GCE A LEVEL MARKING SCHEME

SUMMER 2023

A LEVEL
MATHEMATICS
UNIT 4 APPLIED MATHEMATICS B
1300U40-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

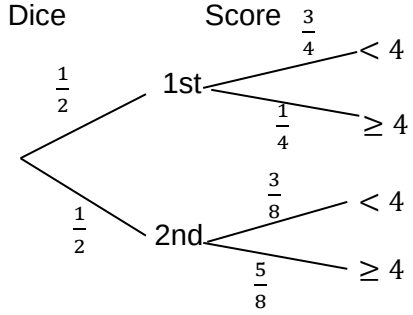
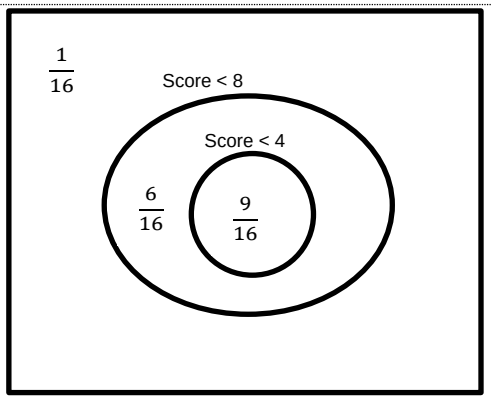
WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE A LEVEL MATHEMATICS

UNIT 4 APPLIED MATHEMATICS B

SUMMER 2023 MARK SCHEME

SECTION A – Statistics

Qu.	Solution	Mark	Notes
1(a)			
	$(\mathrm{P}(score < 4) =)\frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{8}$	M1A1	M1 for either term correct OR for sight of both $\frac{3}{4}$ and $\frac{3}{8}$. A1 for fully correct calculation (products and addition with no errors).
	$= \frac{9}{16}\ (0.5625)$	A1	CAO (3sf or greater required if using decimals)
(b)			FT their $\frac{9}{16}$ for possible B1M1A1
		B1	For sight of $\frac{1}{16}\ (0.0625)$ or $\frac{15}{16}\ (0.9375)$ or $\frac{15}{16} - \text{their}\frac{9}{16}$ (accept $\frac{6}{16}$ from $\frac{15}{16} - \frac{9}{16}$ only)
		M1	2 nested regions with at least 1 correct probability. M0A0 if $(< 4) \cap (< 8)'$ has a non-zero probability on their diagram with two intersecting circles.
		A1	Fully correct diagram. Must have box for final A1. Omission of box can earn possible B1M1. If two overlapping circles used, must have 0 in $(< 4) \cap (< 8)'$ region for A1.
		Total [6]	

Qu.	Solution	Mark	Notes
2	$(\mathrm{P}(+) = 0.122)$ $0.04 \times 0.95 + pd + (0.96 - p) \times 0.02 = 0.122$ $(0.038 + pd + 0.0192 - 0.02p = 0.122)$ $(pd - 0.02p = 0.0648)$	M1A1	May be seen later. M1 for attempt to form an equation in p and d OR in terms of d and 0.09 using $\mathrm{P}(+) = 0.122$ (at least two correct products) Must use 0.122 – sight of LHS alone earns M0 A1 for a fully correct equation in p and d OR d and 0.09, for example $0.04 \times 0.95 + 0.09d + 0.87 \times 0.02 = 0.122$
	$\mathrm{P}(\text{no fungus}\|+) = \frac{(0.96-p)\times 0.02}{0.122} = \frac{87}{610}$ $0.0192 - 0.02p = 0.0174$ $0.02p = 0.0018$ $p = 0.09$	M1A1 A1	M1 for attempt at conditional probability: Numerator must be of the form $f(p)$ or written as $P(\text{no fungus} \cap +)$ to earn the M1 AND one of numerator or denominator must be correct along with equating to $\frac{87}{610}$ to earn M1. OR sight of $\frac{87}{610} \times 0.122$ or 0.0174 or $\frac{87}{5000}$ earns M1. NOTE: $\frac{87}{610} = \frac{k}{0.122}$ with $k \neq 0.0174$ earns M0, for example $\frac{87}{610} = \frac{0.02}{0.122}$ earns M0. A1 for fully correct equation including p. ***ag** Convincing – at least one step between M1A1 and $p = 0.09$.
	ALTERNATIVE to show $p = 0.09$ $P(+\|\text{no fungus}) = \frac{P(+ \cap \text{no fungus}}{(\text{no fungus})}$ $0.02 = \frac{0.0174}{P(\text{no fungus})}$ $P(\text{no fungus}) = \frac{0.0174}{0.02} = 0.87$ $P(\text{has fungus}) = 1 - 0.87 = 0.13$ $p = 0.13 - 0.04 = 0.09$	(M1) (A1) (A1)	
	$0.09d - 0.02 \times 0.09 = 0.0648$ $d = 0.74$	m1 A1 **Total [7]**	Dependent on first M1 only. FT their equation. Must use $p = 0.09$. NOTE: If candidates find $p = 0.09$ first, then proceed with the $P(+)$ equation, they automatically earn this mark in conjunction with the first M1A1. CAO Use of $p = 0.09$ to find $d = 0.74$ without deriving p earns possible M1A1M0A0A0m1A1.

Qu.	Solution	Mark	Notes
3(a)	$18f(x) = 0.9$ OR $f(x) = \frac{1}{20}$	M1	
	$\frac{1}{20}(d-1) = 1$	A1	
	OR $\frac{19-1}{d-1} = 0.9$ OR $\frac{d-19}{d-1} = 0.1$	(M1A1)	May be implied via use of a diagram.
	$d = 21$	A1	CAO Correct answer with no working earns full marks. Correct answer from incorrect working scores M0A0A0. SC1 for either of the following: $d = \frac{19-1}{0.9}$ **leading to** $d = 20$ or $\frac{19}{d-1} = 0.9$ **leading to** $d = \frac{199}{9} = 22.\dot{1}$ (condone 22.1 or better)
(b)	$\text{mean}\left(= \frac{1+21}{2}\right) = 11$	B1	FT their $d > 19$ (from a uniform distribution only).
	$\text{SD}\left(= \sqrt{\frac{(21-1)^2}{12}}\right) = \frac{10\sqrt{3}}{3}$ (5.7735)	B1	FT their $d > 19$ (from a uniform distribution only).
		Total [5]	

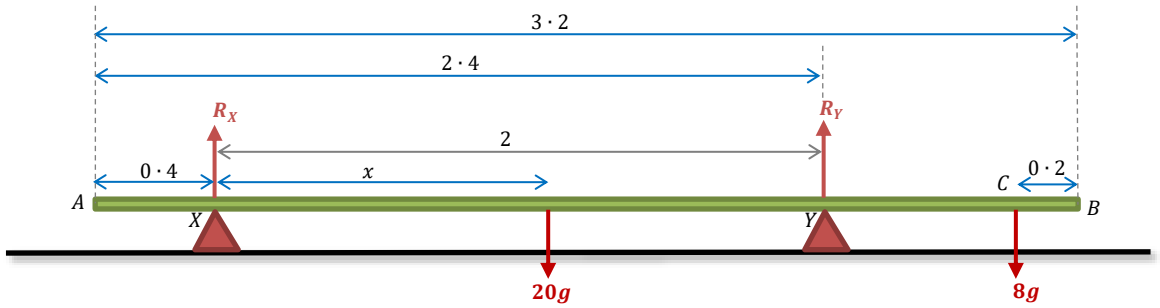
Qu.	Solution	Mark	Notes
4 (a)	Sight of $\frac{805-\mu}{\sigma}$ or $\frac{795-\mu}{\sigma}$	M1	Attempt at standardising. May be implied by either correct equation.
	$P\left(Z \geq \frac{805-\mu}{\sigma}\right) = 0.11$ OR $P\left(Z \leq \frac{805-\mu}{\sigma}\right) = 0.89$ OR $P\left(Z \leq \frac{795-\mu}{\sigma}\right) = 0.2$	m1	m1 for at least one correct probability statement. May be implied by either correct equation.
	Sight of 1.2265 AND -0.8416	B1	CAO May be implied by both correct equations.
	$\frac{805-\mu}{\sigma} = 1.2265$	A1	1.227 from tables
	$\frac{795-\mu}{\sigma} = -0.8416$	A1	–0.842 from tables
	$\frac{795-\mu}{-0.8416} = \frac{805-\mu}{1.2265}$ OR $805 - 1.2265\sigma = 795 + 0.8416\sigma$	m1 (m1)	m1 for eliminating one variable Dependent on M1
	$\mu = 799$	A1	CAO Accept awrt 799.0 or 799.1 (when rounding to 1dp)
	$\sigma = 4.84$	A1	CAO Accept awrt 4.8 or 4.9 (when rounding to 1dp) Correct answers with no working throughout earn no marks.
(b)	(Let μ be the population mean mass of the loaves in grams)		Note: A two tailed test can score B0B1M1A1m1A0.
	$H_0: \mu = 400$ $\quad$ $H_1: \mu < 400$	B1	Allow other letters if defined. Allow worded hypotheses (must refer to population) B0 for $H_0: \text{mean} = 400$, must imply/refer to population. B0 for omission of μ, or $\bar{x}$ in place of μ. B0 for a non-strict inequality in H_1.
	$\bar{X} \sim \text{N}(400, \frac{9^2}{15})$ (under H_0)	B1	Distribution of $\bar{X}$ si. FT their hypotheses for 2nd B1 only.
	Method 1 (p-value with calculator): $\text{P}(\bar{X} < 397 \mid H_0)$	M1	M1 for attempt to find $P(\bar{X} < 397)$ with correct mean and variance for $\bar{X}$. Condone omission of given H_0.
	$= 0.098352$	A1	Condone 0.098.
	Since $0.0983 > 0.05$ there is insufficient evidence to reject H_0.	m1	Dependent on M1. Correct comparison required. Context may imply do not reject H_0. NOTE: the p-value for a 2-tailed test is 0.1967, or may see 0.09352 compared with 0.025. Do not allow comparison with 0.025 if using a one-tailed test.

Qu.	Solution	Mark	Notes
4 (b)	**Method 2 (*p*-value with standardising):** $\mathrm{P}\left(Z < \frac{397-400}{\frac{9}{\sqrt{15}}}\right) = \mathrm{P}(Z < -1.29)$ $(= 1 - 0.90147)$ $= 0.09853$ Since $0.0985 > 0.05$ there is insufficient evidence to reject H_0.	(M1) (A1) (m1)	M1 for either probability statement seen. Sight of -1.29 earns M1 only if using the p-value method. Must use $\sigma_{\bar{X}} = 9/\sqrt{15}$ for M1. May evaluate $P\left(Z < -\frac{\sqrt{15}}{3}\right) = 0.098352$ using the calculator. Dependent on M1. Correct comparison required. Context may imply do not reject H_0. NOTE: the p-value for a 2-tailed test is 0.19706, or may see 0.09853 compared with 0.025. Do not allow comparison with 0.025 if using a one-tailed test.
	Method 3 (critical value with calculator): CV = 396.18 (CR is $\bar{X} < 396.18$) Since $397 > 396.18$ there is insufficient evidence to reject H_0.	(M1A1) (m1)	M1 implied by correct answer from calculator or for correctly standardising: $\frac{\mathrm{CV}-400}{\frac{9}{\sqrt{15}}} = -1.645$. Must use $\sigma_{\bar{X}} = 9/\sqrt{15}$ for M1. Dependent on M1. FT their CV. Correct comparison required. NOTE: the CVs for a 2-tailed test are 395.45 and 404.55
	Method 4 (critical value with standardising): $\mathrm{TS} = \frac{397-400}{\frac{9}{\sqrt{15}}}$ $= -1.29$ Since $-1.29 > -1.645$ there is insufficient evidence to reject H_0.	(M1) (A1) (m1)	Must use $\sigma_{\bar{X}} = 9/\sqrt{15}$ for M1. -1.29 earns M1A1 if used as a TS. Dependent on M1. FT their TS. Correct comparison required. NOTE: for a 2-tailed test, comparison with -1.96 is required
	There is insufficient evidence to suggest that the mean mass of the loaves has decreased.	A1	CSO. Do not allow categorical statements (condone categorical statements if "insufficient evidence" seen in m1 statement). Allow equivalent statements, e.g. there is insufficient evidence to support the customer's suspicion. Condone weight in place of mass. A0 for conclusion not in context (e.g. mean has decreased). A0 for a two-tailed conclusion (e.g. mass has not changed).
(c)	Valid assumption. e.g. Assume that the 15 loaves constitute a random sample. e.g. Assume the 15 loaves are independent. e.g. There wasn't a fault in production.	E1 **Total [15]**	Ignore spurious comments. Condone "the masses of the loaves are (still) normally distributed". Comments such as "this isn't a bad batch" or "this isn't a faulty batch" are considered equivalent to "constitute a random sample", and earn E1. E0 for "weighed correctly" E0 for "uses the same ingredients" or "uses the same recipe" (E0 for any comment that implies the methodology remains the same) E0 for "all loaves made by new management" E0 for "they calculated the mean correctly"

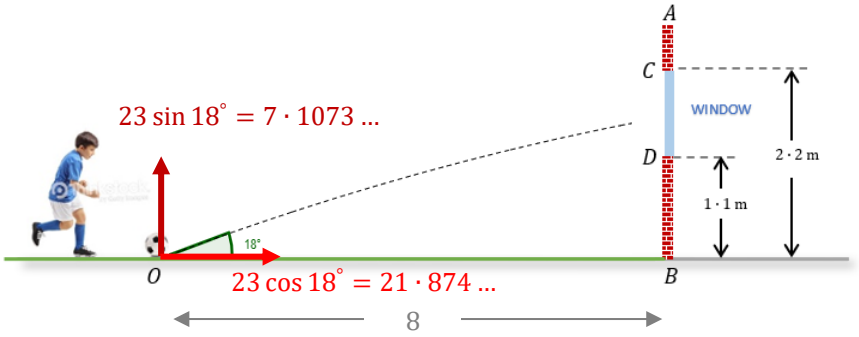
Qu.	Solution	Mark	Notes
5(a)	(Let ρ denote the population correlation coefficient between per capita daily calories from sugar consumption and percentage of the population with the disease.) $H_0: \rho = 0$ $\quad$ $H_1: \rho \neq 0$	B1	Allow other letters if defined. Allow worded hypotheses provided the population is stated or implied. B0 for H_0: correlation = 0. Population correlation must be stated or implied. B0 for omission of ρ, or r in place of ρ. B0 for a one-tailed test.
	$\text{TS} = 0.893$	B1	Labelled as TS or used in correct comparison
	CV = ($\pm$) 0.5368 (at the 1% level) (CR is $R < -0.5368$ or $R > 0.5368$)	B1	Accept use of different significance levels. OR Award this mark for stating "the test statistic is greater than all critical values".
	Since $\text{TS} > 0.5368$, there is sufficient evidence to reject H_0.	B1	FT their CV. Correct comparison required (may be implied through a diagram).
	This suggests that there is a link between the disease and sugar consumption.	E1	CSO Do not allow categorical statements (condone categorical statements if "sufficient evidence" seen in m1 statement). Statement must match two tailed hypothesis. E0 for a one-tailed conclusion, e.g. there is evidence of a positive correlation between…
(b)	Valid comment on correlation in 2000 and 2020. e.g. Although there is (evidence of) a (strong) positive correlation in 2000 there seems to be no correlation in 2020.	E1	
	Valid comment on the ambiguity of the studies. e.g. This would imply that sugar consumption is only coincidentally correlated with the disease in 2000. e.g. There must be a different cause for the disease other than sugar. e.g. n is too small to make meaningful comparisons. e.g. different samples lead to different conclusions, which may explain the difference. e.g. the link between the disease and sugar consumption has diminished over the 20 years.	E1	E1 for identifying a drawback of the conclusions or identifying a reason for the change. E0 for "this is because there is no link between the disease and sugar consumption" would substantiating with a valid reason.
	OR Valid comment on third factor correlated with sugar consumption and the disease in 2000 no longer relevant in 2020. e.g. An additive may have been added to many products containing sugar in 2000 which was contributing to the prevalence of the disease but has been banned by 2020. e.g. Advancements in medical care for the disease from 2000 to 2020 are likely to explain the change in relationship.	(E2)	E1 for correctly identifying a possible confounding variable. E1 for explaining why this may lead to the change in relationship seen.
		Total [7]	

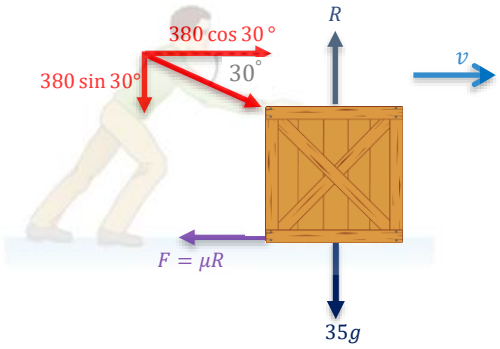
SECTION B – Differential Equations and Mechanics

Q6	Solution	Mark	Notes
(a)	(i) $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1	Correct differentiation of at least one component, $\mathbf{i}, \mathbf{j}$ retained
	$\mathbf{v} = (3t^2 - 14t)\mathbf{i} + (4t - 15)\,\mathbf{j}$ (ms^{-1})	A1	cao
	(ii) $\mathbf{i}$ coefficient $=$ $\mathbf{j}$ coefficient of v	M1	Used FT their $\mathbf{v}$ throughout
	$3t^2 - 14t = 4t - 15$ $3t^2 - 18t + 15 = 0$ $(t^2 - 6t + 5 = 0)$ Forming and solving quadratic	m1	
	$t = 5$ (or $t = 1$)	A1	cao
	At $t = 5$, $\mathbf{v} = 5\mathbf{i} + 5\,\mathbf{j}$ (ms^{-1}) (i.e. NE)	A1	cao
		[6]	At $t = 1$, $\mathbf{v} = -11\mathbf{i} - 11\,\mathbf{j}$ (i.e. SW) must be clearly discounted
(b)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	M1	Correct differentiation of at least one component, $\mathbf{i}, \mathbf{j}$ retained FT their $\mathbf{v}$ from (a)
	$\mathbf{a} = (6t - 14)\mathbf{i} + 4\,\mathbf{j}$	A1	
	At $t = 7$, $\mathbf{a} = 28\mathbf{i} + 4\,\mathbf{j}$ (ms^{-2})	A1	
		[3]	
	Total for Question 6	**9**	

Q7	Solution	Mark	Notes
(a)			
(i)	Resolve vertically $R_X + R_Y = 20g + 8g$ (= 196 + 78 · 4 = 274 · 4) $R_X + 4R_X = 28g$ OR $\frac{1}{4}R_Y + R_Y = 28g$ $R_X = \frac{28}{5}g = 5 \cdot 6g$ (N) $= 54 \cdot 88$ (N) $R_Y = 4 \times \frac{28}{5}g = 22 \cdot 4g$ (N) $= 219 \cdot 52$ (N)	M1 A1 A1	Dim. correct, all forces/terms cao cao, both values
(ii)	Moments about X $20g \times x + 8g \times 2 \cdot 6 = R_Y \times 2$ $196 \times x + 78 \cdot 4 \times 2 \cdot 6 = R_Y \times 2$ $196 \times x + 203 \cdot 84 = R_Y \times 2$ $x = 1 \cdot 2$ $\therefore\ 0 \cdot 4 + 1 \cdot 2 = 1 \cdot 6$ (m) = midpoint	M1 A1 B1 A1 **[7]**	Dim. correct equation attempted, oe, no missing forces Correct equation with unknown distance, FT R_Y (if substituted) Any correct moment with pivot clearly indicated cao, convincing
(b)	Not possible **and** Reason: Weight may act at the centre of a non-uniform rod	E1 **[1]**	
	Total for Question 7	**8**	

Further Notes			
(a) (ii)	Alternative Moments Solutions	B1	Any correct moment with pivot clearly indicated
	Moments about A $R_X \times 0 \cdot 4 + R_Y \times 2 \cdot 4 = 20g \times x' + 8g \times 3$ Substitution of both R_X and R_Y leading to correct midpoint $x' = 1 \cdot 6$	M1 A1 A1	Dim. correct equation, oe, no missing forces Correct equation with unknown distance FT R_i (if substituted) Nothing to add in this case so **bod**
	Moments about Y $R_X \times 2 + 8g \times 0 \cdot 6 = 20g \times x'$ Substitution of R_X leading to correct midpoint $x' = 0 \cdot 8$ midpoint $= 0 \cdot 8 + 0 \cdot 8 = 1 \cdot 6$	M1 A1 A1	Dim. correct equation, oe, no missing forces Correct equation with unknown distance FT R_i (if substituted)
	Moments about B $R_X \times 2 \cdot 8 + R_Y \times 0 \cdot 8 = 20g \times x' + 8g \times 0 \cdot 2$ Substitution of both R_X and R_Y leading to correct midpoint $x' = 1 \cdot 6$	M1 A1 A1	Dim. correct equation, oe, no missing forces Correct equation with unknown distance FT R_i (if substituted) Nothing to add in this case so **bod**
	Moments about C $R_X \times 2 \cdot 6 + R_Y \times 0 \cdot 6 = 20g \times x'$ Substitution of both R_X and R_Y leading to correct midpoint $x = 1 \cdot 4$ midpoint $= 1 \cdot 4 + 0 \cdot 2 = 1 \cdot 6$	M1 A1 A1	Dim. correct equation, oe, no missing forces Correct equation with unknown distance FT R_i (if substituted)
	Moments about Centre of mass $R_X \times x' + 8g \times (2 \cdot 6 - x') = R_Y \times (2 - x')$ Substitution of both R_X and R_Y leading to correct midpoint $x' = 0$	M1 A1 A1	Dim. correct equation, oe, no missing forces Correct equation with unknown distance FT R_i (if substituted)

Q8	Solution	Mark	Notes
			
(a)	Initial horizontal velocity $= 23\cos 18^\circ$ $(= 21\cdot 874\ldots)$	B1	si
	Initial vertical velocity $= 23\sin 18^\circ$ $(= 7\cdot 107\ldots)$	B1	si
	Time to reach AB,		
	$t = \frac{8}{23\cos 18^\circ}$ $(= 0\cdot 3657\ldots)$	M1	$8 = 23\cos 18^\circ \times t$
	Working vertically using $s = ut + \frac{1}{2}at^2$, with $u = \pm 23\sin 18^\circ, a = \pm g, t = \frac{8}{23\cos 18^\circ}$	M1	
	$s = (\mp 23\sin 18^\circ)\left(\frac{8}{23\cos 18^\circ}\right) + \frac{1}{2}(\pm 9\cdot 8)\left(\frac{8}{23\cos 18^\circ}\right)^2$ $s = (\mp 7\cdot 107\ldots)(0\cdot 365\ldots) + \frac{1}{2}(\pm 9\cdot 8)(0\cdot 365\ldots)^2$	A1	$23\sin 18^\circ$ and g **opposing**
	$s = \mp 1\cdot 94\ldots$		
	Ball strikes window since $1\cdot 1 < 1\cdot 9 < 2\cdot 2$	A1	Convincing (or between C and D)
	Window breaks since		
	minimum speed $= 23\cos 18^\circ = 21\cdot 87\ldots > 21$	B1	Must compare to 21
		[7]	
(b)	Any sensible reason. For example, • Does not consider dimensions of ball • Assumes ball is a particle • Does not consider mass/weight • Assumes motion is in one vertical plane • Resistance is ignored (Air or wind)	E1	
		[1]	
	Total for Question 8	**8**	

Q9	Solution	Mark	Notes
			$R = 35g + 380\sin 30^\circ$ $F = \mu R = (35g + 380\sin 30^\circ)\mu$ $35g = 343$ $380\cos 30° = 380 \times \frac{\sqrt{3}}{2}$ $= 190\sqrt{3} = 329 \cdot 08965 \ldots$ $380\sin 30^\circ = 190$
(a)	N2L horizontally with $a = 0$ $380\cos 30^\circ - F = 0$ $F = 380\cos 30^\circ \quad (F = 190\sqrt{3} = 329 \cdot 08 \ldots)$	M1 A1	Dim. Correct, no extra terms
	Resolving vertically $R = 35g + 380\sin 30^\circ$ $R = 35g + 190 = 533$	M1 A1	Dim. correct, all forces, no extra
	Use of $F = \mu R$ $\mu = \frac{190\sqrt{3}}{533}$ Accept $\mu = \frac{190\sqrt{3}}{35g+190} = \frac{38\sqrt{3}}{7g+38}$	M1 A1 **[6]**	Convincing
(b)	Normal reaction will be smaller so friction will be smaller, therefore non-zero net force causing an acceleration.	E1 **[1]**	
	Total for Question 9	**7**	

Q10	Solution	Mark	Notes
a)	(i) $a = \frac{k}{v}$ $\quad (k = av)$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{k}{v}$	B1	Allow $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{-k}{v}$
	At $t = 1$, $v = 5$ and $a = 1 \cdot 8 \quad \Rightarrow \quad k = 9$	B1	
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{9}{v}$		
	(ii) $\int v\,\mathrm{d}v = \pm\int 9\,\mathrm{d}t$ OR $\int v\,\mathrm{d}v = \pm\int k\,\mathrm{d}t$	M1	Separating variables **and** an attempt to integrate
	$\frac{1}{2}v^2 = 9t\ (+C)$ OR $\frac{1}{2}v^2 = kt\ (+C)$	A1	Both sides correct (inc. $-k$ case)
	At $t = 1$, $v = 5 \quad \Rightarrow \quad C = \frac{7}{2}$	m1	Used (only if $+C$ is present)
	$\frac{1}{2}v^2 = 9t + \frac{7}{2}$		
	$v^2 = 18t + 7$	A1	Convincing
		[6]	
b)	$v = \frac{9}{v} \quad \Rightarrow \quad v^2 = 9 \quad \Rightarrow \quad v = \pm 3$	M1	Allow $v = \frac{k}{v} \quad \Rightarrow \quad v^2 = k$
	$18t + 7 = 9$		
	$t = \frac{1}{9}$	A1	cao
		[2]	
	Total for Question 10	**8**	