

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 6 June 2023

Afternoon (Time: 2 hours)

Paper
reference

9MA0/01

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P72804A

©2023 Pearson Education Ltd.
N:1/1/1/1/1/




Pearson

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x - 5)}{3} dx$$

writing each term in simplest form.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area for the answer to Question 1. The page contains 28 horizontal lines.

(Total for Question 1 is 4 marks)



P 7 2 8 0 4 A 0 3 4 4

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \quad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0 \tag{2}$$

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3 \tag{4}$$



3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\vec{OA}| = \sqrt{38}$ (1)

(b) Find the smallest value of a for which

$$|\vec{OB}| > |\vec{OA}|$$
(2)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 3 marks)



P 7 2 8 0 4 A 0 7 4 4

4. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2} \cos x$
- the curve has a stationary point with x coordinate α
- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places. (3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P , giving your answer in the form $y = mx + c$, where m and c are constants to be found. (2)

Lined area for student working.



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Large ruled area for writing answers.

(Total for Question 4 is 5 marks)



5. A continuous curve has equation $y = f(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

(3)

Given also that the sum of all the y values in the table is 97.2

(b) find the value of a and the value of b

(3)



Question 5 continued

Lined area for writing answers, consisting of approximately 30 horizontal lines.

(Total for Question 5 is 6 marks)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



6.

$a = \log_2 x$

$b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$

(1)

(b) $\log_2(x^2 + 8x)$

(2)

(c) $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

(3)



Question 6 continued

Lined writing area for Question 6.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 6 marks)

7. The function f is defined by

$$f(x) = 3 + \sqrt{x - 2} \quad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f (1)

(b) Find f^{-1} (3)

The function g is defined by

$$g(x) = \frac{15}{x - 3} \quad x \in \mathbb{R} \quad x \neq 3$$

(c) Find $gf(6)$ (2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a) \quad (2)$$



Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area for the answer to Question 7 continued.



P 7 2 8 0 4 A 0 1 5 4 4

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



8.

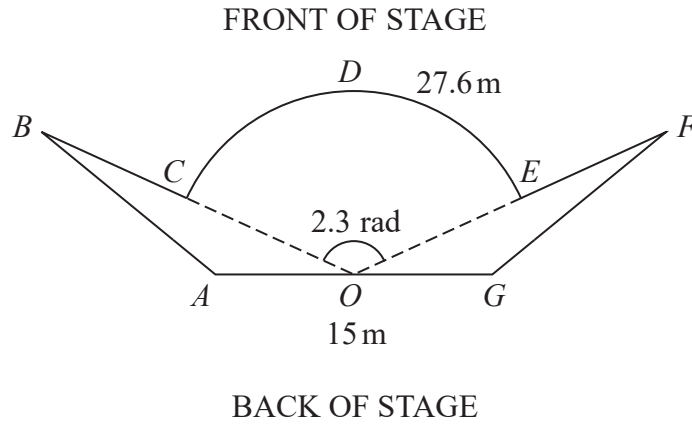


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles ABO and GFO joined to a sector $OCDEO$ of a circle, centre O , where

- angle $COE = 2.3$ radians
- arc length $CDE = 27.6$ m
- AOG is a straight line of length 15 m

(a) Show that $OC = 12$ m. (2)

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places. (2)

Given that the total length of the front of the stage, $BCDEF$, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre. (6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area for the answer to Question 8.



P 7 2 8 0 4 A 0 1 9 4 4

Question 8 continued

Lined writing area for the answer.

(Total for Question 8 is 10 marks)

DO NOT WRITE IN THIS AREA



10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

- (a) Find in terms of k ,
 - (i) the coordinates of the centre of C
 - (ii) the radius of C (3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

- (b) Find the range of possible values of k . (6)



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined writing area for the answer to Question 10.

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Lined area for writing the answer to Question 10.

(Total for Question 10 is 9 marks)



P 7 2 8 0 4 A 0 2 7 4 4

11.

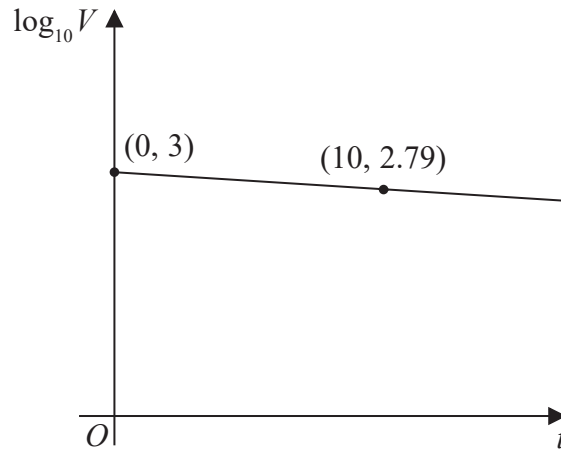


Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t .

The line passes through the points $(0, 3)$ and $(10, 2.79)$ as shown.

Using these points,

- (a) find the initial value of the phone, (2)
- (b) find a complete equation for V in terms of t , giving the exact value of a and giving the value of b to 3 significant figures. (3)

Exactly 2 years after it was bought, the value of the phone was £320

- (c) Use this information to evaluate the reliability of the model. (2)



Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



12.

$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$
- assume that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model. (3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$. (1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model. (2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate. (1)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 13 continued

Lined writing area for the answer to Question 13.



14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



15. A curve has equation $y = f(x)$, where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

(ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)



Question 15 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area consisting of horizontal lines for writing.



Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

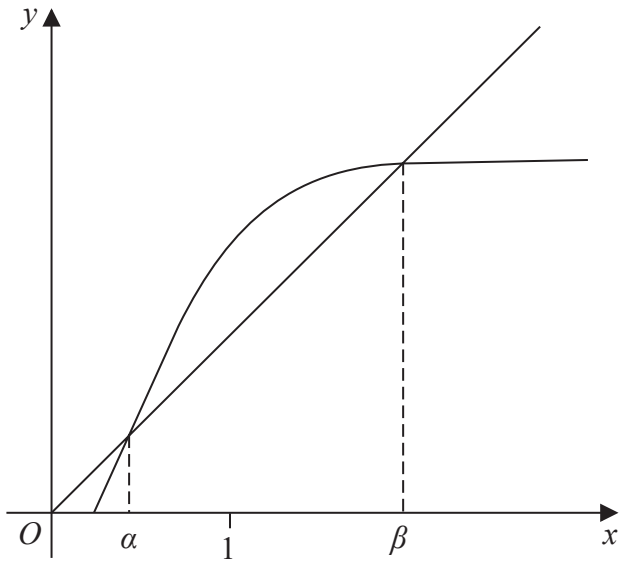
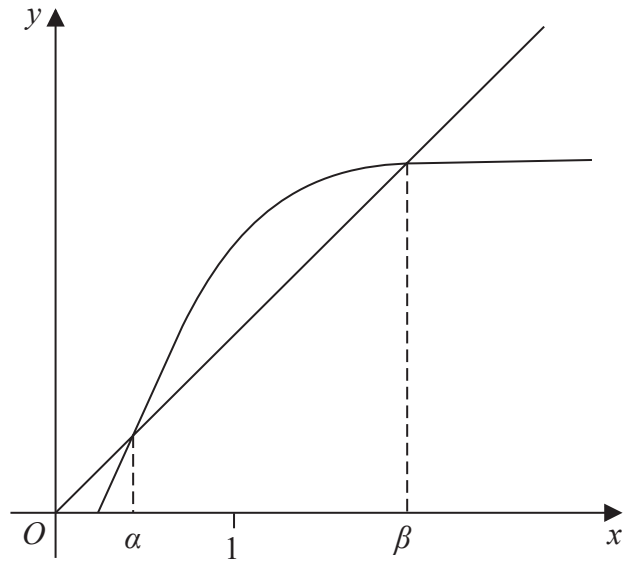


Diagram 1



copy of Diagram 1

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

Question 15 continued

Lined area for writing the answer to Question 15.



P 7 2 8 0 4 A 0 4 3 4 4

Question 15 continued

Lined writing area for the answer to Question 15.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 15 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS

