

## Wednesday 14 June 2023 – Afternoon

### AS Level Further Mathematics B (MEI)

#### Y412/01 Statistics a

Time allowed: 1 hour 15 minutes



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator



#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

#### ADVICE

- Read each question carefully before you start your answer.

- 1 Ryan has 6 one-pound coins and 4 two-pound coins. Ryan decides to select 3 of these coins at random to donate to a charity. The total value, in pounds, of these 3 coins is denoted by the random variable  $X$ .

(a) Show that  $P(X = 3) = \frac{1}{6}$ . [2]

The table below shows the probability distribution of  $X$ .

$r$	3	4	5	6
$P(X = r)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

- (b) Draw a graph to illustrate the distribution. [2]

- (c) **In this question you must show detailed reasoning.**

Find each of the following.

- $E(X)$
- $\text{Var}(X)$  [5]

Ryan's friend Sasha decides to give the same amount as Ryan does to the charity plus an extra three pounds. The random variable  $Y$  represents the **total** amount of money, in pounds, given by Ryan and Sasha.

- (d) Determine each of the following.

- $E(Y)$
- $\text{Var}(Y)$  [3]

- 2 A group of friends live by the sea. Each day they look out to sea in the hope of seeing a dolphin. The probability that they see a dolphin on any day is 0.15. You should assume that this probability is not affected by whether or not they see a dolphin on any other day.
- (a) Explain why you can use a geometric distribution to model the number of days that it takes for them to first see a dolphin. [1]
  - (b) Find the probability that they see a dolphin for the first time on the fifth day. [1]
  - (c) Find the probability that they do not see a dolphin for at least 10 days. [1]
  - (d) Determine the mean and the variance of the number of days that it takes for them to see a dolphin. [3]
- 3 At a pottery which manufactures mugs, it is known that 5% of mugs are faulty. The mugs are produced in batches of 20. Faults are modelled as occurring randomly and independently. The number of faulty mugs in a batch is denoted by the random variable  $X$ .
- (a) Determine  $P(X \geq 2)$ . [2]
  - (b) Find  $\text{Var}(X)$ . [1]

Independently of the mugs, the pottery also manufactures cups, and it is known that 7% of cups are faulty. The cups are produced in batches of 30. Faults are modelled as occurring randomly and independently. The number of faulty cups in a batch is denoted by the random variable  $Y$ .

- (c) Determine the standard deviation of  $X + Y$ . [3]

When 10 batches of cups have been produced, a sample of 15 cups is tested to ensure that the handles of the cups are properly attached.

- (d) Explain why it might not be sensible to select a sample of 15 cups from the same batch. [1]

- 4 At a parcel delivery company it is known that the probability that a parcel is delivered to the wrong address is 0.0005. On a particular day, 15 000 parcels are delivered. The number of parcels delivered to the wrong address is denoted by the random variable  $X$ .
- (a) Explain why the binomial distribution and the Poisson distribution could both be suitable models for the distribution of  $X$ . [3]
- (b) Use a Poisson distribution to find each of the following.
- $P(X = 5)$
  - $P(X \geq 8)$
- [3]

You are given that 15 000 parcels are delivered each day in a 5-day working week.

- (c) (i) Determine the probability that at least 40 parcels are delivered to the wrong address during the week. [2]
- (ii) Determine the probability that at least 8 parcels are delivered to the wrong address on each of the 5 days in the week. [2]
- 5 Two practice GCSE examinations in mathematics are given to all of the students in a large year group. A teacher wants to check whether there is a positive relationship between the marks obtained by the students in the two examinations. She selects a random sample of 20 students. Summary data for the marks obtained in the first and second practice examinations,  $x$  and  $y$  respectively, are as follows.

$$\sum x = 565 \quad \sum y = 724 \quad \sum x^2 = 17103 \quad \sum y^2 = 29286 \quad \sum xy = 21635$$

The teacher decides to carry out a hypothesis test based on Pearson's product moment correlation coefficient.

- (a) **In this question you must show detailed reasoning.**
- Calculate the value of Pearson's product moment correlation coefficient. [4]
- (b) Carry out the test at the 5% significance level. [5]
- (c) Given that the teacher did not draw a scatter diagram before carrying out the test, comment on the validity of the test. [1]

6 An eight-sided dice has its faces numbered 1, 2, ..., 8.

(a) In this part of the question you should assume that the dice is fair.

(i) State the probability that, when the dice is rolled once, the score is at least 6. [1]

(ii) Show that the probability that the score is within 2 standard deviations of its mean is 1. [4]

(b) A student thinks that the dice may be biased. To investigate this, the student decides to roll the dice 80 times and then carry out a  $\chi^2$  goodness of fit test of a uniform distribution. The spreadsheet below shows the data for the test, where some of the values have been deliberately omitted.

	A	B	C	D
1	Score	Observed frequency	Expected frequency	Chi-squared contribution
2	1	14	10	1.6
3	2	4	10	3.6
4	3	10	10	0
5	4	15	10	
6	5	6	10	1.6
7	6	11	10	0.1
8	7	7	10	0.9
9	8		10	0.9

(i) Explain why all of the expected frequencies are equal to 10. [1]

(ii) Determine the missing values in each of the following cells.

- B9

- D5

[3]

(iii) In this question you must show detailed reasoning.

Carry out the  $\chi^2$  test at the 5% significance level.

[6]

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