Pearson Edexcel Level 3 GCE

Monday 23 June 2025

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/4D**

Further Mathematics

Advanced

PAPER 4D: Decision Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator, Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
- – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end

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P 7 5 6 9 2 A

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1. Ravi can choose one of three options, A, B or C, when playing a game. The profit, in pounds, associated with each outcome, and the corresponding probabilities, are shown in the decision tree in Figure 1.

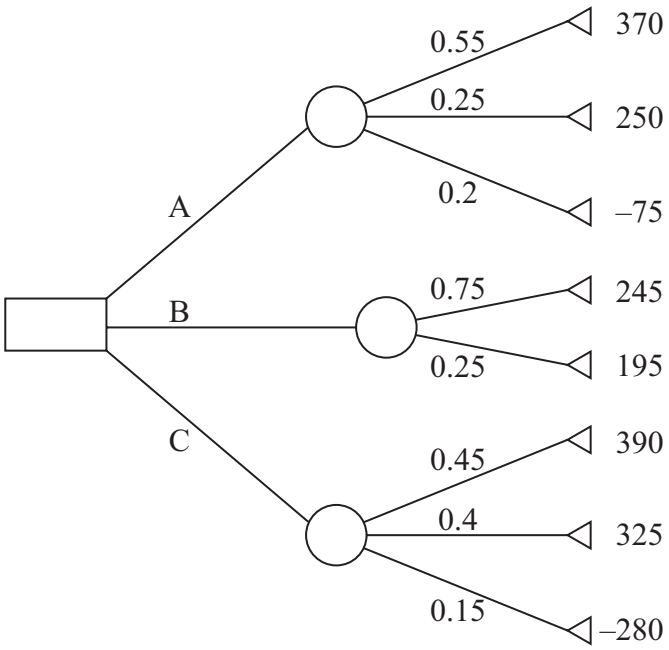

Figure 1

Calculate the optimal EMV to determine Ravi's best course of action. You must make your working clear.

(Total for Question 1 is 3 marks)

2. Five workers, A, B, C, D and E, are available to complete four tasks, P, Q, R and S.

Each worker can be assigned to at most one task, and each task must be done by at most one worker.

Worker B cannot be assigned to task R.

The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S
A	25	32	43	28
B	41	37	–	38
C	43	35	37	39
D	40	33	37	41
E	37	38	43	40

The Hungarian algorithm is to be used to find an allocation that minimises the total time to complete all four tasks.

(a) Explain how the table should be modified so that the Hungarian algorithm can be applied.

(2)

(b) Use the Hungarian algorithm to obtain an allocation that minimises the total time.

(6)

(c) Calculate the least total time to complete all four tasks.

(1)

(Total for Question 2 is 9 marks)

3. The table below shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to three demand points, P, Q and R. It also shows the stock held at each supply point and the number of units required at each demand point.

A minimum cost solution is required.

	P	Q	R	Supply
A	27	23	25	35
B	29	30	28	41
C	29	33	26	29
D	32	34	36	45
Demand	57	31	62	

(a) Use the north-west corner method to obtain an initial solution to this transportation problem.

(1)

(b) Perform one iteration of the stepping-stone method to obtain an improved solution. You must make your method clear by showing the route and stating the

- shadow costs
- improvement indices
- entering cell and exiting cell

(4)

(c) Formulate the transportation problem as a linear programming problem. You must define your decision variables and make the objective function and constraints clear.

(5)

(Total for Question 3 is 10 marks)

4.

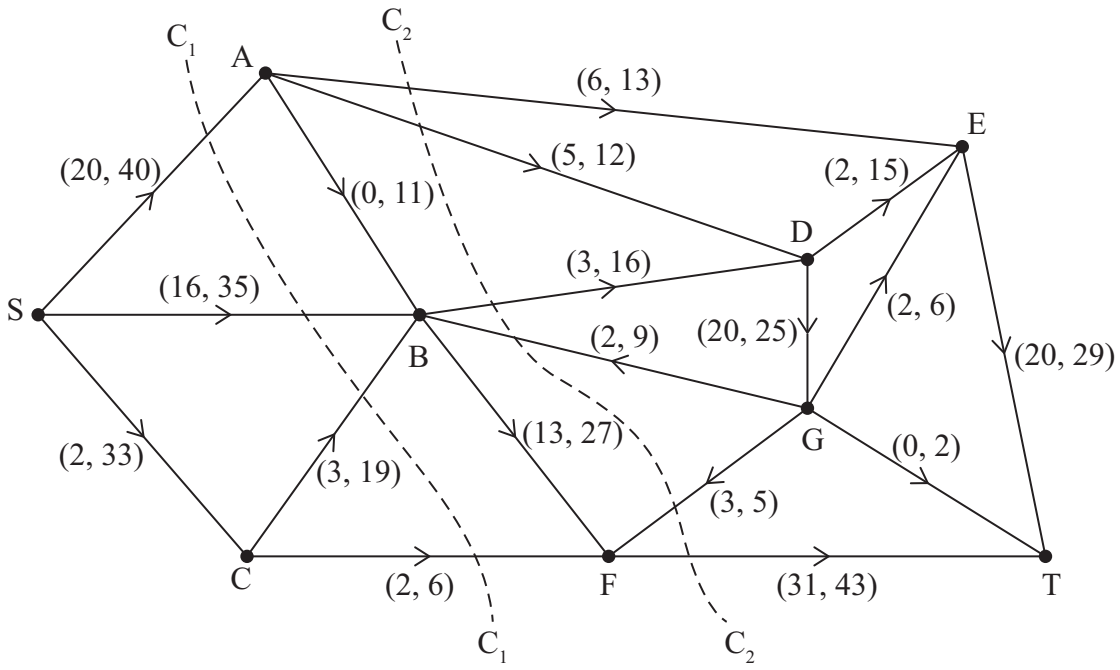

Figure 2

Figure 2 shows a capacitated, directed network. The network represents a system of pipes through which fluid can flow.

The weights on the arcs show the lower and upper capacities for the corresponding pipes, in litres per second.

(a) Calculate the capacity of

(i) cut C_1

(ii) cut C_2

(2)

(b) Using only the capacities of cuts C_1 and C_2, state what can be deduced about the maximum flow through the system.

(1)

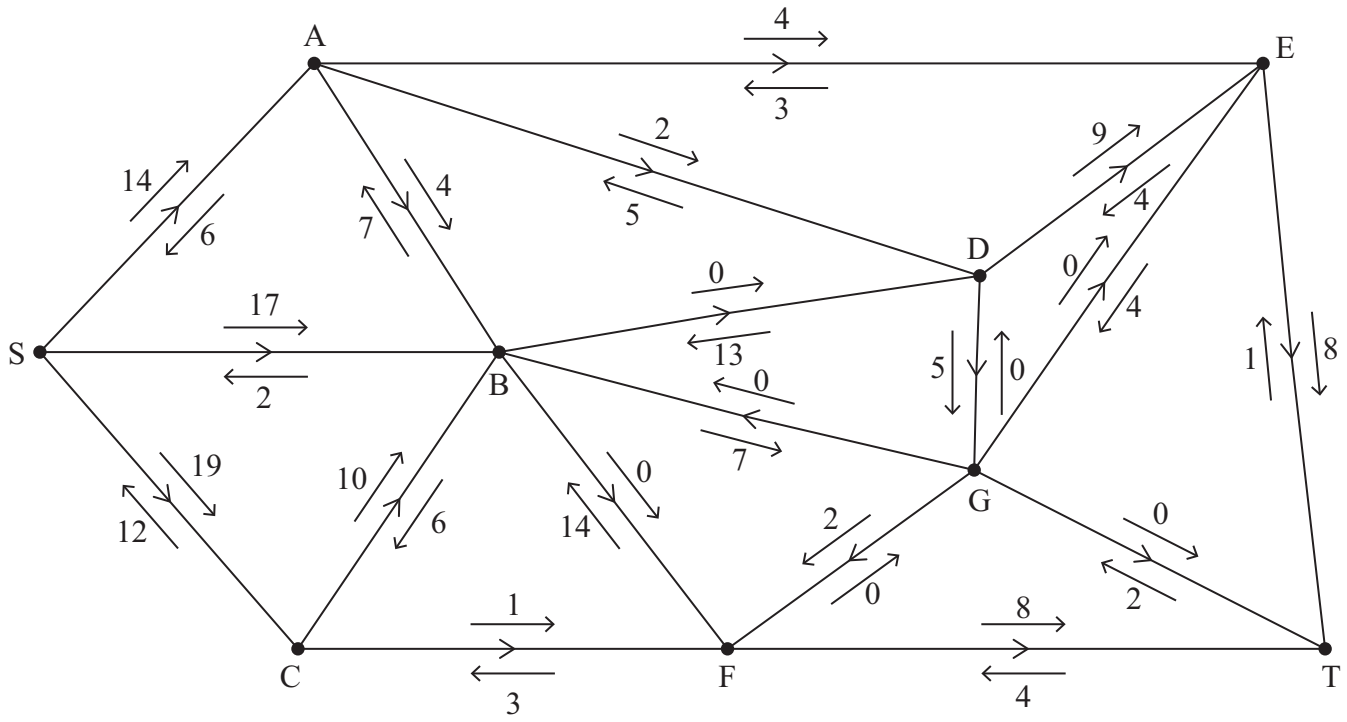

Figure 3

A scientist monitors the system of pipes in which a fluid now flows from the source, S, to the sink, T.

The scientist initialises the labelling procedure for this system. The excess and potential backflows for this are shown on the arrows either side of each arc in Figure 3.

(c) State the value of the initial flow.

(1)

(d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(3)

(e) Use your answer to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book.

(1)

(f) Prove that the answer to part (e) is optimal.

(3)

(Total for Question 4 is 11 marks)

5. Anvi makes boats during the winter months. She can make up to five boats each month.

If she builds more than three boats in any one month, she must hire an assistant at a cost of £400 for that month.

In any month in which boats are made, the overhead costs are £60 for each boat made that month.

A maximum of three boats can be held in storage at the end of each month, at a cost of £70 per boat per month.

Boats must be delivered at the end of each month.

The order schedule for boats is

Month	November	December	January	February	March
Number ordered	2	5	6	4	3

There are no boats in storage at the beginning of November.
Anvi plans to have no boats left in storage after the end of the March delivery.

(a) Use dynamic programming to determine the production schedule that minimises the costs given above. Complete the working in the table provided in the answer book and state the minimum cost.

(12)

The assistant asks to work in November, and Anvi agrees.

(b) Given that more than three boats are now made in November, state the minimum additional cost of producing all the required boats to the schedule shown above.

(1)

(Total for Question 5 is 13 marks)

6.

		Player B		
		Option P	Option Q	Option R
Player A	Option X	4	–2	–5
	Option Y	–1	1	3

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(a) Verify that there is no stable solution to this game.

(2)

Let player A play option X with probability p

(b) Use a graphical method to find the optimal value of p and hence find the best strategy for player A in this game.

(6)

A third option, Z, is added to player A's options. Option Z has the pay-offs shown in the matrix below.

		Player B		
		Option P	Option Q	Option R
Player A	Option X	4	–2	–5
	Option Y	–1	1	3
	Option Z	4	–1	1

Player A now intends to make a random choice between options X, Y and Z, choosing option X with probability x, option Y with probability y and option Z with probability z

Player A decides to use the Simplex algorithm to find the optimal values of x, y and z

(c) Determine an initial Simplex tableau to solve this 3×3 game, making your variables clear.

(4)

In the optimal Simplex tableau for this 3×3 game, $x = 0$ and $y = \frac{5}{7}$

(d) By considering the values of the two games, determine how much better the 3×3 game (with option Z) is for player A compared with the 2×3 game (without option Z).
You must make your reasoning clear.

(3)

(Total for Question 6 is 15 marks)

7. A sequence $\{u_n\}$, where $n \geqslant 0$, satisfies the first order recurrence relation (*)

$$u_{n+1} = \alpha u_n - \beta(k^n) \qquad (*)$$

where α, β, k are non-zero constants and $\alpha \neq k$

(a) Determine, in terms of α, β, k and n, the general solution of this recurrence relation.

(4)

Given that $u_0 = L$ and $u_T = 0$, where T is a constant,

(b) show that a particular solution of the recurrence relation is

$$u_n = \frac{Lk^n\left(1 - \left(\frac{k}{\alpha}\right)^{T-n}\right)}{1 - \left(\frac{k}{\alpha}\right)^{T}}$$

(5)

Taylor borrows a sum of money, L, to buy a house.

Taylor arranges a loan which enables them to increase their repayment as their income grows.

The terms of the loan require Taylor's repayments to be made once a year.

On each anniversary of the loan being taken out, 6% interest is charged on the outstanding balance and Taylor makes an annual repayment of the total loan.

Taylor's total annual repayment of the loan will increase by 4% each year and the whole debt will be cleared in exactly 40 years.

The recurrence relation (*) can be used to model this situation.

(c) (i) State the value of α

(ii) State the value of k

(iii) State the relevance of β in (*)

(3)

(d) Show that after 30 years, according to the model, the outstanding debt is larger than the original loan.

(2)

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

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Answer Book
Do not return the question paper with the answer book.

Total Marks

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1.

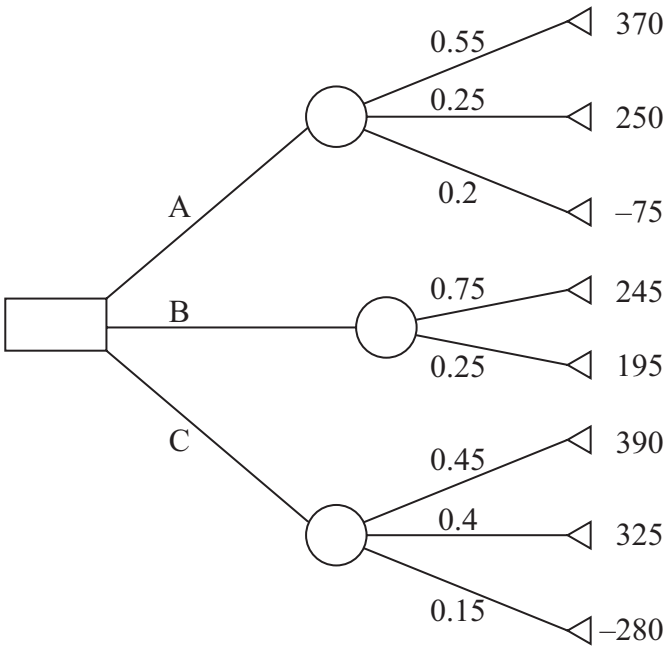

Figure 1

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Question 1 continued

(Total for Question 1 is 3 marks)

2 (a)

	P	Q	R	S
A	25	32	43	28
B	41	37	–	38
C	43	35	37	39
D	40	33	37	41
E	37	38	43	40

You may not need to use all of these tables or all the rows/columns in each table.

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
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Question 2 continued

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
C					
D					
E					

	P	Q	R	S	
A					
B					
C					
D					
E					

(Total for Question 2 is 9 marks)

3.

	P	Q	R	Supply
A	27	23	25	35
B	29	30	28	41
C	29	33	26	29
D	32	34	36	45
Demand	57	31	62	

You may not need to use all of these tables.

	P	Q	R
A			
B			
C			
D			

	P	Q	R
A			
B			
C			
D			

	P	Q	R
A			
B			
C			
D			

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Question 3 continued

	P	Q	R
A			
B			
C			
D			

	P	Q	R
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B			
C			
D			

(Total for Question 3 is 10 marks)

4.

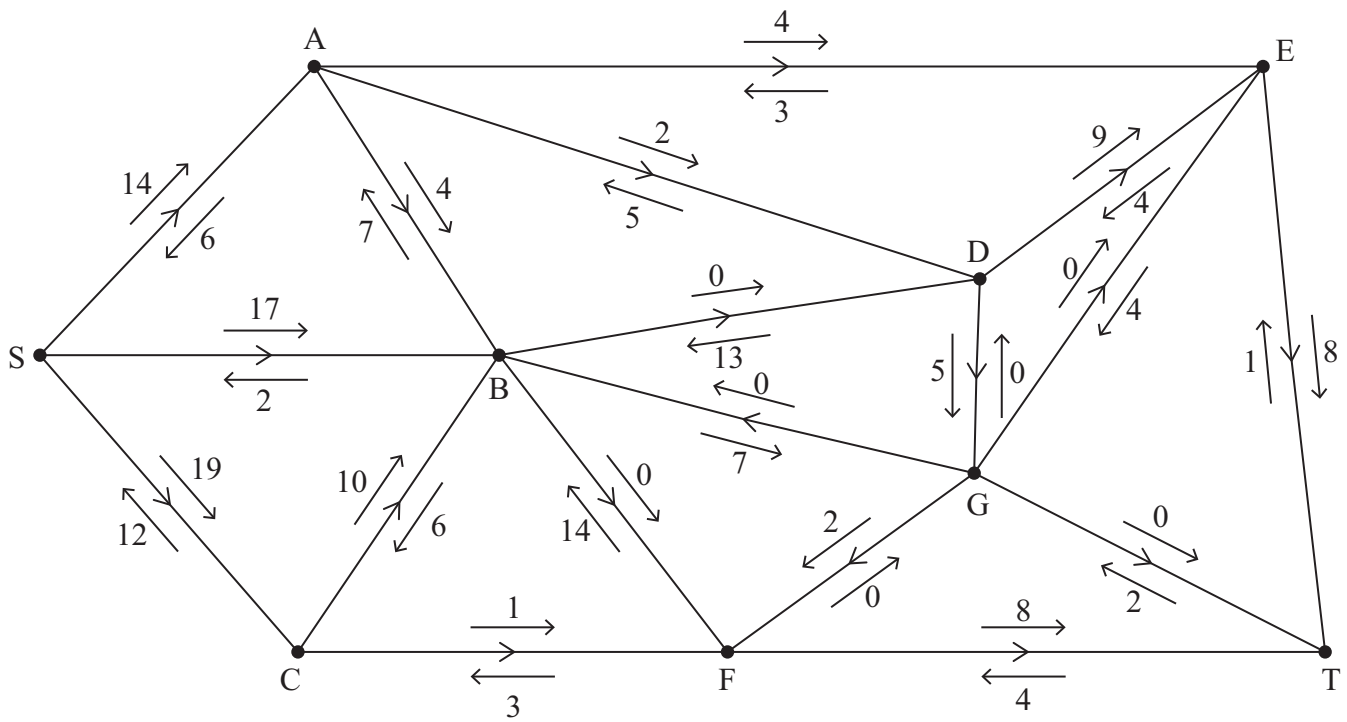

Figure 3

Question 4 continued

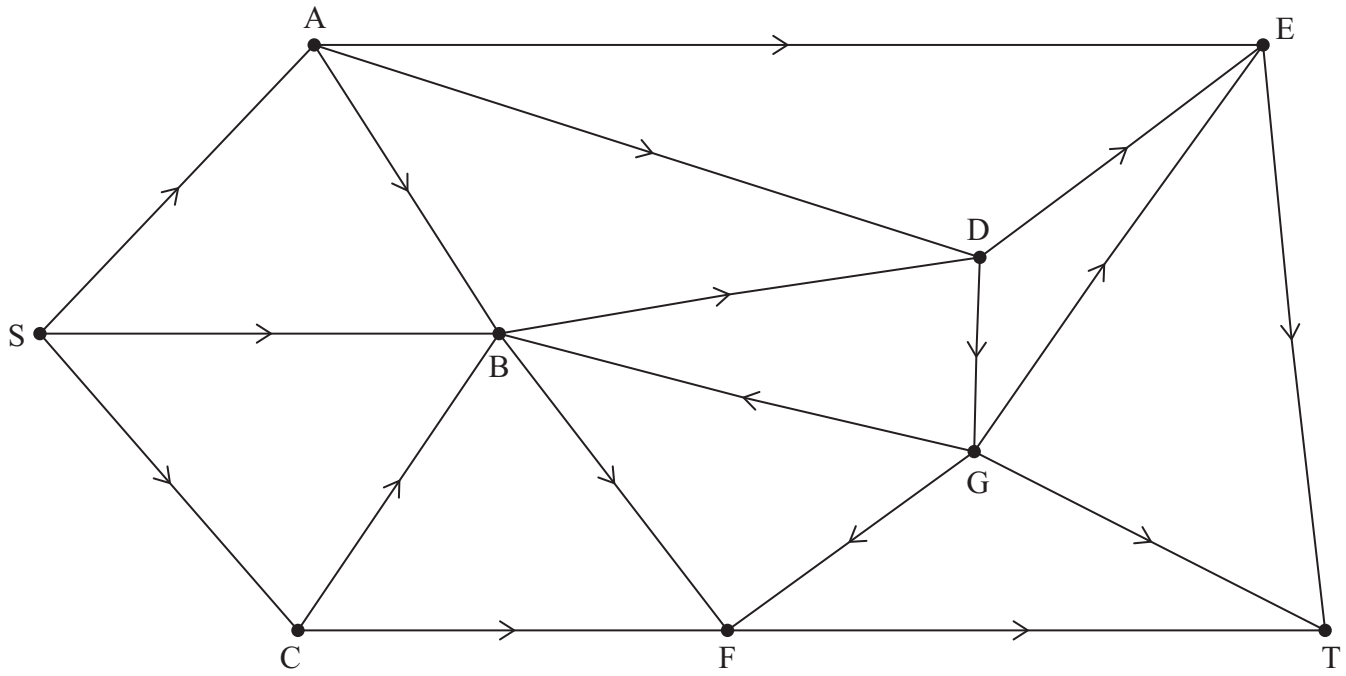

Diagram 1

(Total for Question 4 is 11 marks)

5.

Stage	State	Action	Destination	Value	
March	3	0	0	210	= 210*
	2	1	0	140 + 60	= 200*
	1	2	0	70 + 120	= 190*
	0	3	0	180	= 180*

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Question 5 continued

Stage	State	Action	Destination	Value

Month	November	December	January	February	March
Number made					

Minimum cost: _____

(Total for Question 5 is 13 marks)

6.

		Player B		
		Option P	Option Q	Option R
Player A	Option X	4	–2	–5
	Option Y	–1	1	3

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Question 6 continued

The Simplex tableau for part (c) can be found on page 14.

Question 6 continued

b.v.									Value
P									

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Question 6 continued

(Total for Question 6 is 15 marks)

7.

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Question 7 continued

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Question 7 continued

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