Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel GCE
In Further Mathematics (8FM0)
Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

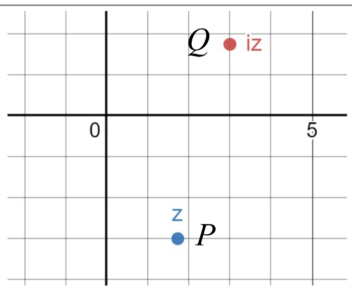
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives

answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1 (a)	$z=\sqrt{3}-3\mathrm{i}$ so $r=\sqrt{\left(\sqrt{3}\right)^2+\left(\pm 3\right)^2}$ or $\sqrt{12}$ or $2\sqrt{3}$ or awrt 3.46 $\theta=-\dfrac{\pi}{3}$ or -60 or awrt -1.05	**B1**	1.1b
	$\{z=\}\sqrt{12}\left(\cos\left(-\dfrac{\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{3}\right)\right)$ or $2\sqrt{3}\left(\cos\left(-\dfrac{\pi}{3}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{3}\right)\right)$ or $\{z=\}\sqrt{12}\left(\cos\left(\dfrac{\pi}{3}\right)-\mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right)$ or $\{z=\}2\sqrt{3}\left(\cos\left(\dfrac{\pi}{3}\right)-\mathrm{i}\sin\left(\dfrac{\pi}{3}\right)\right)$	**B1**	1.1b
		(2)	
(b)	$\left\{\mathrm{i}z=3+\sqrt{3}\mathrm{i}\right\}$		
		**M1** **A1**	1.1b 1.1b
		(2)	
(c)	Rotation	**B1**	1.1a
	$\dfrac{\pi}{2}$ or 90° (anticlockwise) about the origin or $\dfrac{3\pi}{2}$ or 270° clockwise about the origin	**B1**	1.1b
		(2)	
	Alternatives must have scored M1A1 in (b)		
	Reflection / Matrix / Translation	**B1**	1.1a
	Reflection: $\arg z=-\dfrac{\pi}{12}$ or $y=\left(\sqrt{3}-2\right)x$ Matrix: $\begin{pmatrix}\mathrm{i} & 0\\ 0 & \mathrm{i}\end{pmatrix}$ or $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ Translation: $\begin{pmatrix}3-\sqrt{3}\\ \sqrt{3}+3\end{pmatrix}$	**B1**	1.1b
		(2)	
		(6 marks)	

Notes:

(a) **B1:** Correct modulus or correct argument may be correct to 3 s.f for this mark **B1:** Fully correct must be exact values for this mark, angle must be in radians
(b) **M1:** If no coordinates given then it must be in the correct quadrant **and** either z below the line $y = -x$ **or** iz below the line $y = x$ **or** the correct coordinate shown. If incorrect coordinates shown then M0 **A1:** Both z and iz drawn correctly on an Argand diagram. with iz below the line $y = x$ and z below the line $y = -x$ **or** correct coordinates indicated If there are no coordinates and the points line close to the lines $y = x$ and $y = -x$ send to review
(c) Note must be a single transformation, there is no follow through **B1:** Deduces rotation **B1:** Correct angle (radians or degrees), direction and centre **Alternative: must be from a correct (b)** **B1:** Deduces reflection or matrix or translation **B1:** Correct line or matrix or vector

Question	Scheme	Marks	AOs
2(a)	States or uses $(z+5)$ factor	**B1**	1.2
	$(z+5)(4z^2-...)$	**M1**	1.1b
	$(z+5)(4z^2-32z+65)$	**A1**	1.1b
		(3)	
(b)	$z=\dfrac{-(-32)\pm\sqrt{(-32)^2-4(4)(65)}}{2(4)}=...$ Completing the square $4(z\pm 4)^2+q=0\Rightarrow z=...$ where $q>0$	**M1**	1.1b
	$z=\left\{\dfrac{32\pm\sqrt{-16}}{8}=\right\}\dfrac{32\pm 4\mathrm{i}}{8}=\dfrac{8\pm\mathrm{i}}{2}$ *	**A1***	2.1
		(2)	
	Alternative Uses the roots to find the equation $\left(z-\dfrac{8+\mathrm{i}}{2}\right)\left(z-\dfrac{8-\mathrm{i}}{2}\right)=z^2-8z+\dfrac{64}{4}=0$ This could come from using the sum and product of roots	**M1**	1.1b
	$4z^2-32z+65=0$ therefore roots are $\dfrac{8\pm\mathrm{i}}{2}$	**A1***	2.1
		(2)	
(c)	$-2, \dfrac{10\pm\mathrm{i}}{4}$ or $-2, \dfrac{5}{2}\pm\dfrac{\mathrm{i}}{4}$ or $-2, 2.5\pm 0.25\mathrm{i}$	**B1** **B1**	2.2a 2.2a
		(2)	
			(7 marks)

Notes:

(a)

B1: Recalls the fact that if $f(-5)=0$ then $(z+5)$ is a factor

M1: Finds the quadratic factor with a $4z^2$ term, any method. Condone using a factor of $(z-5)$

A1: Correct factorised expression for $f(z)$

(b)

M1: Uses a correct method to show where the **complex roots** have come from.

Note just uses the calculator is M0

A1: Completes the method to show the complex roots. **There needs to be an intermediate line of work with either 4i or $4\sqrt{-1}$ term**

Note: $z = \dfrac{32 \pm \sqrt{-16}}{8} = \dfrac{8 \pm \mathrm{i}}{2}$ is M1 A0 **Alternative** **M1**: uses the roots and expands **A1:** Achieves the correct equation with = 0 no errors and a conclusion
(c) **B1:** Deduces one correct root **B1**: Deduces all 3 correct roots

Question	Scheme	Marks	AOs
3 (a)	Determinant $=-a+b$	**B1**	1.1b
	$\left\{\mathbf{M}^{-1}=\right\}\dfrac{1}{b-a}\begin{pmatrix}-1 & -b\\ 1 & a\end{pmatrix}$	**M1** **A1**	1.1b 1.1b
		(3)	
(b)	$\begin{pmatrix}a & b\\ -1 & -1\end{pmatrix}+\dfrac{1}{b-a}\begin{pmatrix}-1 & -b\\ 1 & a\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ Which may include $a-\dfrac{1}{b-a}=1$ or $-1+\dfrac{1}{b-a}=0$ or $ab-a^2-1=b-a$ or $-b+a+1=0$ $b-\dfrac{b}{b-a}=0$ or $-1+\dfrac{a}{b-a}=1$ or $b^2-ab-b=0$ or $-b+a+a=b-a$	**M1**	3.1a
	Solves any two simultaneous equations to achieves a value for *a* and a value for *b* e.g., $a-1=1\Rightarrow a=...\Rightarrow b=...$	**dM1**	2.1
	$a=2, b=3$ only	**A1**	1.1b
		(3)	
	Alternative $\mathbf{M}^2+\mathbf{I}=\mathbf{M}$ $\begin{pmatrix}a^2-b & ab-b\\ -a+1 & -b+a\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}a & b\\ -1 & -1\end{pmatrix}$ Leading to any two equations which may include $-a+1=-1$ or $-b+1+1=-1$	**M1**	3.1a
	Solves to achieves a value for *a* and a value for *b* e.g., $a-1=1\Rightarrow a=...\Rightarrow b=...$	**dM1**	2.1
	$a=2, b=3$	**A1**	1.1b
		(3)	
			(6 marks)

Notes:

(a)
B1: Correct determinant
M1: Correct method to find the inverse matrix using their determinant
A1: Fully correct inverse matrix, isw

(b)
M1: A complete method to form two simultaneous equations in *a* and *b*, using $\mathbf{M}+\mathbf{M}^{-1}=\mathbf{I}$ and their inverse matrix
dM1: Solves their **two different** equations to achieve a value for *a* and a value for *b*. Must come from a valid attempt at $\mathbf{M}+\mathbf{M}^{-1}=\mathbf{I}$.

A1: Correct values for *a* and *b*

Alternative

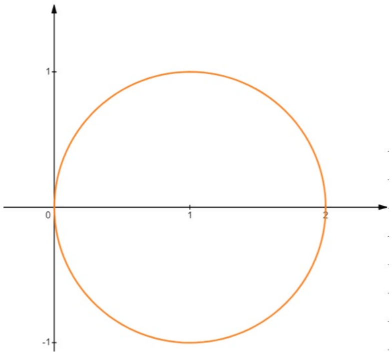
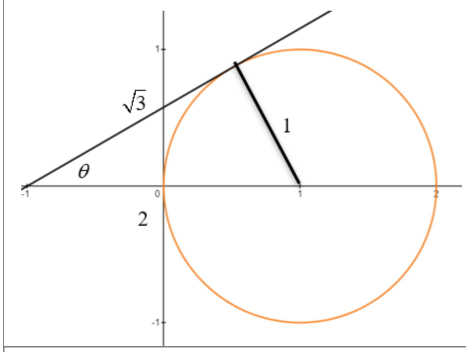
M1: Forms the equation $\mathbf{M}^2 + \mathbf{I} = \mathbf{M}$ and form two equations

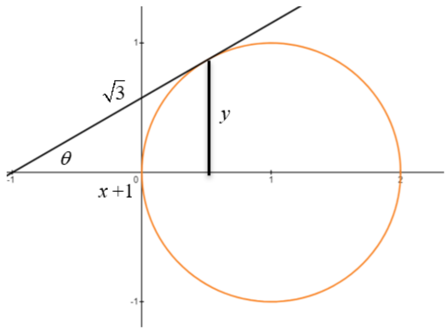
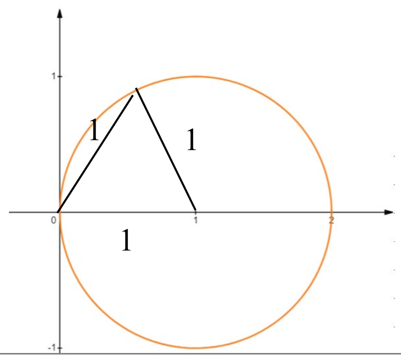
dM1: Solves their equations to achieve a value for *a* and a value for *b*

A1: Correct values for *a* and *b*

Question	Scheme	Marks	AOs
4(i)(a)	Stretch	**B1**	1.1a
	Scale factor 2 parallel to x-axis	**B1**	2.5
		(2)	
(i)(b)	Any line parallel to the x-axis or $y = k$ or the y-axis or $x = 0$	**B1**	1.1b
		(1)	
(ii)	Area triangle $= \frac{1}{2}\times(8-2)(6-3) = 9$ Area triangle for example $\frac{1}{2}\begin{vmatrix} 2 & 3 & 8 & 2 \\ 3 & 6 & 3 & 3 \end{vmatrix} = \frac{1}{2}\left[2\times6+3\times3+8\times3-(3\times3+8\times6+2\times3)\right]$ Area triangle $= \frac{1}{2}(6)\left(\sqrt{10}\right)\sin 71.57$ from a valid attempt to find the angle and sides	**M1**	3.1a
	$\begin{vmatrix} \cos 2\theta & 0 \\ 1 & \tan 2\theta \end{vmatrix} = \cos 2\theta \tan 2\theta$	**B1**	1.1b
	their $'\cos 2\theta \tan 2\theta' = \frac{4.5}{\text{their '9'}}$	**M1**	1.1b
	$\cos 2\theta \tan 2\theta = \frac{4.5}{\text{their } 9} \Rightarrow \cos 2\theta \times \frac{\sin 2\theta}{\cos 2\theta} = \frac{4.5}{\text{their } 9} \Rightarrow \sin 2\theta = k$ where $-1 < k < 1$	**dM1**	3.1a
	$\sin 2\theta = \frac{1}{2} \Rightarrow \theta = \ldots$ or $\sin 2\theta = -\frac{1}{2} \Rightarrow \theta = \ldots$	**ddM1**	1.1b
	Any two of **<u>Coming from</u>** $\sin 2\theta = \frac{1}{2}$ 15°, 75°, 195°, 255° or $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ or awrt 0.26, 1.31, 3.40, 4.45 **<u>Coming from</u>** $\sin 2\theta = -\frac{1}{2}$ 105°, 165°, 285°, 345° or $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ or awrt , 1.83, 2.88, 4.97, 6.02	**A1**	1.1b
	All eight of 15°, 75°, 195°, 255°, 105°, 165°, 285°, 345°	**A1**	2.3
		(7)	
			(10 marks)

Notes:
(i)(a) **B1:** See scheme, enlargement is B0 **B1:** See scheme, if they mention of y-axis must be correct e.g. scale factor 1, stays the same. Condone defining the stretch as from/about the origin and in the x- axis, along the x-axis **(i)(b)** **B1:** See scheme, any incorrect answer stated B0
(ii) **M1:** Correct method to find the area of the triangle T **B1:** Correct determinant of the matrix **M1:** Sets their determinant = 4.5 divided by their area of the triangle T **dM1:** Dependent on previous method mark. Uses the identity $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ to achieve $\sin 2\theta = k$ where $-1 < k < 1$ **ddM1:** Dependent on previous method marks. Solves $\sin 2\theta = k$ where $-1 < k < 1$ to find a value of θ **A1:** Any two correct values in degrees or radians **A1:** All eight correct values and no others **must be in degrees**

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>5(a)</td><td>
Circle
Circle with clear indication that the centre is (1, 0) and the radius is 1</td><td>M1
A1</td><td>1.2
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
$\sin\theta = \frac{1}{2} \Rightarrow \theta = \ldots$</td><td>M1</td><td>3.1a</td></tr>
<tr><td></td><td>Alternative 1 May use $m = \tan\theta$
$y = \tan\theta(x+1)$ and $(x-1)^2 + y^2 = 1$
$x^2 - 2x + 1 + \left[\tan\theta(x+1)\right]^2 = 1$
$x^2 - 2x + 1 + x^2\tan^2\theta + 2x\tan^2 x + \tan^2\theta = 1$
$\left(\tan^2\theta + 1\right)x^2 + \left(2\tan^2\theta - 2\right)x + \tan^2\theta = 0$
$b^2 - 4ac = \left(2\tan^2\theta - 2\right)^2 - 4\left(\tan^2\theta + 1\right)\left(\tan^2\theta\right) = 0$
$\tan^2\theta = \frac{1}{3} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \ldots$</td><td></td><td></td></tr>
<tr><td></td><td>$\{\theta =\}\frac{\pi}{6}$ or 30 or $\arg(z+1) = \frac{\pi}{6}$ or 30</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$\{\theta =\} -\frac{\pi}{6}$ or $\frac{11\pi}{6}$ or -30 or 330 or
$\arg(z+1) = -\frac{\pi}{6}$ or $\frac{11\pi}{6}$ or -30 or 330</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
</table>

(c)	 $\sin\theta = \dfrac{y}{\sqrt{3}} \Rightarrow y = ...$ $\cos\theta = \dfrac{x+1}{\sqrt{3}} \Rightarrow x = ...$ **Alternative 1** $(x-1)^2 + y^2 = 1$ and $y = \tan\{\text{their } \theta\}(x+1) \Rightarrow y = ...\left\{\dfrac{\sqrt{3}}{3}x + \dfrac{\sqrt{3}}{3}\right\}$ $(x-1)^2 + \left(\dfrac{\sqrt{3}}{3}x + \dfrac{\sqrt{3}}{3}\right)^2 = 1$ leading to $4x^2 - 4x + 1 = 0 \Rightarrow x = ...$ and $y = \ldots$ **Alternative 2** $z + 1 = \text{their}\sqrt{3}\left(\cos \text{their}\theta + \mathrm{i}\sin \text{their}\theta\right)$ leading to $z = \ldots$	**M1**	3.1a
	$z = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\mathrm{i}$	**A1**	1.1b
	$z = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$	**A1ft**	2.2a
		(3)	
	 Using equilateral triangles to find $z = ...$ $z = 1\left(\cos\left(\pm\dfrac{\pi}{3}\right) + \mathrm{i}\sin\left(\pm\dfrac{\pi}{3}\right)\right)$	**M1**	3.1a
	$z = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}\mathrm{i}$	**A1**	1.1b
	$z = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}$	**A1ft**	2.2a
			(8 marks)

Notes:

(a)
M1: Recalls that the locus represents a circle
A1: Circle with centre (1, 0) and radius 1, ignore any tangents drawn

(b) **M1:** Complete method to find one possible value of θ **Alternative 1:** condone slips with algebra as long as intention is clear **A1:** One correct value θ **A1:** Deduces the other correct value of θ and no other incorrect angles
(c) **M1:** A correct method to find a complex number using one of their values of θ **A1:** One correct complex number **check special case for coordinates only** **A1ft:** Deduces the other correct complex number**.** Follow through on $z = a + b\text{i}$ so scored for $z = a - b\text{i}$ as long as the method mark is scored **Special case:** If leave as two correct coordinates or give values for x and y then SC M1 A1 A0

Question	Scheme	Marks	AOs
6	$n=1$ LHS = 1 RHS $=\frac{1}{4}(1)^2(2)^2=1$	B1	2.2a
	Assume true for $\boldsymbol{n=k}$ or $\sum_{r=1}^{k} r^3=\frac{1}{4}k^2(k+1)^2$	M1	2.5
	$\sum_{r=1}^{k+1} r^3=\frac{1}{4}k^2(k+1)^2+(k+1)^3$ Or $(k+1)^3=\sum_{r=1}^{k+1} r^3-\frac{1}{4}k^2(k+1)^2$ or $(k+1)^3=\frac{1}{4}(k+1)^2(k+1)^2-\frac{1}{4}k^2(k+1)^2$	M1	2.1
	$\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ Or Multiples out $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ and $\frac{1}{4}(k+1)^2(k+2)^2$ to form a quartic **for each** Or Multiplies out or factorises out $\frac{1}{4}(k+1)^2(k+1)^2-\frac{1}{4}k^2(k+1)^2$	M1	1.1b
	$\frac{1}{4}(k+1)^2\left[k^2+4k+4\right]=\frac{1}{4}(k+1)^2(k+2)^2$ leading to $\frac{1}{4}(k+1)^2(\{k+1\}+1)^2$ Or $k^4+6k^3+13k^2+12k+4=k^4+6k^3+13k^2+12k+4$ And draws conclusion true Or Shows $\frac{1}{4}(k+1)^2(k+1)^2-\frac{1}{4}k^2(k+1)^2=(k+1)^3$ by either multiplying out both sides to reach the same cubic, and draws a conclusion or by factorisation	A1	1.1b
	<u>If true for $n=k$ then true for $n=k+1$</u>, and as also <u>true for $n=1$</u>, so the result is <u>true for all n</u>	A1	2.4
		(6)	
			(6 marks)

Note

B1: Shows the statement is true for $n=1$. Minimum requirement $1=\frac{1}{4}(1)(4)$

M1: Makes the inductive assumption, **assume** true $\boldsymbol{n=k}$. This may appear in the conclusion.

M1: A correct statement for $n = k + 1$

M1: Factorises out $(k+1)^2$ or multiples out $\frac{1}{4}k^2(k+1)^2 + (k+1)^3$ and $\frac{1}{4}(k+1)^2(k+2)^2$ to form a quartic for each

A1: Achieves $\frac{1}{4}(k+1)^2(\{k+1\}+1)^2$ with no omissions or errors. Shows both are sides are $k^4 + 6k^3 + 13k^2 + 12k + 4$ no omissions or errors. and draws a minimal conclusion. Shows $\frac{1}{4}(k+1)^2(k+1)^2 - \frac{1}{4}k^2(k+1)^2 = (k+1)^3$ may expand both sides and draw a minimal conclusion or factorises, no omissions or errors.

Note: they may state aiming for $\frac{1}{4}(k+1)^2(k+2)^2$ and then achieves this which is fine for A1

A1: Makes appropriate concluding sentence covering the points indicated in scheme. Dependent on all previous marks except B1. The statement true for $n = 1$ could appear at the start of the solution

Question	Scheme	Marks	AOs
7(a)	$x=0$ and $y=12$ $0=(12+12)^2(A-12^2)\Rightarrow A=144$	B1	3.3
		(1)	
(b)	$V=\dfrac{\pi}{350}\displaystyle\int_{-12}^{12}(12+y)^2(144-y^2)\ \{\mathrm{d}y\}$	B1	3.4
	$(12+y)^2(144-y^2)=\ldots\{20736+3456y-24y^3-y^4\}$ $V=\left\{\dfrac{\pi}{350}\right\}\displaystyle\int(20736+3456y-24y^3-y^4)\ \mathrm{d}y$ $=\left\{\dfrac{\pi}{350}\right\}\left(20736y+1728y^2-6y^4-\dfrac{1}{5}y^5\right)$ $=\{\pi\}\left(\dfrac{10368}{175}y+\dfrac{864}{175}y^2-\dfrac{3}{175}y^4-\dfrac{1}{1750}y^5\right)$ **Alternative by parts** $(12+y)^2(144-y^2)=(12+y)^3(12-y)$ $V=\displaystyle\int(12+y)^3(12-y)\ \mathrm{d}y$ $=\dfrac{1}{4}(12+y)^4(12-y)+\displaystyle\int\dfrac{1}{4}(12+y)^4\ \mathrm{d}y$ $=\dfrac{1}{4}(12+y)^4(12-y)+\dfrac{1}{20}(12+y)^5$	M1 A1	1.1b 1.1b
	$\left[20736(12)+1728(12)^2-6(12)^4-\dfrac{1}{5}(12)^5\right]-$ $\left[20736(-12)+1728(-12)^2-6(-12)^4-\dfrac{1}{5}(-12)^5\right]$ $=(323481.6)-(-74659.6)$ **Alternative by parts** $\left[\dfrac{1}{4}(12+12)^4(12-12)+\dfrac{1}{20}(12+12)^5\right]-\left[\dfrac{1}{4}(12-12)^4(12+12)+\dfrac{1}{20}(12-12)^5\right]$	M1	3.4
	$=3600\ \mathrm{m}^3$ cao	A1	2.2b
		(5)	
(c)	$350x^2=(13+y)^2(169-y^2)$ Or $411x^2=(13+y)^2(169-y^2)$	B1	3.5c
		(1)	
(d)	e.g. the balloon may not be exactly the same shape as the curve The balloon's material will have some thickness	B1	3.5b

	the balloon is not the same shape as the curve as the balloon does not taper to nothing at the bottom. Balloon may stretch B0 for balloon may not be smooth, comments on the basket		
		(1)	

(8 marks)

Notes:

(a)
B1: Uses $x = 0$ and $y = 12$ in the **full** equation to show that $A = 144$
Note Uses $x = 0$ and $y = -12$ is B0

(b)
B1: Uses the model to set up the volume for the balloon, with limits, the d*y* may be implied
M1: Multiplies out the brackets and integrates $\int x^n \, dx \to x^{n+1}$. Alternatively uses integration by parts the correct way. Condone a slip when multiplying out
A1: Correct integration
M1: Uses the limits of – 12 and 12, subtracts the correct way round.
If the integration is correct this can be evidenced by for example $(323481.6)-(-74659.6)$ or

2903.56 – (– 670.05) if including $\dfrac{\pi}{350}$

If the integration is incorrect we must see the substitution of 12 and –12 into their integrated function
Candidates may use limits of – 12 to 0 and then 0 to 12 and add which is fine.
A1: Correct volume 3600 m^3, unit required and 2 s.f.
Note: No Evidence of integration maximum B1 M0A0 M0A0 if a correct answer stated

(c)
B1: See scheme for equation

(d)
B1: Correct limitation, see scheme, must be about the balloon part not the basket. Balloon might not be smooth is B0

Question	Scheme	Marks	AOs
8(a)	$\sum \frac{1}{2}r(r+1)=\frac{1}{2}\left[\sum r^2+\sum r\right]$	**M1**	3.1a
	$\frac{1}{2}\left[\sum r^2+\sum r\right]=\frac{1}{2}\left[\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)\right]$	**M1** **A1**	1.1b 1.1b
	$=\frac{1}{2}n(n+1)\left[\frac{1}{6}(2n+1)+\frac{1}{2}\right]$ Or $=\frac{1}{2}n\left[\frac{1}{6}(n+1)(2n+1)+\frac{1}{2}(n+1)\right]$ Or $=\frac{1}{6}n(n+1)\left[\frac{1}{2}(2n+1)+\frac{3}{2}\right]$	**dM1**	1.1b
	$=\frac{1}{6}n(n+1)(n+2)$ * cso	**A1***	2.1
		(5)	
(b)	$\frac{1}{6}n(n+1)(n+2)=22n \Rightarrow n^2+3n-130=0 \Rightarrow n=...$ Or $\frac{1}{6}n(n+1)(n+2)=22n \Rightarrow n^3+3n^2-130n=0 \Rightarrow n=...$	**M1**	3.1a
	$n=10$ only	**A1**	3.2a
		(2)	
			(7 marks)

Notes:

(a)
M1: Starts the method by writing the correct sum and splitting into two sums, condone use of n instead of r
M1: Substitutes in at least one standard formula
A1: Fully correct unsimplified expression
dM1: Dependent on the previous method mark. Factorises out at least n.
A1*: Achieves the printed answer with no omission or errors, factorisation of $(n + 1)$ term needed cso

(b)
M1: Sets the printed answer equal to $22n$, forms a quadratic/cubic and solves to find a non zero value for n. If an incorrect equation please check that their value for n is correct for their equation if no method shown.
A1: Selects the correct positive value of n. If $n=-13$ appears this must be rejected

Question	Scheme	Marks	AOs
9(a)	$\dfrac{(1\times 6)+(-2\times -2)+(8\times -4)\pm 1}{\sqrt{1^2+(-2)^2+8^2}}$	**M1**	3.4
	$\dfrac{23}{\sqrt{69}}$ or $\dfrac{1}{3}\sqrt{69}$ or awrt 2.77	**A1**	1.1b
		(2)	
(b)	e.g. not reliable as The source of the water will not be at a single point The water level will vary The ground may not be flat	**B1**	3.2b
		(1)	
(c)	$\mathbf{r}=\begin{pmatrix}6\\-2\\-4\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\8\end{pmatrix}$ and $(6+\lambda)-2(-2-2\lambda)+8(-4+8\lambda)=1 \Rightarrow \lambda=...\left\{\dfrac{1}{3}\right\}$ Or Their $\dfrac{1}{3}\sqrt{69}$ divided by $\sqrt{1^2+(-2)^2+(8)^2}$ leading to a value for λ	**M1**	3.1b
	$\mathbf{r}=\begin{pmatrix}6\\-2\\-4\end{pmatrix}+\text{their}\,\dfrac{1}{3}\begin{pmatrix}1\\-2\\8\end{pmatrix}=...$	**dM1**	1.1b
	$\left(\dfrac{19}{3},-\dfrac{8}{3},-\dfrac{4}{3}\right)$ or $\left(6\dfrac{1}{3},-2\dfrac{2}{3},-1\dfrac{1}{3}\right)$ only	**A1**	1.1b
		(3)	
(d)	Compares their answer to (a) with 2.52, **must give some idea about the difference between the values** and draws an appropriate conclusion Answer from (a) • Less than 2.2 or more than 2.8 should be saying a poor model • Between 2.2 and 2.4 or **between 2.6 and 2.8 either good or bad model** • Between 2.4 and 2.6 should be saying a good model	**B1**	3.5a
			(7 marks)
Notes:			
(a)			

M1: Uses the shortest distance formula $\dfrac{(1\times 6)+(-2\times -2)+(8\times -4)\pm 1}{\sqrt{1^2+(-2)^2+8^2}}$ condoning a sign slip with d and no modulus for this mark. May find the answer to (c) first and then find the distance. It must be a correct method, scoring at least M1M1 in (c) **A1:** Correct exact answer or awrt 2.77 isw
(b) **B1:** See scheme Note: the position of the water may change is B0
(c) This may be seen in part (a) to score any marks at least the final coordinate must be stated in (c) or 'see (a)' **M1:** A complete method to find the value of λ **dM1:** Dependent on the previous method mark. Uses their value of λ in the equation of the line $\mathbf{r}=\begin{pmatrix}6\\-2\\-4\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\8\end{pmatrix}=...$ to find a coordinate **A1:** Correct coordinate
(d) **B1:** See scheme, must have an answer to (a) so score this mark

Question	Scheme	Marks	AOs
10(a)	$\begin{pmatrix}1\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}a\\3\\5\end{pmatrix}=a+3-5=0\Rightarrow a=...$	**M1**	1.1b
	$a=2$	**A1**	1.1b
		(2)	
(b)	e.g. $\begin{vmatrix}1 & 1 & -1\\2 & 3 & 5\\1 & b & 13\end{vmatrix}=1(39-5b)-1(26-5)-1(2b-3)=0$	**M1** **A1**	3.1a 1.1b
	$b=3$	**A1**	1.1b
	Uses the planes to eliminate one variable to form two equations e.g $\left.\begin{array}{r}2x+2y-2z=6\\2x+3y+5z=4\end{array}\right\}\Rightarrow -y-7z=2$ and $\left.\begin{array}{r}x+y-z=3\\x+3y+13z=c\end{array}\right\}\Rightarrow -2y-14z=3-c$	**dM1**	3.1a
	Uses their equations to form and solve an equation for c e.g. $3-c=4\Rightarrow c=...$	**ddM1**	1.1b
	$c=-1$ cso	**A1**	1.1b
		(6)	
	Alternative 1 Uses the planes to eliminate one variable to form two equations e.g $\left.\begin{array}{r}2x+2y-2z=6\\2x+3y+5z=4\end{array}\right\}\Rightarrow -y-7z=2$ and $\left.\begin{array}{r}x+y-z=3\\x+by+13z=c\end{array}\right\}\Rightarrow (1-b)y-14z=3-c$	**M1** **A1** **A1**	3.1a 1.1b 1.1b
	$1-b=-2\Rightarrow b=...$ or $3-c=4\Rightarrow c=...$	**dM1**	3.1a
	$1-b=-2\Rightarrow b=...$ and $3-c=4\Rightarrow c=...$	**ddM1**	1.1b
	$b=3$ and $c=-1$ cso	**A1**	1.1b
		(6)	
	Alternative 2 eliminates one variable to form two equations	**M1** **A1**	3.1a 1.1b

	e.g. $\left.\begin{array}{l}2x+2y-2z=6\\2x+3y+5z=4\end{array}\right\}\Rightarrow -y-7z=2\Rightarrow y=-2-7z$ $x+y-z=3\Rightarrow x+(-2-7z)-z=3\Rightarrow x=5+8z$ Or Selects values for the coordinates to find two points that lie on the line of intersection e.g. $z=0\Rightarrow x+y=3$ and $2x+3y=4\Rightarrow(5,-2,0)$ $z=1\Rightarrow x+y-1=3$ and $2x+3y-5=4\Rightarrow(13,-9,1)$	**A1**	1.1b
	$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}8\\-7\\1\end{pmatrix}$ $\begin{pmatrix}8\\-7\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\b\\13\end{pmatrix}$ leading to $8-7b+13=0\Rightarrow b=...$ Or $\begin{pmatrix}13\\-9\\1\end{pmatrix}-\begin{pmatrix}5\\-2\\0\end{pmatrix}=\begin{pmatrix}8\\-7\\1\end{pmatrix}$ then $\begin{pmatrix}8\\-7\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\b\\13\end{pmatrix}$ leading to $8-7b+13=0\Rightarrow b=...$	**dM1**	3.1a
	$(5,-2,0)$ lies on Π_3 leading to $5-2b=c$ Uses their b to find a value for c	**ddM1**	1.1b
	$b=3$ $c=-1$ cso	**A1**	1.1b
		(6)	
(8 marks)			
Notes:			
(a) **M1:** Finds the dot product of the normal vectors, sets equal to 0 and finds a value for a. Condone a sign slip. **A1:** Correct value for a			
(b) **M1:** Finds the determinant of a 3 by 3 matrix and sets = 0, this may be implied **A1:** Correct determinant **A1:** Correct value for b **dM1:** Uses the equations of the three planes to find two equations by eliminating the same variable. Condone a slip on one coefficient or constant term **ddM1:** Correctly uses their equations to form and solve an equation for c			

A1: Correct value for c

Alternative 1

M1: Uses the planes to eliminate one variable to form two equations

A1: One correct equation

A1: two correct equations

dM1: Sets up an equation for b or c by comparing coefficients and solves to find a value. Condone a slip on one coefficient or constant term

ddM1: Solves equations to find values for b and c

A1: Correct value for b and c cso

Alternative 2

M1: Uses the planes to eliminate one variable to form two equations or substitutes in two values for one coordinate and solves to find two coordinates that lie on the line of intersection

A1: One correct equation or coordinate

A1: Two correct equations or coordinates

dM1: $\begin{pmatrix} 8 \\ -7 \\ 1 \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 1 \\ b \\ 13 \end{pmatrix}$ leading to $8-7b+13=0 \Rightarrow b=...$ or uses their coordinates to find the direction vector and uses that it is perpendicular to $\begin{pmatrix} 1 \\ b \\ 13 \end{pmatrix}$ to find a value for b

ddM1: $(5,-2,0)$ lies on Π_3 leading to $5-2b=c$

A1: Correct values for b and c

Note for 10 (b) using Alternative 1

Eliminating x gives	Eliminating y gives	Eliminating z gives
$y+7z=-2$ $(b-1)y+14z=c-3$ $(2b-3)y+21z=2c-4$	$x-8z=5$ $(b-1)x+(-b-13)z=3b-c$ $(2b-3)x+(5b-39)z=4b-3c$	$7x+8y=19$ $14x+(13+b)y=39+c$ $21x+(39-5b)y=52-5c$

Question	Scheme	Marks	AOs
11	$\alpha+(\alpha+3\beta)+(\alpha+6\beta)=21$ and $\alpha(\alpha+3\beta)(\alpha+6\beta)=91$	**M1**	1.1b
	$3\alpha+9\beta=21\Rightarrow\alpha+3\beta=7\Rightarrow\alpha=7-3\beta$ $(7-3\beta)(7-3\beta+3\beta)(7-3\beta+6\beta)=91$ $(7-3\beta)^3+9(7-3\beta)^2\beta+18(7-3\beta)\beta^2=91$ Or $3\beta=7-\alpha$ and $\alpha^3+9\alpha^2\beta+18\alpha\beta^2=91$ leading to $\alpha^3+3\alpha^2(7-\alpha)+2\alpha(7-\alpha)^2=91$	**M1**	3.1a
	$343-63\beta^2=91$ or $49-9\beta^2=13$ Or $7\alpha^2-98\alpha+91=0$	**A1**	1.1b
	$49-9\beta^2=13\Rightarrow\beta=...\{\pm2\}$ or $7\alpha^2-98\alpha+91=0\Rightarrow\alpha=...\{1,13\}$	**dM1**	1.1b
	$\alpha=7-3(\pm2)=...\{1,13\}$ Or $\beta=\dfrac{7-'\alpha'}{3}$	**ddM1**	1.1b
	Roots = 1, 7, 13 only	**dddM1** **A1**	1.1b 1.1b
		(7)	
	Alternative $\alpha+(\alpha+3\beta)+(\alpha+6\beta)=21$	**M1**	1.1b
	$\alpha+3\beta=7$ therefore one root must be 7	**M1** **A1**	3.1a 1.1b
	$(x-7)(x^2-14x+13)$	**dM1**	1.1b
	$(x-7)(x-1)(x-13)$	**ddM1**	1.1b
	Roots = 1, 13	**dddM1** **A1**	1.1b 1.1b
			(7 marks)

Notes:

M1: Sets the sum of the roots = 21 and the product of the roots = 91
M1: Forms an equation in one variable using their sum of roots and product of roots equations
A1: Correct simplified quadratic equation

dM1: Dependent on the previous method mark, solves their equation to find **real roots**, may be a cubic if slips earlier

ddM1: Dependent on previous method mark. Finds the value of the other variable

dddM1: Dependent on previous method mark. Uses their values of α and β to find the value of the roots

A1: Correct roots

Alternative

M1: Sets the sum of the roots = 21

M1: Simplifies and deduces one root

A1: One root = 7

dM1: Factorises into linear and quadratic, achieves correct first and last term

ddM1: Factorises their quadratic, which must be factorisable

dddM1: Uses their factorised quadratic to find the remining 2 roots

A1: Correct roots 1 and 13

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