



# Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE  
In A Level Further Mathematics (9FM0)  
Paper 3D Further Decision 1

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\checkmark$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper
  - $\square$  The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
  5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.  
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

| Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Scheme                                                                                                                                             | Marks          | AOs                  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|----------------|----------------------|
| <b>1(a)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Bin 1: <u>4.7</u> <u>2.9</u> <u>1.4</u> <b>2.8</b><br>Bin 2: <u>5.5</u> <b>5.8</b><br>Bin 3: <b>3.8</b> <b>6.5</b><br>Bin 4: 5.1 6.3<br>Bin 5: 4.1 | M1<br>A1<br>A1 | 1.1b<br>1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                    | <b>(3)</b>     |                      |
| <b>(b)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Bin 1: <u>6.5</u> <u>6.3</u><br>Bin 2: <u>5.8</u> <u>5.5</u> 1.4<br>Bin 3: <u>5.1</u> <u>4.7</u> 2.9<br>Bin 4: <u>4.1</u> <u>3.8</u> 2.8           | M1<br>A1       | 1.1b<br>1.1b         |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                    | <b>(2)</b>     |                      |
| <b>(5 marks)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                    |                |                      |
| <b>Notes:</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                    |                |                      |
| <p><b>a1M1:</b> First four items placed correctly (the boxed values) and at least eight values placed in bins – condone cumulative totals for M1 only</p> <p><b>a1A1:</b> First eight items placed correctly (the boxed <b>and</b> bold values)</p> <p><b>a2A1:</b> CSO (no additional/repeated values)</p> <p><b>b1M1:</b> First eight items placed correctly (the underlined values) and at least ten values placed in bins (no additional/repeated values) - condone cumulative totals for M1 only</p> <p><b>b1A1:</b> CSO</p> |                                                                                                                                                    |                |                      |

| Question | Scheme                                                                                        |     |     |     |     |     |     |     |     |        | Marks                               | AOs                                      |          |
|----------|-----------------------------------------------------------------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|--------|-------------------------------------|------------------------------------------|----------|
| 2(a)     | e.g. Middle right                                                                             |     |     |     |     |     |     |     |     |        | M1<br><br>A1<br><br>A1 ft<br><br>A1 | 1.1b<br><br>1.1b<br><br>1.1b<br><br>1.1b |          |
|          |                                                                                               |     |     |     |     |     |     |     |     | Pivots |                                     |                                          |          |
|          | 271                                                                                           | 828 | 182 | 845 | 904 | 523 | 536 | 028 | 747 | 135    |                                     |                                          | 523      |
|          | 828                                                                                           | 845 | 904 | 536 | 747 | 523 | 271 | 182 | 028 | 135    |                                     |                                          | 904, 028 |
|          | 904                                                                                           | 828 | 845 | 536 | 747 | 523 | 271 | 182 | 135 | 028    |                                     |                                          | 536, 182 |
|          | 904                                                                                           | 828 | 845 | 747 | 536 | 523 | 271 | 182 | 135 | 028    |                                     |                                          | 845      |
|          | 904                                                                                           | 845 | 828 | 747 | 536 | 523 | 271 | 182 | 135 | 028    |                                     |                                          | 747      |
|          | 904                                                                                           | 845 | 828 | 747 | 536 | 523 | 271 | 182 | 135 | 028    |                                     |                                          |          |
|          |                                                                                               |     |     |     |     |     |     |     |     |        |                                     |                                          |          |
|          | Middle left                                                                                   |     |     |     |     |     |     |     |     |        |                                     |                                          |          |
|          |                                                                                               |     |     |     |     |     |     |     |     |        |                                     |                                          | Pivots   |
|          | 271                                                                                           | 828 | 182 | 845 | 904 | 523 | 536 | 028 | 747 | 135    |                                     |                                          | 904      |
|          | 904                                                                                           | 271 | 828 | 182 | 845 | 523 | 536 | 028 | 747 | 135    |                                     |                                          | 523      |
|          | 904                                                                                           | 828 | 845 | 536 | 747 | 523 | 271 | 182 | 028 | 135    |                                     |                                          | 845, 182 |
|          | 904                                                                                           | 845 | 828 | 536 | 747 | 523 | 271 | 182 | 028 | 135    |                                     |                                          | 536, 028 |
|          | 904                                                                                           | 845 | 828 | 747 | 536 | 523 | 271 | 182 | 135 | 028    |                                     |                                          | 828      |
|          | 904                                                                                           | 845 | 828 | 747 | 536 | 523 | 271 | 182 | 135 | 028    |                                     |                                          |          |
|          |                                                                                               |     |     |     |     |     |     |     |     |        | (4)                                 |                                          |          |
| (b)      | In the worst case, the first pass requires (n – 1) comparisons, the second (n – 2), and so on |     |     |     |     |     |     |     |     |        | M1                                  | 3.1a                                     |          |
|          | $\sum_{r=1}^{n-1} r = \frac{1}{2}n(n-1)$                                                      |     |     |     |     |     |     |     |     |        | A1                                  | 2.2a                                     |          |
|          |                                                                                               |     |     |     |     |     |     |     |     |        | (2)                                 |                                          |          |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                        |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|--------------------|--------------------|--------------|-----------|-----------|---|---|---|---|---|---|-------|---|---|---|---|---|-------|---|---------------|-----|---------------|-----|-----|-------|---------------|----------|------|---------------|----------|------|-----------------|-------------|------|----|-----------------|-------------|------------------|------------|------|---|-----|------------------|--------------------|--------------|-------|--|---|-----|--------------------|--------------|------|-------|------------------------|--------------|
| (c)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <table><tr><td><math>a</math></td><td><math>b</math></td><td><math>c</math></td><td><math>d</math></td><td>decimal</td><td>3 sig fig</td><td></td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>Row 1</td></tr><tr><td></td><td></td><td>1</td><td>2</td><td>2</td><td>Row 2</td></tr><tr><td></td><td>2</td><td>2</td><td><math>\frac{5}{2}</math></td><td>2.5</td><td>2.5</td><td>Row 3</td></tr><tr><td></td><td>3</td><td>6</td><td><math>\frac{8}{3}</math></td><td>2.666...</td><td>2.67</td><td>Row 4</td></tr><tr><td></td><td>4</td><td>24</td><td><math>\frac{65}{24}</math></td><td>2.708333...</td><td>2.71</td><td>Row 5</td></tr><tr><td></td><td>5</td><td>120</td><td><math>\frac{163}{60}</math></td><td>2.71666...</td><td>2.72</td><td>Row 6</td></tr><tr><td></td><td>6</td><td>720</td><td><math>\frac{1957}{720}</math></td><td>2.7180555...</td><td>2.72</td><td>Row 7</td></tr></table> | $a$                    | $b$                | $c$                | $d$          | decimal   | 3 sig fig |   | 1 | 1 | 1 | 1 | 1 | Row 1 |   |   | 1 | 2 | 2 | Row 2 |   | 2             | 2   | $\frac{5}{2}$ | 2.5 | 2.5 | Row 3 |               | 3        | 6    | $\frac{8}{3}$ | 2.666... | 2.67 | Row 4           |             | 4    | 24 | $\frac{65}{24}$ | 2.708333... | 2.71             | Row 5      |      | 5 | 120 | $\frac{163}{60}$ | 2.71666...         | 2.72         | Row 6 |  | 6 | 720 | $\frac{1957}{720}$ | 2.7180555... | 2.72 | Row 7 | <b>M1</b><br><b>A1</b> | 1.1b<br>1.1b |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | $a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $b$                    | $c$                | $d$                | decimal      | 3 sig fig |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1                      | 1                  | 1                  | 1            | Row 1     |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                        | 1                  | 2                  | 2            | Row 2     |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2                      | 2                  | $\frac{5}{2}$      | 2.5          | 2.5       | Row 3     |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 3                      | 6                  | $\frac{8}{3}$      | 2.666...     | 2.67      | Row 4     |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 4                      | 24                 | $\frac{65}{24}$    | 2.708333...  | 2.71      | Row 5     |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 5                      | 120                | $\frac{163}{60}$   | 2.71666...   | 2.72      | Row 6     |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 6                      | 720                | $\frac{1957}{720}$ | 2.7180555... | 2.72      | Row 7     |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| Output is $\frac{1957}{720}$ (accept 2.7180555... or 2.72 if working to 3 s.f.)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | <b>A1</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 2.2a                   |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| <b>SC Accept</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                        |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| <table><tr><td><math>a</math></td><td><math>b</math></td><td><math>c</math></td><td><math>d</math></td><td>decimal</td><td>3 sig fig</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr><tr><td></td><td>2</td><td>1</td><td>2</td><td>2</td><td>2</td></tr><tr><td></td><td>3</td><td>2</td><td><math>\frac{5}{2}</math></td><td>2.5</td><td>2.5</td></tr><tr><td></td><td>4</td><td>6</td><td><math>\frac{8}{3}</math></td><td>2.666...</td><td>2.67</td></tr><tr><td></td><td>5</td><td>24</td><td><math>\frac{65}{24}</math></td><td>2.708333...</td><td>2.71</td></tr><tr><td></td><td>6</td><td>120</td><td><math>\frac{163}{60}</math></td><td>2.71666...</td><td>2.72</td></tr><tr><td></td><td></td><td>720</td><td><math>\frac{1957}{720}</math></td><td>2.7180555...</td><td>2.72</td></tr></table> | $a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $b$                    | $c$                | $d$                | decimal      | 3 sig fig | 1         | 1 | 1 | 1 | 1 | 1 |   | 2     | 1 | 2 | 2 | 2 |   | 3     | 2 | $\frac{5}{2}$ | 2.5 | 2.5           |     | 4   | 6     | $\frac{8}{3}$ | 2.666... | 2.67 |               | 5        | 24   | $\frac{65}{24}$ | 2.708333... | 2.71 |    | 6               | 120         | $\frac{163}{60}$ | 2.71666... | 2.72 |   |     | 720              | $\frac{1957}{720}$ | 2.7180555... | 2.72  |  |   |     |                    |              |      |       |                        |              |
| $a$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $b$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | $c$                    | $d$                | decimal            | 3 sig fig    |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1                      | 1                  | 1                  | 1            |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1                      | 2                  | 2                  | 2            |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 2                      | $\frac{5}{2}$      | 2.5                | 2.5          |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 6                      | $\frac{8}{3}$      | 2.666...           | 2.67         |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 24                     | $\frac{65}{24}$    | 2.708333...        | 2.71         |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 120                    | $\frac{163}{60}$   | 2.71666...         | 2.72         |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 720                    | $\frac{1957}{720}$ | 2.7180555...       | 2.72         |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (3)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                        |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| (d)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\frac{\left  \frac{1957}{720} - e \right }{e} \times 100 = 0.00832(\%) \quad \text{If using 2.72 error} = 0.0632(\%)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | <b>B1</b><br><b>ft</b> | 3.2b               |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | (1)                    |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |
| (10 marks)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                        |                    |                    |              |           |           |   |   |   |   |   |   |       |   |   |   |   |   |       |   |               |     |               |     |     |       |               |          |      |               |          |      |                 |             |      |    |                 |             |                  |            |      |   |     |                  |                    |              |       |  |   |     |                    |              |      |       |                        |              |

## Notes:

**a1M1:** Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing any other item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). Condone one error or omission in the list.

**If only choosing one pivot per iteration then max of M1 only**

Bubble sort is not a MR and scores M0.

If the list is sorted into ascending order, they can score M1 A1 A0 A0 for a fully correct sort even if reversed (if any error seen this can score M1 only).

If 028 is written as 28 at any stage this will lose the final CSO mark

The choice of pivots may only be seen once they have been used

**a1A1:** First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct)

**a2A1ft:** Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

If they alternate MR and ML pivots for the first and second passes, the pivots for the third pass must be consistent with their second pass

**a3A1:** CSO – so must have chosen and used 747 (MR) or 828 (ML) as a pivot for the fifth pass

**b1M1:** Considering the total number of comparisons in the worst case (possibly will see e.g.  $(n - 1) + (n - 2) + \dots + 2 + 1$  ] which scores this mark (brackets not needed and sufficient terms to make sum clear. Accept a clear list without + signs for this mark)

**b1A1:** Using the standard series result that  $\sum r = \frac{1}{2}n(n + 1)$  with  $n = (n - 1)$  to get the correct quadratic expression for the total number of comparisons when sorting  $n$  values using bubble sort (allow any letter) Accept  $(n - 1) + (n - 2) + \dots + 2 + 1 = \frac{1}{2}n(n - 1)$  for both marks (sight of  $\frac{1}{2}n(n - 1)$  with no working scores both marks) (ISW if correct expression incorrectly expanded)

**Note: Rows are shown above, and they may be spread across multiple rows in the table. Each row ends when the appropriate value in column d is seen**

**c1M1:** At least four rows of cells completed with a correct first and second row – condone repeated values in all columns or a single value in each row (Note SC accept Column B with 1, 2, 3, 4, 5, 6 with no empty cell between 1 and 2)

**c1A1:** CAO – the values in the third, fourth and fifth rows correct (accept either full recurring decimals or decimals consistently rounded to 3 significant figures)

**c2A1:** CAO - correct output following a correct seventh row (their output must be clearly indicated e.g. on the Output line or identified in or near the table) If a value is written on the output line, this takes precedence over the table.

**d1B1ft:** Follow through their answer to (c) provided that % error is  $< 0.1\%$  (must be given to at least 3 significant figures) (condone missing %) (accept - 0.00832%) (if a numerical approximation to  $e$  is used, it must be given to at least 3 d.p. – so 2.718 or better) (accept  $8.32 \times 10^{-3}$ ). If incorrect working is seen, even if followed by a correct answer, this is B0.

$$\text{Note } \frac{\left| \frac{1957}{720} - e \right|}{\frac{1957}{720}} \times 100 = 0.00832\% \text{ scores B0}$$

Sort Ascending Max M1 A1 A0 A0 e.g Middle right

|            |            |            |            |            |            |            |            |            |            | Pivots   |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|----------|
| 271        | 828        | 182        | 845        | 904        | <b>523</b> | 536        | 028        | 747        | 135        | 523      |
| 271        | 182        | <b>028</b> | 135        | <u>523</u> | 828        | 845        | <b>904</b> | 536        | 747        | 028, 904 |
| <u>028</u> | 271        | <b>182</b> | 135        | <u>523</u> | 828        | 845        | <b>536</b> | 747        | <u>904</u> | 182, 536 |
| <u>028</u> | 135        | <u>182</u> | 271        | <u>523</u> | <u>536</u> | 828        | <b>845</b> | 747        | <u>904</u> | 845      |
| <u>028</u> | <u>135</u> | <u>182</u> | <u>271</u> | <u>523</u> | <u>536</u> | 828        | <b>747</b> | <u>845</u> | <u>904</u> | 747      |
| <u>028</u> | <u>135</u> | <u>182</u> | <u>271</u> | <u>523</u> | <u>536</u> | <u>747</u> | 828        | <u>845</u> | <u>904</u> |          |

Middle left

|            |            |            |            |            |            |            |     |            |            | Pivots   |
|------------|------------|------------|------------|------------|------------|------------|-----|------------|------------|----------|
| 271        | 828        | 182        | 845        | <b>904</b> | 523        | 536        | 028 | 747        | 135        | 904      |
| 271        | 828        | 182        | 845        | <b>523</b> | 536        | 028        | 747 | 135        | <u>904</u> | 523      |
| 271        | <b>182</b> | 028        | 135        | <u>523</u> | 828        | <b>845</b> | 536 | 747        | <u>904</u> | 182, 845 |
| <b>028</b> | 135        | <u>182</u> | 271        | <u>523</u> | 828        | <b>536</b> | 747 | <u>845</u> | <u>904</u> | 028, 536 |
| <u>028</u> | 135        | <u>182</u> | <u>271</u> | <u>523</u> | <u>536</u> | <b>828</b> | 747 | <u>845</u> | <u>904</u> | 828      |
| <u>028</u> | <u>135</u> | <u>182</u> | <u>271</u> | <u>523</u> | <u>536</u> | 747        | 828 | <u>845</u> | <u>904</u> |          |

| Question   | Scheme                                                      |    |    |    |    |    |    |    | Marks    | AOs          |    |     |
|------------|-------------------------------------------------------------|----|----|----|----|----|----|----|----------|--------------|----|-----|
| 3(a)       |                                                             | A  | B  | C  | D  | E  | F  | G  | B2, 1, 0 | 1.1b<br>1.1b |    |     |
|            | A                                                           | -  | 9  | ∞  | 25 | 35 | ∞  | 11 |          |              |    |     |
|            | B                                                           | ∞  | -  | 40 | 8  | ∞  | 35 | ∞  |          |              |    |     |
|            | C                                                           | ∞  | 40 | -  | 17 | ∞  | ∞  | 47 |          |              |    |     |
|            | D                                                           | ∞  | 8  | 17 | -  | 11 | 31 | ∞  |          |              |    |     |
|            | E                                                           | 35 | ∞  | ∞  | 11 | -  | 14 | ∞  |          |              |    |     |
|            | F                                                           | ∞  | 35 | ∞  | 31 | 14 | -  | 12 |          |              |    |     |
|            | G                                                           | 11 | ∞  | 47 | ∞  | 24 | 12 | -  |          |              |    |     |
|            |                                                             |    |    |    |    |    |    |    | (2)      |              |    |     |
| (b)        | Fourth iteration:                                           |    |    |    |    |    |    |    | M1       | 2.1          |    |     |
|            |                                                             | A  | B  | C  | D  | E  | F  | G  |          |              |    |     |
|            | A                                                           | -  | 9  | 34 | 17 | 28 | 44 | 11 |          |              |    |     |
|            | B                                                           | ∞  | -  | 25 | 8  | 19 | 35 | 72 |          |              |    |     |
|            | C                                                           | ∞  | 25 | -  | 17 | 28 | 48 | 47 |          |              |    |     |
|            | D                                                           | ∞  | 8  | 17 | -  | 11 | 31 | 64 |          |              |    |     |
|            | E                                                           | 35 | 19 | 28 | 11 | -  | 14 | 46 |          |              |    |     |
|            | F                                                           | ∞  | 35 | 48 | 31 | 14 | -  | 12 | A1       | 1.1b         |    |     |
|            | G                                                           | 11 | 20 | 45 | 28 | 24 | 12 | -  |          |              |    |     |
|            | Fifth iteration:                                            |    |    |    |    |    |    |    |          |              |    |     |
|            |                                                             | A  | B  | C  | D  | E  | F  | G  |          |              |    |     |
|            | A                                                           | -  | 9  | 34 | 17 | 28 | 42 | 11 |          |              | M1 | 2.1 |
|            | B                                                           | 54 | -  | 25 | 8  | 19 | 33 | 65 |          |              |    |     |
|            | C                                                           | 63 | 25 | -  | 17 | 28 | 42 | 47 |          |              |    |     |
|            | D                                                           | 46 | 8  | 17 | -  | 11 | 25 | 57 | A1       | 1.1b         |    |     |
|            | E                                                           | 35 | 19 | 28 | 11 | -  | 14 | 46 |          |              |    |     |
|            | F                                                           | 49 | 33 | 42 | 25 | 14 | -  | 12 |          |              |    |     |
|            | G                                                           | 11 | 20 | 45 | 28 | 24 | 12 | -  | A1       | 2.2a         |    |     |
|            |                                                             |    |    |    |    |    |    |    |          |              |    |     |
|            |                                                             |    |    |    |    |    |    |    |          |              |    |     |
|            |                                                             |    |    |    |    |    |    |    | (5)      |              |    |     |
| (c)(i)     | NNA starting at B: B – D – E – F – G – A – C – B            |    |    |    |    |    |    |    | B1       | 2.2a         |    |     |
| (ii)       | Time taken: 8 + 11 + 14 + 12 + 11 + 34 + 25 = 115 (minutes) |    |    |    |    |    |    |    | B1       | 1.1b         |    |     |
| (iii)      | Actual villages visited: BDEFGAB <u>DC</u> <u>DB</u>        |    |    |    |    |    |    |    | B1       | 2.2a         |    |     |
|            |                                                             |    |    |    |    |    |    |    | (3)      |              |    |     |
| (10 marks) |                                                             |    |    |    |    |    |    |    |          |              |    |     |

**(10 marks)**

**Notes:**

**a1B1:** At least three rows and three columns correct (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

**a2B1:** CAO

**b1M1:** No change in the fourth row and fourth column with at least two values reduced correctly (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

**b1A1:** CAO for the fourth iteration (changes in bold and blue shading)

**b2M1:** No change in the fifth row and fifth column with at least two values reduced correctly (follow through from their previous iteration) (condone at most 2 empty cells but accept consistent blanks for the leading diagonal)

**b2A1:** At least eight values reduced correctly (from the values shown in bold and orange shading)

**b3A1:** CSO for the fourth and fifth iterations

(if correct changes for the fifth iteration are shown in bold and orange shading)

**ci1B1:** CAO (Nearest neighbour cycle starting at B – must include return to B) (condone the use of arcs BD DE EF FG GA AC CB – must be written in this manner and not AG or BC)

**cii2B1:** Correct time

**ciii3B1:** Correct route – must be stated as nodes

| Question   | Scheme                                                                                                                                          | Marks                                                                                                                                     | AOs                                                             |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| 4(a)       | <p>(i) Route from J to K via A: JEBACAGFK<br/> (ii) Length of shortest route from J to K via A is 76 (m)</p>                                    | <p><b>M1</b><br/> <b>A1</b><br/> (ACBG<br/> D)<br/> <b>A1</b><br/> (FKE)<br/> <b>A1ft</b><br/> (HJ)</p> <p><b>A1</b><br/> <b>A1ft</b></p> | <p>1.1b<br/> 1.1b<br/> 1.1b<br/> 1.1b</p> <p>2.2a<br/> 2.2a</p> |
|            |                                                                                                                                                 | (6)                                                                                                                                       |                                                                 |
| (b)(i)     | $BC + D(GF)K = 6 + 18 = 24$<br>$B(FG)D + C(GF)K = 15 + 26 = 41$<br>$B(F)K + C(G)D = 23 + 10 = 33$<br>Edges to traverse twice are BC, DG, FG, FK | <p><b>M1</b><br/> <b>A1</b><br/> <b>A1</b><br/> <b>A1</b></p>                                                                             | <p>3.1b<br/> 1.1b<br/> 1.1b<br/> 2.2a</p>                       |
| (ii)       | Length of route = $268 + 24 = 292$ (m)                                                                                                          | <b>A1ft</b>                                                                                                                               | 2.2a                                                            |
|            |                                                                                                                                                 | (5)                                                                                                                                       |                                                                 |
| (c)        | E.g. $[24 - ] ("45" - 13 - 11 - 16)$<br>Or $[292 - ] (268 + "45" - (13 + 11 + 16))$<br><br>= 19 (m)                                             | <p><b>M1</b><br/> <br/> <b>A1</b></p>                                                                                                     | <p>3.1b<br/> <br/> 2.2a</p>                                     |
|            |                                                                                                                                                 | (2)                                                                                                                                       |                                                                 |
| (13 marks) |                                                                                                                                                 |                                                                                                                                           |                                                                 |

## Notes:

**In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 22 21 18 in that order (so 22 18 21 is incorrect)**

**It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4,... will be penalised once (see notes below) but 1, 2, 3, 5, 6,... is fine. Errors in the final values and working values are penalised before errors in the order of labelling**

### **If starting at J or K instead of A M0**

**a1M1:** A larger value replaced by a smaller value in at least two of the working value boxes at either B or D or F or H or J or K

**a1A1:** All values at A, C, B, G and D correct and working values in the correct order. Condone lack of 0 in As working value

**a2A1:** All values at F, K and E correct and the working values in the correct order. Penalise order of labelling only once per question (F, K and E must be labelled in that order and F must be labelled after A, C, B, G and D)

**a3A1ft:** All values at H and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working values at H follow from the candidate's final values from the feeds into H (which will come from nodes D and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from B, E and K with the order of labelling determined by the candidate's order of labelled at B, E and K)

**a4A1:** CAO (correct path from J to K via A – JEBCACGFK) (accept in terms of arcs)

**a5A1ft:** ft their final value at J + their final value at K only (if 76 stated and 76 is not the sum of these final values then A0)

**If route and length are written on the wrong lines this can still score both marks**

**bi1M1:** The correct three pairings of the correct four nodes (B, C, D and K)

**bi1A1:** Two rows correct including pairings and totals

**bi2A1:** All three rows correct including pairings and totals

**bi3A1:** Correct repeats (BC, DG, FG, FK)

**bii4A1ft:** Length of route correct on the follow through (268 + smallest repeat) but must be from a choice of 3 totals

**c1M1:** For using their final value at J and the blocked lengths in attempt to work out the length or change in length of the new route (sight of “45” 13 11 and 16) (follow through their final value at J for this mark) (condone use of 40 for 13 + 11 + 16)

**c1A1:** CAO (19)

| Question   | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Marks                  | AOs                         |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|-----------------------------|-----------------|----------------|----------------|----------------|---------------------------|-------|---------|---|---|---|---|---------------|---------------|---|----------------|---------------------------|---|---|---|---|----------------|---------------|---------------|----------------|------------------------|---|---|---|---|----------------|---------------|---------------|----------------|---------------------------|---|---|---|---|-----------------|---------------|---------------|----------------|---------------------------|----------------------------------|-----------------------------------------|
| 5(a)       | <p>The Simplex algorithm cannot be used as the problem has no obvious basic feasible solution since the origin is not in the feasible region</p> <p>Alternatively</p> <p>Simplex can only be used with non-negative values of variables</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | B1                     | 3.5b                        |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (1)                    |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (b)        | <p><math>2(X+4)+2(Y+1)+2(-Z-3)\leq 21\Rightarrow 2X+2Y-2Z\leq 17</math></p> <p><math>2(X+4)-(Y+1)-(-Z-3)\leq 18\Rightarrow 2X-Y+Z\leq 8</math></p> <p><math>-3(X+4)+(Y+1)-2(-Z-3)\leq 1\Rightarrow -3X+Y+2Z\leq 6</math></p> <p><math>Q+10=3(X+4)+4(Y+1)+2(-Z-3)\Rightarrow Q=3X+4Y-2Z</math></p> <p><math>(X\geq 0,Y\geq 0,Z\geq 0)</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | M1<br><br>A1<br><br>A1 | 3.3<br><br>1.1b<br><br>1.1b |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (3)                    |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (c)        | $Q-\frac{5}{4}Z+\frac{15}{8}s_1+\frac{1}{4}s_3=\frac{267}{8}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | B1                     | 3.4                         |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (1)                    |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (d)        | $Q=\frac{5}{4}Z-\frac{15}{8}s_1-\frac{1}{4}s_3+\frac{267}{8}$ so therefore, we can increase the profit by increasing Z                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | B1                     | 2.4                         |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (1)                    |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (e)(i)     | <table><tr><th>b.v.</th><th>X</th><th>Y</th><th>Z</th><th>s<sub>1</sub></th><th>s<sub>2</sub></th><th>s<sub>3</sub></th><th>Value</th><th>Row Ops</th></tr><tr><td>X</td><td>1</td><td>0</td><td>0</td><td><math>\frac{1}{6}</math></td><td><math>\frac{1}{3}</math></td><td>0</td><td><math>\frac{11}{2}</math></td><td><math>r1+\frac{3}{4}\times R2</math></td></tr><tr><td>Z</td><td>0</td><td>0</td><td>1</td><td><math>\frac{1}{18}</math></td><td><math>\frac{4}{9}</math></td><td><math>\frac{1}{3}</math></td><td><math>\frac{13}{2}</math></td><td><math>\frac{4}{9}\times r2</math></td></tr><tr><td>Y</td><td>0</td><td>1</td><td>0</td><td><math>\frac{7}{18}</math></td><td><math>\frac{1}{9}</math></td><td><math>\frac{1}{3}</math></td><td><math>\frac{19}{2}</math></td><td><math>r3+\frac{1}{4}\times R2</math></td></tr><tr><td>Q</td><td>0</td><td>0</td><td>0</td><td><math>\frac{35}{18}</math></td><td><math>\frac{5}{9}</math></td><td><math>\frac{2}{3}</math></td><td><math>\frac{83}{2}</math></td><td><math>r4+\frac{5}{4}\times R2</math></td></tr></table> | b.v.                   | X                           | Y               | Z              | s <sub>1</sub> | s <sub>2</sub> | s <sub>3</sub>            | Value | Row Ops | X | 1 | 0 | 0 | $\frac{1}{6}$ | $\frac{1}{3}$ | 0 | $\frac{11}{2}$ | $r1+\frac{3}{4}\times R2$ | Z | 0 | 0 | 1 | $\frac{1}{18}$ | $\frac{4}{9}$ | $\frac{1}{3}$ | $\frac{13}{2}$ | $\frac{4}{9}\times r2$ | Y | 0 | 1 | 0 | $\frac{7}{18}$ | $\frac{1}{9}$ | $\frac{1}{3}$ | $\frac{19}{2}$ | $r3+\frac{1}{4}\times R2$ | Q | 0 | 0 | 0 | $\frac{35}{18}$ | $\frac{5}{9}$ | $\frac{2}{3}$ | $\frac{83}{2}$ | $r4+\frac{5}{4}\times R2$ | B1<br><br>M1<br><br>A1<br><br>A1 | 1.1b<br><br>2.1<br><br>1.1b<br><br>1.1b |
| b.v.       | X                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Y                      | Z                           | s <sub>1</sub>  | s <sub>2</sub> | s <sub>3</sub> | Value          | Row Ops                   |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| X          | 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 0                      | 0                           | $\frac{1}{6}$   | $\frac{1}{3}$  | 0              | $\frac{11}{2}$ | $r1+\frac{3}{4}\times R2$ |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| Z          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 0                      | 1                           | $\frac{1}{18}$  | $\frac{4}{9}$  | $\frac{1}{3}$  | $\frac{13}{2}$ | $\frac{4}{9}\times r2$    |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| Y          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 1                      | 0                           | $\frac{7}{18}$  | $\frac{1}{9}$  | $\frac{1}{3}$  | $\frac{19}{2}$ | $r3+\frac{1}{4}\times R2$ |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| Q          | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 0                      | 0                           | $\frac{35}{18}$ | $\frac{5}{9}$  | $\frac{2}{3}$  | $\frac{83}{2}$ | $r4+\frac{5}{4}\times R2$ |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (ii)       | $X=\frac{11}{2},Y=\frac{19}{2},Z=\frac{13}{2},Q=\frac{83}{2}$ ( $X=5.5\ Y=9.5\ Z=6.5\ Q=41.5$ )<br>$x=\frac{19}{2},y=\frac{21}{2},z=-\frac{19}{2},P=\frac{103}{2}$ ( $x=9.5\ y=10.5\ z=-9.5\ P=51.5$ )                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | M1<br>A1ft             | 3.4<br>2.2a                 |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | (6)                    |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |
| (12 marks) |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                        |                             |                 |                |                |                |                           |       |         |   |   |   |   |               |               |   |                |                           |   |   |   |   |                |               |               |                |                        |   |   |   |   |                |               |               |                |                           |   |   |   |   |                 |               |               |                |                           |                                  |                                         |

## Notes:

**a1B1:** Correct reason why Simplex cannot be used to solve the LP problem. Please mark positively and award if a correct statement is seen.

SC accept Because  $z \leq -3$  (and variables must be  $\geq 0$ )

**b1M1:** Substituting given equations into all three given inequalities to form inequalities or equations with slack variables in terms of  $X$ ,  $Y$  and  $Z$  (condone at most two slips but see Special Case below)

**b1A1:** At least two of the four expressions simplified correctly (three inequalities and the new objective) (accept any equivalent rearrangement but the objective must be in terms of  $Q$  not  $P$ ) (accept equations with slack variables)

**b2A1:** CAO (must be inequalities)

**c1B1:** CAO (must be in terms of  $Q$ )

**d1B1:** CAO (Must include mention that  $Z$  can be increased but do not award if any incorrect statement seen)

Do not accept just a statement that the objective row contains a negative value and therefore it is not optimal

Note – accept correct recurring decimals in place of fractions or any equivalent fractions

**ei1B1:** Pivot row ( $Z$  row) correct including change of b.v. (ignore row ops)

**ei1M1:** All values in one of the non-pivot rows correct **or** one of the non zero and one columns ( $s_1, s_2, s_3$  or value) correct

**ei1A1:** Row operations used correctly at least twice, i.e. **two** of the non-pivot rows or **two** of the non zero and one columns ( $s_1, s_2, s_3$  or value) – ignore row operations for this mark

**ei2A1:** CAO all values and row operations correctly stated including b.v column (allow alternative numbering of rows as long as this is clear. Condone use of  $r_2$  throughout. Do not accept in terms of b.v.) (Accept in terms of original  $r_2$  so  $r_1 + 1/3r_2$ ,  $4/9r_2$ ,  $r_3 + 1/9r_2$ ,  $r_4 + 5/9r_2$ ) (Row ops for pivot row may be written as  $r_2 \div 9/4$ )

**eii2M1:** Stating (or implying) optimal values of  $X$ ,  $Y$ ,  $Z$  and  $Q$  (see special case)

**eii3A1ft:** CAO for  $x$ ,  $y$ ,  $z$  and  $P$  (the correct 4 values implies both marks) (follow through their values for  $X$ ,  $Y$ ,  $Z$  and  $Q$  from the tableau)

## Special Case

(b) If they make a consistent error when substituting by using an incorrect expression for  $X$ ,  $Y$  or  $Z$  (e.g. substituting  $x = X + 1$  instead of  $x = X + 4$  in all terms) they may score M1 A1 (for at least two of their four expressions correct) A0

(e) If they use the same incorrect substitution and do not state the values of  $X$ ,  $Y$ ,  $Z$  and  $Q$  they may score M1 A0 for the implied values of  $X$ ,  $Y$ ,  $Z$  and  $Q$  from their  $x$ ,  $y$ ,  $z$  and  $P$

| Que<br>stio<br>n | Scheme                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Marks                                                   | AOs                                                             |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|-----------------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|----------|-----|---|---------|---|------|---|---------|---|------|---|---------|---|------------|---|------------|-----------------------------|---------------------------------|
| 6(a)             | <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>-</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>C</td></tr><tr><td>F</td><td>C</td></tr></tbody></table> <table><thead><tr><th>Activity</th><th>IPA</th></tr></thead><tbody><tr><td>G</td><td>A, B, E</td></tr><tr><td>H</td><td>D, G</td></tr><tr><td>I</td><td>A, B, E</td></tr><tr><td>J</td><td>H, I</td></tr><tr><td>K</td><td>A, B, E</td></tr><tr><td>L</td><td>A, B, E, F</td></tr><tr><td>M</td><td>A, B, E, F</td></tr></tbody></table> | Activity                                                | IPA                                                             | A | - | B | - | C | - | D | A | E | C | F | C | Activity | IPA | G | A, B, E | H | D, G | I | A, B, E | J | H, I | K | A, B, E | L | A, B, E, F | M | A, B, E, F | <div>B1</div> <div>B1</div> | <div>1.1b</div> <div>1.1b</div> |
| Activity         | IPA                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| A                | -                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| B                | -                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| C                | -                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| D                | A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| E                | C                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| F                | C                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| Activity         | IPA                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| G                | A, B, E                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| H                | D, G                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| I                | A, B, E                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| J                | H, I                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| K                | A, B, E                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| L                | A, B, E, F                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| M                | A, B, E, F                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                         |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | (2)                                                     |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| (b)              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | <div>M1</div> <div>A1</div> <div>M1</div> <div>A1</div> | <div>1.1b</div> <div>1.1b</div> <div>1.1b</div> <div>1.1b</div> |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | (4)                                                     |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
| (c)              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | <div>M1</div> <div>A1</div> <div>A1</div> <div>A1</div> | <div>2.1</div> <div>1.1b</div> <div>1.1b</div> <div>1.1b</div>  |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |
|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | (4)                                                     |                                                                 |   |   |   |   |   |   |   |   |   |   |   |   |          |     |   |         |   |      |   |         |   |      |   |         |   |            |   |            |                             |                                 |

|     |                 |                |  |                 |                |  |  |      |
|-----|-----------------|----------------|--|-----------------|----------------|--|--|------|
| (d) | <b>Activity</b> | <b>Workers</b> |  | <b>Activity</b> | <b>Workers</b> |  |  |      |
|     | A               | 1              |  | G               | 2              |  |  |      |
|     | B               | 2              |  | H               | 1              |  |  | 3.4  |
|     | C               | 1              |  | I               | 2              |  |  | 1.1b |
|     | D               | 2              |  | J               | 2              |  |  | 1.1b |
|     | E               | 3              |  | K               | 1              |  |  |      |
|     | F               | 1              |  | L               | 1              |  |  |      |
|     |                 |                |  | M               | 1              |  |  |      |

(13 marks)

**Notes:**

**a1B1:** Five correct rows (not including A, B and C)

**a2B1:** All rows correct (accept blanks for A, B and C)

**b1M1:** All top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue value which is a number in a top box greater than the subsequent value

**b1A1:** CAO – top boxes

**b2M1:** All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue value which is a number in a bottom box greater than the previous value. Condone missing 0 and/or their 20 (at the end event) for the M mark only

**b2A1:** CAO – bottom boxes

**c1M1:** Gantt (cascade) chart with at least 9 activities labelled and at least four activities having non-zero float. A scheduling diagram scores M0

**c1A1:** Critical activities (C, E, G, H and J) correct (note this may be seen as separate activities listed alphabetically instead of at the top of the chart)

**c2A1:** At least 4 non-critical activities correct

**c3A1:** CAO All 13 activities present (just once). No errors.

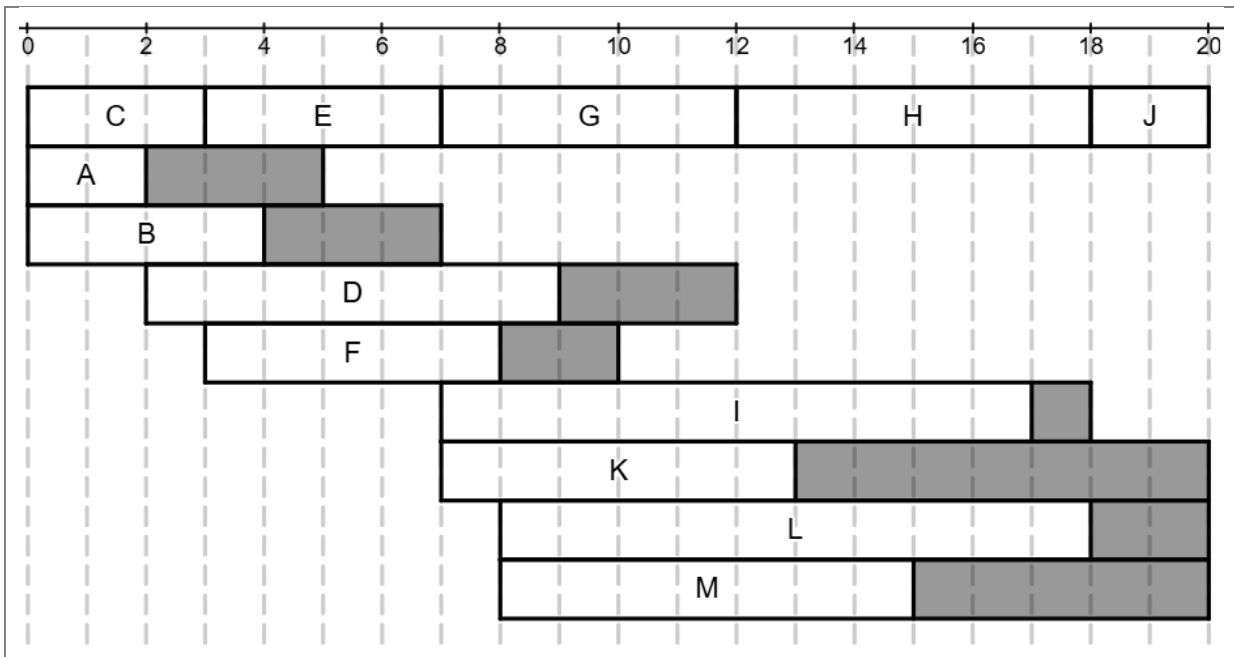
**d1B1:** At least four correct

**d2B1:** At least eight correct

**d3B1:** CAO

For reference these are the early start, late end and length of float for the non-critical activities

| Activity | Start | Late End | Float | Critical | Start | End |
|----------|-------|----------|-------|----------|-------|-----|
| A        | 0     | 5        | 3     | C        | 0     | 3   |
| B        | 0     | 7        | 3     | E        | 3     | 7   |
| D        | 2     | 12       | 3     | G        | 7     | 12  |
| F        | 3     | 10       | 2     | H        | 12    | 18  |
| I        | 7     | 18       | 1     | J        | 18    | 20  |
| K        | 7     | 20       | 7     |          |       |     |
| L        | 8     | 20       | 2     |          |       |     |
| M        | 8     | 20       | 5     |          |       |     |



| Qu         | Scheme                                                                                                                                                         |              |             |       |       |       |       |       |       |       | Marks        | AOs             |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|-------------|-------|-------|-------|-------|-------|-------|-------|--------------|-----------------|
| 7(a)       | $3x + 4y \leq 20 \Rightarrow 3x + 4y + s_1 = 20$<br>$3x + y \leq 8 \Rightarrow 3x + y + s_4 = 8$                                                               |              |             |       |       |       |       |       |       |       | B1           | 2.5             |
|            | $x \geq 1 \Rightarrow x - s_2 + a_1 = 1$<br>$2x + 3y \geq 6 \Rightarrow 2x + 3y - s_3 + a_2 = 6$                                                               |              |             |       |       |       |       |       |       |       | B1           | 2.5             |
|            | $P = 11x + ky - M(a_1 + a_2)$<br>$a_1 + a_2 = 7 - 3x - 3y + s_2 + s_3$<br>$P = 11x + ky - M(7 - 3x - 3y + s_2 + s_3)$                                          |              |             |       |       |       |       |       |       |       | M1           | 2.1             |
|            | $P - (11 + 3M)x - (k + 3M)y + Ms_2 + Ms_3 = -7M$                                                                                                               |              |             |       |       |       |       |       |       |       | A1           | 2.2a            |
|            | e.g.                                                                                                                                                           |              |             |       |       |       |       |       |       |       | M1<br><br>A1 | 3.3<br><br>2.2a |
|            | b.v.                                                                                                                                                           | $x$          | $y$         | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $a_1$ | $a_2$ | Value |              |                 |
|            | $s_1$                                                                                                                                                          | 3            | 4           | 1     | 0     | 0     | 0     | 0     | 0     | 20    |              |                 |
|            | $a_1$                                                                                                                                                          | 1            | 0           | 0     | -1    | 0     | 0     | 1     | 0     | 1     |              |                 |
|            | $a_2$                                                                                                                                                          | 2            | 3           | 0     | 0     | -1    | 0     | 0     | 1     | 6     |              |                 |
|            | $s_4$                                                                                                                                                          | 3            | 1           | 0     | 0     | 0     | 1     | 0     | 0     | 8     |              |                 |
|            | $P$                                                                                                                                                            | $-(11 + 3M)$ | $-(k + 3M)$ | 0     | $M$   | $M$   | 0     | 0     | 0     | $-7M$ |              |                 |
|            |                                                                                                                                                                |              |             |       |       |       |       |       |       |       |              |                 |
|            |                                                                                                                                                                |              |             |       |       |       |       |       |       |       | (7)          |                 |
| (b)        | Using $x = \frac{18}{7}, y = \frac{2}{7}$ or stating $P = \frac{198}{7} + \frac{2}{7}k$                                                                        |              |             |       |       |       |       |       |       |       | B1           | 3.4             |
|            | If optimal after the third iteration, then $\frac{11}{7} - \frac{3}{7}k \geq 0$ and $\frac{33}{7} - \frac{2}{7}k \geq 0$                                       |              |             |       |       |       |       |       |       |       | M1           | 3.1a            |
|            | $(0 <) k \leq \frac{11}{3}$                                                                                                                                    |              |             |       |       |       |       |       |       |       | A1           | 2.2a            |
|            | Maximum $P$ when $k = \frac{11}{3}$ and $P = 11\left(\frac{18}{7}\right) + \frac{11}{3}\left(\frac{2}{7}\right)$                                               |              |             |       |       |       |       |       |       |       | dM1          | 3.4             |
|            | Optimal value of $P$ is $\frac{88}{3}$<br>accept answers from a stated value of k from $(0 <) k \leq \frac{11}{3}$<br>so $\frac{198}{7} < P \leq \frac{88}{3}$ |              |             |       |       |       |       |       |       |       | A1           | 2.2a            |
|            |                                                                                                                                                                |              |             |       |       |       |       |       |       |       | (5)          |                 |
| (12 marks) |                                                                                                                                                                |              |             |       |       |       |       |       |       |       |              |                 |

**Notes:**

**If correct they must use two slack, two surplus and two artificial variables. Accept alternative letters for these as long as the artificial variables are clearly identifiable**

**Please check suffices on  $s$  and  $a$  terms carefully – they may be in a different order – check that these are consistent**

**a1B1:** Correctly re-writing the two  $\leq$  inequalities as equations with slack variables (can be implied by the corresponding two correct rows in Simplex tableau – our row 1 and 4)

**a2B1:** Correctly re-writing one of the  $\geq$  inequalities as an equation with surplus and artificial variables (can be implied by a correct corresponding row in Simplex tableau – our row 2 or 3)

**a3B1:** Correctly re-writing both  $\geq$  inequalities

**a1M1:** Forming an objective of the form  $P = 11x + ky - M(a_1 + a_2)$  and substituting for  $a_1$  and  $a_2$  (we must see the substitution but this does not need to be a correct expression for this mark)

**a1A1:** CAO for new objective (accept equivalent equation with terms in  $x$  and  $y$  collected) (M1 A1 may be implied by a correct objective row in the tableau)

**a2M1:** Any two rows correct on the ft from the candidate's stated equations (ignore b.v. for this mark)

**a2A1:** CAO (including consistent b.v. column) – note that the candidate's order in which the rows appear in the tableau (and choice of slack variable) may be different

A fully correct tableau implies all marks in (a) provided that there are no errors seen in the formation of the objective function

**b1B1:** Either using the correct values of  $x$  and  $y$  in the objective function or stating  $P = \frac{198}{7} + \frac{2}{7}k$

**b1M1:** Considering at least one of the expressions ( $s_3$  or  $s_4$  columns) in the  $P$  row that involve  $k$  and compare with 0 (accept any correct inequality or equals)

**b1A1:** Correct range of values for  $k$  (condone missing  $0 <$ ) but must have considered both possibilities and chosen  $11/3$  (may be implied by subsequent working) (allow  $k = 11/3$  stated as the maximum value)

**b2dM1: Dependent on previous M mark** – using their  $k$  in given  $P$  which must come from a correct inequality or equation (they may choose a value of  $k$  from the correct range e.g.  $k = 3$ )

**b2A1:** CAO - Correct value of  $P$  for their choice of  $k$

(accept answers in the range  $\frac{198}{7} < P \leq \frac{88}{3}$  if  $k = 3$   $P = 204/7$ )

