Rewarding Learning

ADVANCED
General Certificate of Education
2025

Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

WEDNESDAY 4 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A: Mechanics

			AVAILABLE MARKS
1 **(i)**	$\mathbf{a} = \frac{4}{3}t^3\,\mathbf{i} - 5t^2\,\mathbf{j}$		
	When $t = 3$; $\mathbf{a} = \frac{4}{3}(3)^3\,\mathbf{i} - 5(3)^2\,\mathbf{j}$	M1	
	$\mathbf{a} = (36\mathbf{i} - 45\mathbf{j})\ \text{m s}^{-2}$	W1	
(ii)	$\mathbf{v} = \int \left(\frac{4}{3}t^3\,\mathbf{i} - 5t^2\,\mathbf{j}\right)\mathrm{d}t$	M1	
	$\mathbf{v} = \frac{1}{3}t^4\,\mathbf{i} - \frac{5t^3}{3}\,\mathbf{j} + \mathbf{c}$	W1	
	When $t = 0$; $\mathbf{v} = 6\mathbf{i} + 2\mathbf{j} \Rightarrow \mathbf{c} = 6\mathbf{i} + 2\mathbf{j}$	MW1	
	$\mathbf{v} = \frac{t^4}{3}\,\mathbf{i} - \frac{5t^3}{3}\,\mathbf{j} + 6\mathbf{i} + 2\mathbf{j}$		
	$\mathbf{v} = \left(\frac{t^4}{3} + 6\right)\mathbf{i} + (2 - \frac{5t^3}{3})\,\mathbf{j}$	W1	
	When $t = 3$; $\mathbf{v} = \left(\frac{1}{3}(3)^4 + 6\right)\mathbf{i} + (2 - \frac{5}{3}(3)^3)\,\mathbf{j}$		
	$\mathbf{v} = (33\mathbf{i} - 43\mathbf{j})\ \text{m s}^{-1}$	W1	7
2 **(i)**	$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = (m_1 + m_2)\mathbf{v}$		
	$4(5\mathbf{i} + 4\mathbf{j}) + 6(7.5\mathbf{i} + 6\mathbf{j}) = 10\mathbf{v}$	M2 W2	
	$20\mathbf{i} + 16\mathbf{j} + 45\mathbf{i} + 36\mathbf{j} = 10\mathbf{v}$		
	$65\mathbf{i} + 52\mathbf{j} = 10\mathbf{v}$		
	$\mathbf{v} = (6.5\mathbf{i} + 5.2\mathbf{j})\ \text{m s}^{-1}$	W1	
(ii)	$\mathbf{I} = m(\mathbf{v} - \mathbf{u})$		
	$\mathbf{I} = 4[(6.5\mathbf{i} + 5.2\mathbf{j}) - (5\mathbf{i} + 4\mathbf{j})]$	M1	
	$\mathbf{I} = 4[1.5\mathbf{i} + 1.2\mathbf{j}]$		
	$\mathbf{I} = (6\mathbf{i} + 4.8\mathbf{j})$ Ns	MW1	7

	AVAILABLE MARKS

3 **(i)** $s = ut + \frac{1}{2}at^2$

$0 = 60\sin\alpha t - \frac{1}{2}gt^2$ M1

$0 = \left(60\sin\alpha - \frac{1}{2}gt\right)t$

$60\sin\alpha - \frac{1}{2}gt = 0$ W1

$\frac{1}{2}gt = 60\sin\alpha$

$t = \frac{120\sin\alpha}{g}$

Time of flight, $T = \frac{120\sin\alpha}{g}$ W1

Range, $R = 60\cos\alpha \times \frac{120\sin\alpha}{g}$ M1

$R = \frac{7200\sin\alpha\cos\alpha}{g}$ W1

$R = \frac{3600\sin 2\alpha}{g}$ MW1

Alternative solution:

Let t represent the time taken for projectile to reach maximum height from point A.

$v = u + at$

$0 = 60\sin\alpha - gt$ M1

$gt = 60\sin\alpha$

$t = \frac{60\sin\alpha}{g}$ W1

Time of flight, $T = 2t$

$T = \frac{120\sin\alpha}{g}$ W1

Range, $R = 60\cos\alpha \times \frac{120\sin\alpha}{g}$ M1

$R = \frac{7200\sin\alpha\cos\alpha}{g}$ W1

$R = \frac{3600\sin 2\alpha}{g}$ MW1

(ii) $R = \frac{3600\sin 2\alpha}{g}$

$350 = \frac{3600\sin 2\alpha}{g}$ MW1

$\sin 2\alpha = 0.9527\ldots$

1st Solution:

$2\alpha = 72.3219\ldots$

$\alpha = 36.16096\ldots = 36°$ W1

2nd Solution:

$2\alpha = 180 - 72.3219\ldots = 107.678\ldots$

$\alpha = 53.839\ldots = 54°$ W1

(iii) Range will be maximum when $\sin 2\alpha = 1$ MW1

$\alpha = 45°$ MW1 11

		AVAILABLE MARKS

4 **(i)** $v = 0$ when particle is at rest.

$0 = 5\cos\left(\frac{\pi}{2}t\right)$ M1

1st Solution:

$\frac{\pi}{2}t = \frac{\pi}{2}$

$t = 1$ second W1

2nd Solution:

$\frac{\pi}{2}t = 2\pi - \frac{\pi}{2}$

$\frac{\pi}{2}t = \frac{3\pi}{2}$

$t = 3$ seconds W1

(ii) $s = \int 5\cos\left(\frac{\pi}{2}t\right)\mathrm{d}t$ M1

$s = \frac{10}{\pi}\sin\left(\frac{\pi}{2}t\right) + c$ W1

$t = 0 \quad s = 0 \Rightarrow c = 0$ M1

$s = \frac{10}{\pi}\sin\left(\frac{\pi}{2}t\right)$

$t = 0 \quad s = 0$

$t = 1 \quad s = \frac{10}{\pi}\sin\left(\frac{\pi}{2}(1)\right) = \frac{10}{\pi}$

$t = 3 \quad s = \frac{10}{\pi}\sin\left(\frac{\pi}{2}(3)\right) = -\frac{10}{\pi}$ MW1

$t = 3\frac{2}{3} \quad s = \frac{10}{\pi}\sin\left(\frac{\pi}{2}\left(3\frac{2}{3}\right)\right) = -\frac{5}{\pi}$ MW1

Total distance travelled $= \left(\frac{10}{\pi} \times 3\right) + \frac{5}{\pi}$ M1

$= \frac{35}{\pi}$ m W1

Alternative solution:

$s = \int 5\cos\left(\frac{\pi}{2}t\right)\mathrm{d}t$ M1

$s = \left|\int_0^1 v \ \mathrm{d}t\right| + \left|\int_1^3 v \ \mathrm{d}t\right| + \left|\int_3^{3\frac{2}{3}} v \ \mathrm{d}t\right|$ W1

$s = \left|\left[\frac{10}{\pi}\sin\left(\frac{\pi}{2}t\right)\right]_0^1\right| + \left|\left[\frac{10}{\pi}\sin\left(\frac{\pi}{2}t\right)\right]_1^3\right| + \left|\left[\frac{10}{\pi}\sin\left(\frac{\pi}{2}t\right)\right]_3^{3\frac{2}{3}}\right|$ M1

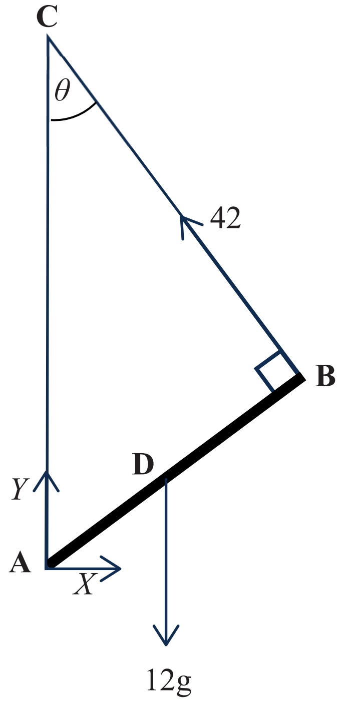
$s = \left|\left[\frac{10}{\pi}\sin\left(\frac{(1)\pi}{2}\right) - \frac{10}{\pi}\sin\left(\frac{(0)\pi}{2}\right)\right]\right| + \left|\left[\frac{10}{\pi}\sin\left(\frac{(3)\pi}{2}\right) - \frac{10}{\pi}\sin\left(\frac{(1)\pi}{2}\right)\right]\right|$

$+ \left|\left[\frac{10}{\pi}\sin\left(\frac{\left(3\frac{2}{3}\right)\pi}{2}\right) - \frac{10}{\pi}\sin\left(\frac{(3)\pi}{2}\right)\right]\right|$ MW1

$s = \left|\left(\frac{10}{\pi} - 0\right)\right| + \left|\left(-\frac{10}{\pi} - \frac{10}{\pi}\right)\right| + \left|-\frac{5}{\pi} - \left(-\frac{10}{\pi}\right)\right|$ M1 MW1

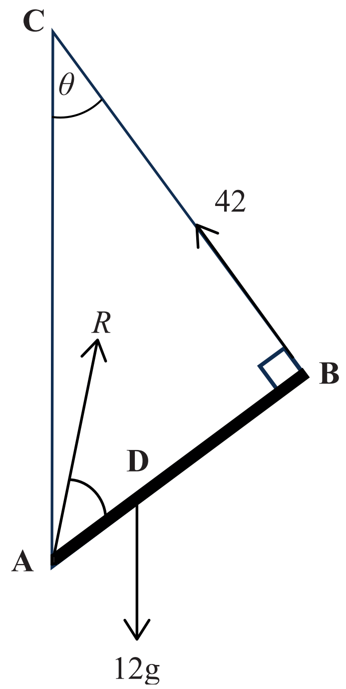
$s = \frac{35}{\pi}$ m W1 10

5 (i)



MW2

Alternative solution



MW2

AVAILABLE MARKS

(ii) AD:DB = 2:3

AD = 4 m, DB = 6 m — MW1

M(A): $12g\cos\theta\,(4) = 42(10)$ — M2 W2

$470.4\cos\theta = 420$

$\cos\theta = \dfrac{25}{28}$ — W1

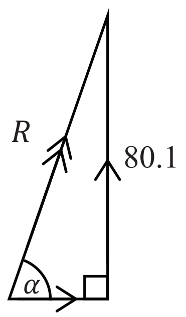
(iii) Resolve horizontally: $X = 42\sin\theta$

$X = 42 \times \dfrac{\sqrt{159}}{28}$ — M1

$X = 18.9142\ldots$ — W1

Resolve vertically: $Y + \left(42 \times \dfrac{25}{28}\right) = 12g$

$Y = 80.1$ — MW1



$R = \sqrt{18.9142\ldots^2 + 80.1^2}$ — M1

$R = 82.3$ N — W1

$\tan\alpha = \dfrac{80.1}{18.9142\ldots}$

$\alpha = 76.7°$ — W1

(iv) The hinge is smooth. — MW1

	AVAILABLE MARKS
	15
Section A	**50**

Section B: Statistics

AVAILABLE MARKS

6 **(i)** $\dfrac{8.9-\mu}{\sigma} = 1.645$ M1 MW1

$8.9-\mu = 1.645\sigma$ W1

(ii) $\dfrac{7.9-\mu}{\sigma} = -0.434397\ldots$ MW1

$7.9-\mu = -0.434397\ldots\sigma$ W1

(iii)

$$\begin{aligned} 8.9-\mu &= 1.645\sigma \\ \underline{7.9-\mu} &\underline{= -0.434397..\sigma} \\ 1 &= 2.079397\ldots\sigma \end{aligned}$$

M1

$\sigma = 0.4809\ldots$

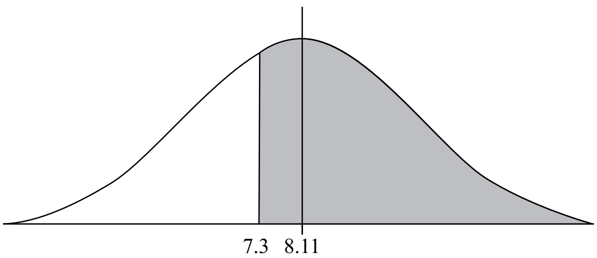
$\sigma = 0.481$ W1

$8.9-\mu = 1.645(0.4809\ldots)$

$\mu = 8.1089\ldots$

$\mu = 8.11$ W1

(iv)



M1

$P(X > 7.3) = 0.954$ MW1 **10**

7 **(i)** This is a two-tailed test because rejection of H_0 could indicate either positive or negative correlation. MW1 W1

(ii) $-0.4683 < r < 0.4683$ MW2

(iii) $r = 0.5217$ lies outside the acceptance region MW1

We reject H_0 MW1

We conclude that there is sufficient evidence at the 5% level of significance to suggest a correlation between an athlete's reaction time and their time taken to run 100 metres. MW1 **7**

8 $H_0 : \mu = 3.8$ MW1
$H_1 : \mu > 3.8$ (one-tailed test) MW1

AVAILABLE MARKS

Let A be the amount of blue-green algae in the lake, so $A \sim N(3.8, 0.7^2)$
For a one-tailed test at the 2.5% level of significance, reject H_0 if $\bar{z} > 1.96$ MW1

$$\bar{z} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

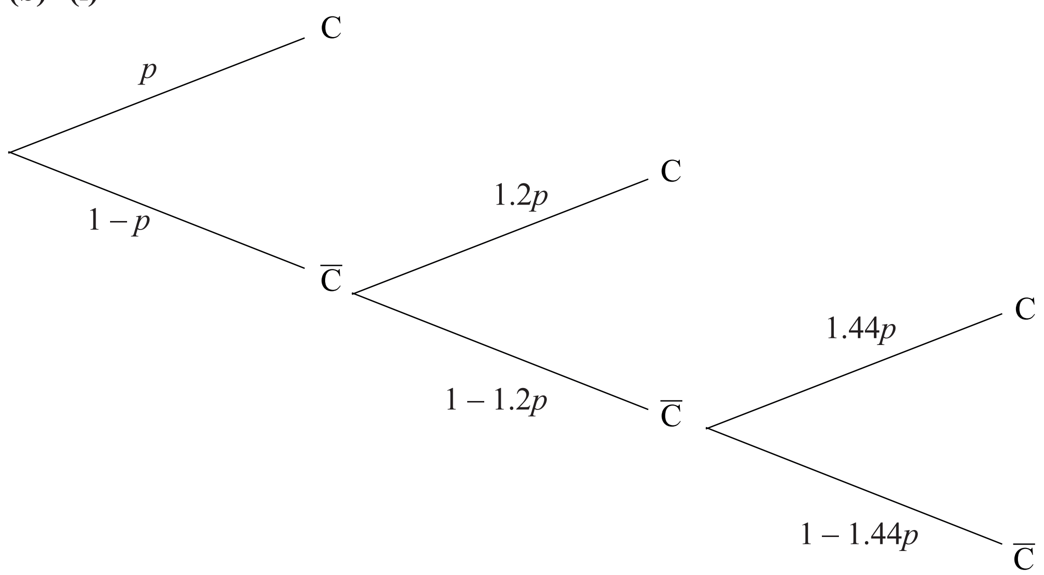
$$= \frac{3.98 - 3.8}{\frac{0.7}{\sqrt{60}}}$$ M1 MW1

$$= 1.992$$ W1

$\bar{z} = 1.992 > 1.96$ MW1
We reject H_0 MW1
We conclude that there is sufficient evidence at the 2.5% level of significance to suggest an increase in the amount of blue-green algae in the lake. MW1 9

			AVAILABLE MARKS
9 **(a)** **(i)**	$P(D \cap \overline{M}) = \frac{16}{505}$	MW1	
(ii)	$P(H \mid M) = \frac{78}{474} = \frac{13}{79}$	M1 WI	

(b) **(i)**



$P(\text{3 attempts}) = (1 - p)(1 - 1.2p)(1.44p)$ M1 MW3

$= (1 - 2.2p + 1.2p^2)(1.44p)$

$1.44p - 3.168p^2 + 1.728p^3 = 0.102p$ M1

$1.728p^2 - 3.168p + 1.338 = 0$ MW1

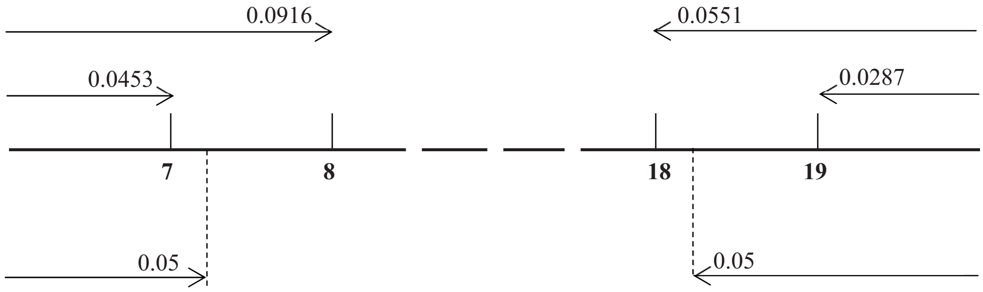
$p = \frac{3.168 \pm \sqrt{(-3.168)^2 - 4(1.728)(1.338)}}{2(1.728)}$ M1

$p = 0.65981..... = 0.660$ W1

(ii) The probability of answering correctly is not constant or there is not a fixed number of trials. MW1 12

10 **(i)** $H_0 : p = 0.25$ MW1
$H_1 : p \neq 0.25$ MW1

(ii) Use $X \sim \text{Bin}(50, 0.25)$ M1
$P(X \leqslant 7) = 0.0453$ and $P(X \leqslant 8) = 0.0916$
$P(X \leqslant 17) = 0.9449$ and $P(X \leqslant 18) = 0.9713$



MW1
MW1

So, the critical region for this test is $X \leqslant 7, X \geqslant 19$ W2

(iii) The actual probability of incorrectly rejecting the null hypothesis is

$0.0453 + 0.0287 = 0.074$ M1 W1

(iv) The test value 17 does not lie in the critical region. MW1
We do not reject H_0 MW1
We conclude that there is insufficient evidence at the 10% level of significance to suggest that the proportion of sixth form students who cannot swim is not 1 in 4 MW1

	AVAILABLE MARKS
	12
Section B	**50**
Total	**100**