Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 4B Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$r = \frac{1610}{\sqrt{314.55 \times 9026}}$	M1	1.1b
	$= 0.9555\ldots$ awrt <u>**0.956**</u>	A1	1.1b
		(2)	
(b)	$b = \frac{1610}{314.55}$	M1	2.1
	$a = \bar{y} - b\bar{x} = 108 - 'b' \times 19.65$	M1	1.1b
	$y = 5.12x + 7.42$*	A1*	1.1b
		(3)	
(c)	Residual = 104 – (5.12(20) + 7.42) = awrt <u>**–5.8**</u>	B1	3.4
		(1)	
(d)	<u>Positive residuals</u> show that the model <u>underestimates</u> the 500m race times of the fastest and slowest children.	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)	**M1:** correct expression for *r* **A1:** awrt **0.956**
(b)	**M1:** correct expression for gradient, or correct formula stated with *b* to more than 3.s.f **M1:** correct expression, or correct formula stated with *a* to more than 3.s.f follow through their *b* ***A1:** correct linear model with ***a* =** awrt **7.42** and ***b* =** awrt **5.12**
(c)	**B1:** correct use of model awrt **–5.8**
(d)	**B1:** correct contextual explanation mentioning positive residuals and underestimate o.e.

Question	Scheme	Marks	AOs
2(a)	$z = 2.3263$	B1	1.1b
	98% CI for μ is: $128 \pm 2.3263 \times \frac{3.5}{\sqrt{80}}$	M1	2.1
	$= (127.0896\ldots, 128.9103\ldots) =$ awrt $(127.09, 128.91)$	A1	1.1b
		(3)	
(b)	$\mathrm{H}_0: \mu_{g(reen)} - \mu_{r(ed)} = 10 \quad \mathrm{H}_1: \mu_{g(reen)} - \mu_{r(ed)} > 10$	B1	2.5
		(1)	
(c)	Normal model: $\bar{G} - \bar{R} \sim \mathrm{N}\left(10, \frac{3.5^2}{80} + \frac{4^2}{n}\right)$	M1 A1	3.3 1.1b
	$z = \frac{(128-117)-10}{\sqrt{\frac{3.5^2}{80}+\frac{4^2}{n}}} > 1.6449$	M1 B1	3.1b 1.1b
	$\frac{1}{1.6449} > \sqrt{\frac{3.5^2}{80}+\frac{4^2}{n}} \rightarrow \left(\frac{1}{1.6449}\right)^2 - \frac{12.25}{80} > \frac{16}{n} \rightarrow n > 73.9...$	M1	1.1b
	***n* = 74**	A1	1.1b
		(6)	
(d)	Sample sizes are large so CLT means even though we do not know the distributions of *G* and *R*, the means $\bar{G}$ and $\bar{G} - \bar{R}$ will be (approximately) normally distributed.	B1	2.4
		(1)	
(e)	[Since 85 > 73.9…] there is significant evidence to support Camilo's belief/the mean weight of green apples is more than 10 grams greater than red apples is supported.	B1ft	2.2b
		(1)	
			(12 marks)

Notes

(a)	**B1:** 2.3263 or better **M1:** for $128 \pm \sqrt{\frac{3.5^2}{80}} \times$ "z value". $\lvert z\rvert > 1$ May be implied by a correct CI **A1:** awrt **127.09** and awrt **128.91**
(b)	**B1:** Both hypotheses (oe) correct with correct notation (if using μ_x and μ_y these must be defined)
(c)	**M1:** setting up normal model with correct mean or variance May be shown in standardisation with correct mean or variance and $\lvert z\rvert > 1$ **A1:** correct variance and mean, may be seen in standardisation **M1:** setting up an equation or inequality by standardising with their mean and their standard deviation and $\lvert z\rvert > 1$ **B1:** Correct critical value 1.6449 or better, allow **1.64 or better** **M1:** rearranging to solve for *n* (may be implied by 73.9…) **A1:** $n = 74$ cao

(d)	**B1:** correct explanation mentioning large sample size and the means being distributed normally
(e)	**B1ft:** drawing a correct inference in context, must be consistent with their value of n

Question	Scheme	Marks	AOs
3(a)	$[\bar{x} = \frac{2268}{9} = 252]$ $s_A^2 = \dfrac{571700 - 9\times 252^2}{8} [= 20.5]$	M1	2.1
	$\chi^2_{8,0.025} = 17.535$ $\chi^2_{8,0.975} = 2.180$	B1	3.3
	$\dfrac{8\times 20.5}{17.535} < \sigma_A^2 < \dfrac{8\times 20.5}{2.180}$	M1	1.1b
	$9.3527... < \sigma_A^2 < 75.2293...$	A1	1.1b
		(4)	
(b)	$H_0 : \sigma_A^2 = \sigma_B^2,\ H_1 : \sigma_A^2 \neq \sigma_B^2$	B1	2.5
	$\dfrac{s_A^2}{s_B^2} = \dfrac{\text{"20.5"}}{12.7} = 1.61...$	M1	3.3
	$F_{8,\,10}(0.05) = 3.07$	B1	1.1b
	There is insufficient evidence to suggest that the variances are different.	A1	2.2b
		(4)	
			(8 marks)

Notes

(a)	**M1:** correct expression for s_A^2 **B1:** for selecting χ^2 with v = 8. May indicated in notation or either of 17.535, 2.180 **M1:** setting up 95% CI with their sample variance, chi-squared values, and 8 **A1:** correct CI with awrt **9.35** and awrt **75.2**
(b)	**B1:** both hypotheses correct using σ or σ^2 **M1:** using the *F*-distribution as the model eg $\dfrac{s_A^2}{s_B^2}$ **B1:** awrt **3.07** **A1:** Drawing a correct inference following through their CV and value for Allow $\sigma_B^2 = \sigma_A^2$ Allow standard deviation instead of variance NB: Allow candidates to use their $\dfrac{s_B^2}{s_A^2}$ with $\dfrac{1}{3.35}$ = awrt 0.299 for the final M1B1A1 in (b)

Question	Scheme	Marks	AOs
4(a)	$W = E_1 + E_2 + ... + E_6 + S \sim \text{N}(6\times 60 + 24, 6\times 3^2 + 1.8^2)$ $[W \sim \text{N}(384, 57.24)]$	M1M1	3.3 1.1b
	P(W < 387) = 0.6541…. awrt **<u>0.654</u>**	A1	3.4
		(3)	
(b)	$X = E_1 + E_2 + ... + E_{12} + L \sim \text{N}(12\times 60 + 40, 12\times 3^2 + 2.1^2)$ $[X \sim \text{N}(760, 112.41)]$	M1	1.1b
	$2W \sim \text{N}(\text{"384"}\times 2, 2^2 \times \text{"57.24"})[= \text{N}(768, 228.96)]$	M1	1.1b
	$X - 2W \sim \text{N}(760 - 768, 112.41 + 228.96) \to \text{N}(-8, 341.37)$	M1A1	3.3 1.1b
	P(X – 2W > 0) = 0.3325… awrt **<u>0.333</u>**	A1	3.4
		(5)	
			(8 marks)

Notes

(a)	**M1:** setting up normal model with correct mean **M1:** use of correct variance **A1:** awrt **0.654**
(b)	**M1:** setting up normal model for large box of eggs, sight of 12 × 60 + 40 **M1:** sight or use of 2 times their mean from (a) and 4 times their variance from (a) **M1:** setting up distribution for the difference in weights, with mean of their “760” – “768” **A1:** correct mean and variance **A1:** awrt **0.333**

Question	Scheme	Marks	AOs
5(a)	e.g. for each person the scores before and after are not independent	B1	2.4
		(1)	
(b)	The differences in scores are normally distributed.	B1	2.4
		(1)	
(c)	*d*: 4 –3 4 2 2 6 3 0	M1	3.1b
	$\bar{d} = \pm\, 2.25$ $\quad s_d = \sqrt{7.642857....} = 2.76457\ldots$	M1	1.1b
	$H_0 : \mu_d = 0 \quad H_1 : \mu_d > 0$	B1	3.3
	$t = \pm \dfrac{"\pm 2.25"}{\frac{"2.76..."}{\sqrt{8}}}$	M1	1.1b
	$= \pm\ 2.3019\ldots.$ **awrt** $\pm$ **2.30**	A1	1.1b
	Critical value $t_7 = \pm\, 1.895$	B1	1.1b
	2.30 > 1.895, therefore (reject H_0) there is sufficient evidence to: support an increase in the percentage of words recalled or support psychologist's claim	A1ft	2.2b
		(7)	
			(9 marks)

Notes	
(a)	**B1:** correct explanation
(b)	**B1:** correct modelling assumption
(c)	**M1:** for setting up paired *t*-test with at least 5 correct differences consistent in direction **M1:** complete method for $\bar{d}$ and s_d or s_d^2 **B1:** Use of *t*-distribution for differences with both hypotheses correct in terms of μ / μ_d Allow $\mu_d < 0$ if candidate does test with negative test statistic **M1:** method for finding test statistic with their values **A1:** awrt $\pm$ **2.30** **B1:** correct critical $\pm$ **1.895** (or better) with compatible sign **A1ft:** drawing a correct inference in context Must be consistent with their CV and a standardised test statistic from *t*-distribution Must mention "percentage" and "words" or "psychologist's claim" **SC:** If they have carried out a *t*-test for two independent samples they can potentially score: M0M0B0M1A0B1A1ft If they do follow this path, they can score: 3rd M1 for standardising with difference of their means and a pooled variance 2nd B1 for a CV of 1.761 A1ft for a correct contextual conclusion for their test statistic and 1.761

Question	Scheme	Marks	AOs
6(a)	$\text{F}(2)=\frac{2}{3}$ and $\text{F}(4)=1$		
	$k(2-4a)=\frac{2}{3}$ $\quad k(4-16a)=1$	M1	2.1
	$\frac{2}{3(2-4a)}=\frac{1}{(4-16a)}\to a=\frac{1}{10},\quad k=\frac{5}{12}$	M1A1	1.1b 1.1b
	$\frac{5}{12}\left(m-\frac{1}{10}m^2\right)=0.5\to m^2-10m+12=0$	M1M1	3.1a 1.1b
	$m=5-\sqrt{13}$ $\quad$ ($m=5+\sqrt{13}$ reject)	A1	1.1b
		(6)	
(b)	$\text{f}(x) = \frac{\text{d}}{\text{d}x}\left(\text{F}(x)\right)$	M1	1.1b
	$\text{f}(x) = \begin{cases} "\frac{5}{12}"(1-"0.2"x) & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$	A1ft	1.1b
		(2)	
(c)	Mode is X = 0…	B1	2.2a
	…since f(x) is linear with negative gradient. or …f(x) is a decreasing function	dB1	2.4
		(2)	
			(10 marks)

Notes

(a)	**M1:** using $\text{F}(2)=\frac{2}{3}$ and $\text{F}(4)=1$ to form two correct expressions in a and k **M1:** solving simultaneously to find a value of a or k **A1:** both $a=\frac{1}{10}$ and $k=\frac{5}{12}$, may be implied by correct equation or median **M1:** setting up F(m) = 0.5 to form a 3TQ **M1:** using an appropriate method to solve quadratic, resulting in a value of m **A1:** selecting the correct value for m
(b)	**M1:** differentiating F(x), at least one term correct for their a and k **A1ft:** correct ft expression with correct limits
(c)	**B1:** deducing that the mode is X = 0 **dB1:** correct reasoning May be given via a diagram of a linear function $0\leq x\leq 4$ with: • a negative gradient • that intersects the y-axis • is never negative in y

Question	Scheme		Marks	AOs
7(a)	$\mathrm{f}(r)=\frac{1}{8}$ for $2 \leqslant r \leqslant 10$ [0 otherwise]		B1	1.1b
			(1)	
(b)	$\mathrm{E}(4\pi R^2)=4\pi\mathrm{E}(R^2)$	$\int_2^{10} 4\pi r^2(\tfrac{1}{8})\mathrm{d}r$	M1	1.1b
	E(R) = 6	$=\left[\tfrac{1}{2}\pi\left(\frac{r^3}{3}\right)\right]_2^{10}$	M1	1.1b
	$\mathrm{E}(4\pi R^2)$ $=4\pi(\mathrm{Var}(R)+[\mathrm{E}(R)]^2)$ $=4\pi(\tfrac{16}{3}+\text{"}6\text{"}^2)$	$=\tfrac{1}{2}\pi\left(\frac{10^3}{3}-\frac{2^3}{3}\right)$	M1	2.1
	$=\frac{496\pi}{3}$ *		A1*	2.2a
			(4)	
(c)	$\mathrm{E}(\tfrac{4}{3}\pi R^3)=\int_2^{10}\tfrac{4}{3}\pi r^3(\tfrac{1}{8})\,\mathrm{d}r$		B1ft	1.1b
	$=\left[\tfrac{1}{6}\pi\left(\frac{r^4}{4}\right)\right]_2^{10}$		M1	2.1
	$=\tfrac{1}{6}\pi\left(\frac{10^4}{4}-\frac{2^4}{4}\right)$		dM1	1.1b
	$=416\pi$		A1	2.2a
			(4)	
			(9 marks)	

Notes

(a)	**B1:** correct probability density function with limits Allow any letter but must be consistent
(b)	**M1:** use of E(aR) = aE(R) / correct integral with their f(r) (limits not needed) **M1:** E(R) = 6 seen or implied / use of integration to find E(SA) **M1:** use of $\mathrm{E}(R^2)=\mathrm{Var}(R)+[\mathrm{E}(R)]^2$ / correct integration of their f(r), limits shown **A1*:** exact value from correct working
(c)	follow through their pdf from (a) for first 3 marks of (c) **B1ft:** correct integral for E(V) including limits, **M1:** integration of their pdf from (a) to find E(V) **dM1:** (dep on previous M1) use of correct limits, may be implied **A1:** 416π (allow awrt 1310 to 3 sf)

Question	Scheme	Marks	AOs
8(a)	$Y \sim B(n, p)$ $E(Y) = np$	M1	3.3
	$E(\hat{p}_1) = \frac{np + 3np - 2np}{2n} = p$	M1	3.4
	$E(\hat{p}_2) = \frac{2np + 3np + np}{6n} = p$, therefore, both are unbiased.	A1cso	1.1b
		(3)	
(b)	$\text{Var}(\hat{p}_1) = \frac{np(1-p) + 9np(1-p) + 4np(1-p)}{4n^2}$	M1	2.1
	$\text{Var}(\hat{p}_1) = \frac{7p(1-p)}{2n}$	A1	1.1b
		(2)	
(c)	[Both are unbiased so…] as $\text{Var}(\hat{p}_2) < \text{Var}(\hat{p}_1)$	M1	2.4
	$\hat{p}_2$ is the better estimator.	A1ft	2.2a
		(2)	
(d)	$\hat{p}_3$ is unbiased so $b = a + 4$	M1	1.1b
	Require $\text{Var}(\hat{p}_3) < \text{Var}(\hat{p}_2)$ so $\frac{a^2 + 10}{b^2} < \frac{7}{18}$	M1	2.1
	$18(a^2 + 10) < 7(a + 4)^2 \rightarrow 11a^2 - 56a + 68 < 0$	M1	1.1b
	$2 < a < \frac{34}{11}$	A1ft	1.1b
	$a = 3$ and $b = 7$	A1	2.2a
		(5)	
			(12 marks)

Notes	
(a)	**M1:** use of mean of np **M1:** use of binomial model (may be implied by use of np) to find either expected value **A1cso:** correct solution with both expectations evaluated to p and correct conclusion
(b)	**M1:** use of sum of variances, all terms positive and sight or use of 3^2 and 2^2 **A1:** correct expression for $\text{Var}(\hat{p}_1)$
(c)	**M1:** correct explanation, ft their $\text{Var}(\hat{p}_1)$ **A1ft:** correct deduction, consistent with their $\text{Var}(\hat{p}_1)$
(d)	**M1:** use of $E(\hat{p}_3) = p$ to set up an equation in a and b only **M1:** setting up an inequality $\text{Var}(\hat{p}_3) < \text{Var}(\hat{p}_2)$ using a correct expression for $\text{Var}(\hat{p}_3)$ **M1:** eliminating one variable and setting up 3TQ inequality **A1ft:** correct solution to their inequality, must choose “inside” region **A1:** correctly deducing the values of a and b

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