1 : p > 0.2$	MW2	
(iii) The significance level of a test is the probability of incorrectly rejecting the null hypothesis.	M2	
(iv) Let A = number of answers which Daniel gets correct.		
Under H_0 : $A \sim \text{Bin}(40, 0.2)$	MW1	
$P(A \geq 12) = 1 - P(A \leq 11)$	M1	
$= 1 - 0.91249\ldots$	MW1	
$= 0.0875$	W1	
Since $0.0875 > 0.05$ we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance to suggest that Daniel was doing better than guessing his answers. Hence the teacher's suspicion is justified.	MW3	12
	Total	**100**