GCE AS MARKING SCHEME

SUMMER 2023

AS
MATHEMATICS
UNIT 2 APPLIED MATHEMATICS A
2300U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

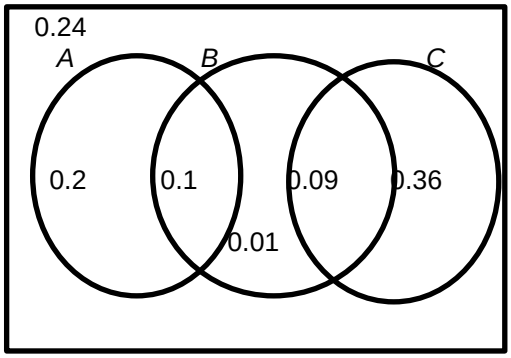
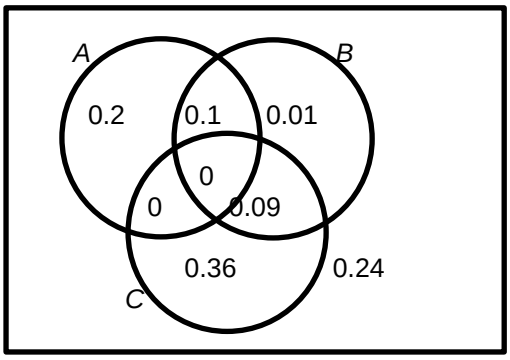
WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS MATHEMATICS

UNIT 2 APPLIED MATHEMATICS A

SUMMER 2023 MARK SCHEME

SECTION A – Statistics

Qu.	Solution	Mark	Notes
1(a)	$\mathrm{P}(A \cup B) = 0.3 + 0.2 - 0.1$	M1	Use of addition formula $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
	$\mathrm{P}(A \cup B) = 0.4$	A1	Condone missing labels
(b)	$\mathrm{P}(B \cap C) = 0.2 \times 0.45$	M1	Use of $P(B \cap C) = P(B)P(C)$
	$\mathrm{P}(B \cap C) = 0.09$	A1	Condone missing labels
(c)			*FT their 0.09 for possible B1B1B1B1 (provided all valid probabilities)*
		B1	B1 for three overlapping circles with A, B and C labelled with *A&B* and *B&C* intersecting and NOT *A&C*, or $\mathrm{P}(A \cap C) = 0$ (condone box omitted)
		B1	B1 for 0.1 and *0.09* seen within a valid Venn diagram.
		B1	B1 for any two of *0.24*, 0.2, *0.01*, *0.36* Allow written with correct notation: $P(A' \cap B' \cap C') = 0.24$, $P(A \cap B' \cap C') = 0.2$ $P(A' \cap B \cap C') = 0.01$ $P(A' \cap B' \cap C) = 0.36$
		B1	B1 for all of 0.24, 0.2, 0.01 and 0.36 within a fully valid Venn diagram inclusive of box.
	OR may see		
			
		(B1)	B1 for three overlapping circles with A, B and C labelled with $\mathrm{P}(A \cap B \cap C) = 0$ and $P(A \cap B' \cap C) = 0$ (condone box omitted)
		(B1)	B0 if these are blank (must contain 0)
		(B1)	B1 for 0.1 and *0.09* seen within a valid Venn diagram.
		(B1)	B1 for any two of *0.24*, 0.2, *0.01*, *0.36*
			B1 for all of 0.24, 0.2, 0.01 and 0.36 within a fully valid Venn diagram inclusive of box.
(d)	Use of Venn diagram to find $P((B \cup C)')$ directly.		$\mathrm{P}(B \cup C')$ is M0A0
	$\mathrm{P}((B \cup C)') = 0.2 + 0.24$	M1	FT their "0.2" and "0.24" provided two valid probabilities added to give an answer between 0 and 1.
	$\mathrm{P}((B \cup C)') = 0.44$	A1	FT (c) provided valid probability.
	OR (use of Venn diagram to find $P(B \cup C)$)		
	Use of $1 - \mathrm{P}(B \cup C) = 1 - (0.1 + 0.01 + 0.09 + 0.36)$	(M1)	FT their "0.09" provided valid prob with attempt to subtract from 1. Final answer between 0 and 1 to earn M1.
	$\mathrm{P}((B \cup C)') = 1 - 0.56 = 0.44$	(A1)	
	OR (use of addition formula to find $P(B \cup C)$)		
	Use of $1 - P(B \cup C) = 1 - (0.2 + 0.45 - 0.09)$	(M1)	FT their "0.09" provided valid prob with attempt to subtract from 1. Final answer between 0 and 1 to earn M1.
	$\mathrm{P}((B \cup C)') = 1 - 0.56 = 0.44$	(A1)	
		Total [10]	

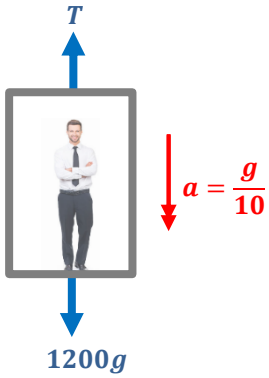
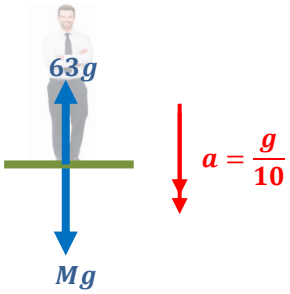
Qu.	Solution	Mark	Notes
2	(Discrete) uniform (distribution). Because **each card** has the same probability (1/20) of being selected.	B1 E1	Continuous uniform earns B0. B0E1 is possible for correct reason E0 for the probability is constant. Must refer to each card.
		Total [2]	
3	$P(8,16) + P(4,16) + P(4,8) + P(16,16) + P(8,8) + P(4,4)$ $= 0.5 \times 0.2 + 0.3 \times 0.2 + 0.3 \times 0.5 + 0.2^2 + 0.5^2 + 0.3^2$ $= 0.69$	M1 m1A1 A1	M1 for at least three correct combinations, si m1 addition of products for their combinations A1 fully correct calculation (no terms omitted) cao
	OR P(4, anything) + P(8, 8 or 16) + P(16,16) $= 0.3 \times 1 + 0.5 \times 0.7 + 0.2 \times 0.2$ $= 0.69$	(M1) (m1A1) (A1)	M1 for at least two correct combinations, si. m1 addition of products for their combinations A1 for fully correct calculation (no terms omitted) cao
	OR $1 -$ ((P(8, 4) + P(16, 4) + P(16, 8)) $1 - (0.5 \times 0.3 + 0.2 \times 0.3 + 0.2 \times 0.5)$ $= 0.69$	(M1) (m1A1) (A1)	M1 for at least two correct combinations excluded, si. Sight of 0.31 implies M1. m1 FT their combinations, must include '1-' with addition products for their combinations A1 for fully correct calculation (no terms omitted) cao
		Total [4]	

Qu.	Solution	Mark	Notes
4(a)	Let the random variable X be the number of emissions in a 3-second period.		
	$X \sim \text{Po}(12)$	B1	si
	$\begin{aligned} \text{P}(X \geq 10) &= 1 - \text{P}(X \leq 9) \\ &= 1 - 0.2424 \\ &= 0.7576 \end{aligned}$	M1 A1	Written, or used for their stated Poisson distribution. cao 3sf or better
(b)	Let the random variable Y be the number of sources that emit at least 10 radioactive particles in a 3-second interval.		
	$Y \sim \text{B}(n, p)$	M1	si (binomial distribution with valid numerical n and valid p)
	$Y \sim \text{B}(9, 0.7576)$	A1	FT $0 <$'their p'< 1 in (a), si
	$\text{P}(Y \leq 5) = 0.1518$	A1	cao 3sf or better (award A1 for 0.151 from $p = 0.758$)
		Total [6]	

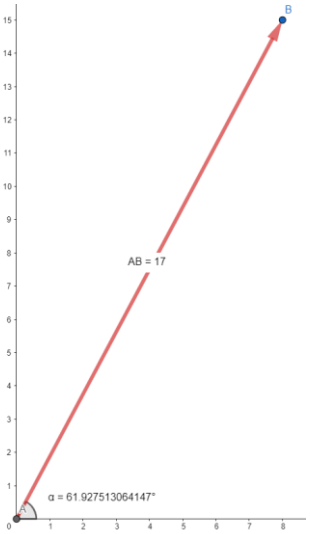
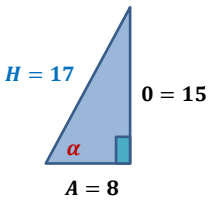
Qu.	Solution	Mark	Notes
5(a)	Valid explanation e.g. The doctor is only interested in a reduction in the **proportion** of his patients who take too little exercise. e.g. The behaviour change program is designed to reduce the **proportion** of patients who take too little exercise.	E1	Reference to upper tail E0 E0 for only referencing or implying increase/decrease in amount of exercise.
(b)(i)	Valid explanation e.g. the p- value is the probability of 40 or fewer patients taking too little exercise after the programme, given that there has been no change. e.g. p-value is the probability of 40 or fewer patients taking too little exercise after the programme, given that $p = 0.34$. e.g. p-value is the probability of 40 or fewer patients taking too little exercise after the programme, given H_0 is true.	E1	oe must be in context. E0 for "given that there has been no change" or equivalent omitted.
(ii)	$H_0: p = 0.34 \quad H_1: p < 0.34$ (Let the random variable X represent the number of patients who take too little exercise.) Under H_0, $X \sim \mathrm{B}(156, 0.34)$ **Method 1 (p-value):** $\mathrm{P}(X \leq 40) = 0.015488$ Since 0.015488 > 0.01, there is insufficient evidence to reject H_0. **Method 2 (critical region):** The CR is $X \leq k$ where $P(X \leq k) \leq 0.01$. $P(X \leq 39) = 0.009768$, $P(X \leq 40) = 0.015488$ Hence, $k = 39$. The CR is $X \leq 39$. Since 40 is not in the CR, there is insufficient evidence to reject H_0. There is insufficient evidence at the 1% level to suggest the behaviour change programme has had the desired effect.	B1 B1 M1A1 m1 (M1) (A1) (m1) A1 **Total [8]**	Allow use of θ or π, or other letters if defined. Do not allow $x = 0.34$. Allow worded hypotheses or use of 34%. B0 for 0.34%. B0 for omission of p or a non-strict inequality in H_1. si (e.g. from correctly calculated $P(X \leq 40)$ or $P(X = 40)$) M0A0m0A0 for $\mathrm{P}(X = k)$, e.g. sight of $P(X = 40) = 0.00572$ earns M0 M1 for attempt to find $\mathrm{P}(X \leq 40)$ si. FT their '$\mathrm{P}(X \leq 40)$' – probability needed and correct comparison with 0.01. Do not need to see insufficient evidence for this mark. Either probability (evaluating $P(X = 39)$ or $P(X = 40)$ earns M0A0m0A0). CV or CR identified (condone as probability statement, i.e. $P(X \leq 39)$). Award M1A1 for correct CV/CR with no working. FT their calculated CV CSO Must have fully correct solution. Do not allow categorical statements without reference to insufficient evidence or suggests within solution. Method 1: Withhold this mark if the p-value is not compared with the significance level. Method 2: Withhold this mark if $P(X \leq 39)$ given as CR or failing to consider both probs. Withhold this mark if the test statistic is not compared with the CV.

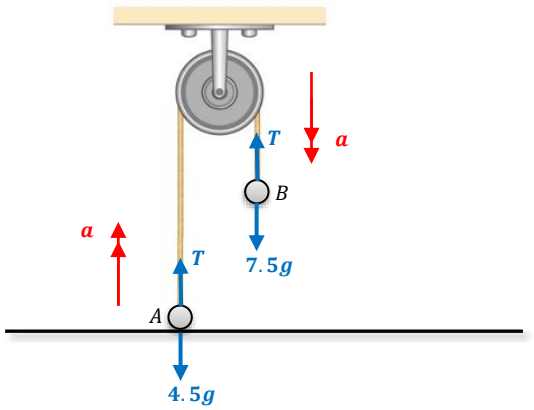
Qu.	Solution	Mark	Notes
6(a)	Select students numbered 195, 023, 432, 271	B1	Must reject 752 (outside range) and second 195 (appears twice). Allow any order.
(b)	Valid reasons. e.g. Different samples can lead to different conclusions. e.g. Clare's school may be in a different area. e.g. Poppi and Clare are at different stages of schooling. e.g. One school may be Welsh medium, whilst the other may be English medium.	E2	E1 for each – do not award E2 for the same comment written in two different ways. E0 for Clare's sample is larger than Poppi's. E0 for "use a bigger sample".
(c)(i)	Valid reason. e.g. 10.8% can speak Welsh in Region B compared to 9.7% in Region A.	B1	B0 for answers based on the number of people rather than percentages (e.g. more people speak Welsh in Region B than Region A). Condone percentages not quoted (e.g. a greater percentage / proportion in Region B can speak Welsh compared to Region A).
(ii)	Valid reason. e.g. Only 82.6% have no skills in Welsh in Region A compared to 83.7% in Region B. e.g. 3.6% have other skills in Welsh compared to 2.2% in Region B. e.g. 4.1% understand spoken Welsh in Region A compared to 3.3% in Region B.	B1	B0 for answers based on the number of people rather than percentages Condone percentages not quoted
(d)(i)	$\bar{x}\left(=\frac{\Sigma x}{n}=\frac{53.9}{7}\right)=7.7$ (minutes) Appropriate use of calculator $\sigma = 2.56$ (minutes) OR $\sigma=\sqrt{\frac{\Sigma x^2}{n}-\bar{x}^2}=\sqrt{\frac{7.2^2+5.4^2+7.4^2+4.6^2+13.2^2+8.4^2+7.7^2}{7}-7.7^2}=\sqrt{\frac{461.01}{7}-7.7^2}$ = 2.56 (minutes) OR $\sigma=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}=\sqrt{\frac{(7.2-7.7)^2+(5.4-7.7)^2+(7.4-7.7)^2+(4.6-7.7)^2+(13.2-7.7)^2+(8.4-7.7)^2+(7.7-7.7)^2}{7}}=\sqrt{\frac{45.98}{7}}$ $= 2.56$ (minutes)	B1 M1 A1 (M1) (A1) (M1) (A1)	Allow embedded $\bar{x}$ within a shown calculation (either alternative method). Allow labelled as μ. Allow 2.6 Equivalents are $\sqrt{\frac{\Sigma x^2}{n}-\frac{(\Sigma x)^2}{n}}$ or $\sqrt{\frac{S_{xx}}{n}}$ FT their mean provided positive term within square root for M1 Allow 2.6 FT their mean for M1 Allow 2.6 SC1 for $\sigma^2 = 6.57$ or $s^2 = 7.66$ Accept calculation of s^2 leading to an answer of 2.768 for M1A1
(ii)	Valid comparison of means in context with interpretation: e.g. The children who can speak Welsh can concentrate for longer **on average / in general** than those who have no skills in Welsh. e.g. The children who cannot speak Welsh concentrate for less time **on average / in general** than those who can speak Welsh. Valid comparison of standard deviations in context with interpretation: e.g. The length of time children who have no skills in Welsh can concentrate for is **more variable / less consistent / more spread out** than those who can speak Welsh. e.g. The length of time children who speak Welsh can concentrate for is **less variable / more consistent / less spread out** than those who have no skills in Welsh. However, one should not overly heed these comparisons due to the small sample size.	E1 E1 (E1) **Total [10]**	Must be an interpretation not simply comparing values. FT their $\bar{x}$. E0 for omission of "on average". E0 for the mean concentration time for those who speak Welsh is higher than the mean concentration time for those who do not speak Welsh oe. Must be an interpretation not simply comparing values. FT their σ. E0 for the standard deviation for those who speak Welsh is less than the standard deviation for those who do not speak Welsh oe. Only following on from a comparison of means or standard deviations. BOD for "1st group" in place of "children who speak Welsh".

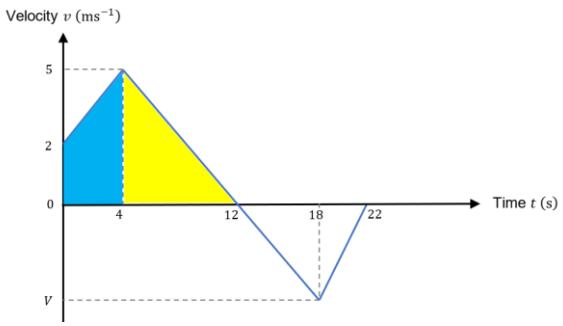
SECTION B – Mechanics

Q7	Solution	Mark	Notes
			$1200g = 11\,760$ $a = \frac{g}{10} = 0 \cdot 98$
a)	N2L applied to lift, downwards positive $1200g - T = 1200a \quad (11760 - T = 1200a)$ $1200g - T = 1200\left(\frac{g}{10}\right) \quad (11760 - T = 1176)$ $T = 1080g = 10\,584$ (N)	M1 A1 A1 **[3]**	Dimensionally correct equation T and $1200g$ opposing Correct equation $T = 1200\left(\frac{9g}{10}\right)$ cao
			$63g = 617 \cdot 4$ $a = \frac{g}{10} = 0 \cdot 98$
b)	N2L applied to person only, downwards positive $Mg - R = Ma$ $Mg - 63g = Ma \quad (9 \cdot 8M - 617 \cdot 4 = Ma)$ $Mg - 63g = M\left(\frac{g}{10}\right) \quad (9 \cdot 8M - 617 \cdot 4 = 0 \cdot 98M)$ $M = 70$	M1 A1 A1 **[3]**	Dimensionally correct equation $R(= 63g)$ and Mg opposing Correct equation $\frac{9}{10}M = 63$ cao
	Total for Question 7	6	

Q8	Solution	Mark	Notes
	At starting position $(t = 0)$, $s = 0$ $4t^2 - t^3 = 0$ $t^2(4 - t) = 0$	M1	Used
	$t = 4$ (or $(t = 0)$)	A1	
	$v = \frac{ds}{dt} = 8t - 3t^2$	M1	Attempt to differentiate
	At $t = 4$, $v = 8(4) - 3(4)^2$ $v = -16$	m1	
	speed $= 16$ (ms^{-1})	A1	cao
		[5]	
	Total for Question 8	**5**	

Q9	Solution	Mark	Notes
a)	$\mathbf{R} = \mathbf{F} + \mathbf{G} + \mathbf{H}$ $= (2\mathbf{i} - 13\mathbf{j}) + (-6\mathbf{i} + 8\mathbf{j}) + (12\mathbf{i} + k\mathbf{j})$ $= 8\mathbf{i} + (k - 5)\mathbf{j}$ $\lvert\mathbf{R}\rvert = 17$ $\sqrt{8^2 + (k-5)^2} = 17$ $k - 5 = \pm 15$ $k = -10, 20$	M1 A1 M1 A1 **[4]**	 cao, both values
b)	For larger $k = 20$, $\mathbf{R} = 8\mathbf{i} + (20 - 5)\mathbf{j} = 8\mathbf{i} + 15\mathbf{j}$  Let α be angle that $\mathbf{R}$ makes with the $\mathbf{i}$ vector Accept any of the following ratios $\sin\alpha = \frac{15}{17}$ $\left(= \frac{8}{17}\right)$ $\cos\alpha = \frac{8}{17}$ $\left(= \frac{15}{17}\right)$ $\tan\alpha = \frac{15}{8}$ $\left(= \frac{8}{15}\right)$ $(90 - 28 \cdot 0(724\ldots)^\circ)$ $\alpha = 61 \cdot 9(275\ldots)^\circ$	 M1 A1 **[2]**	$\sin\alpha = \frac{15}{17}$ $\cos\alpha = \frac{8}{17}$ $\tan\alpha = \frac{15}{8}$  FT their derived value of k $\sin\alpha = \frac{k-5}{17}$ $\left(= \frac{8}{17}\right)$ $\cos\alpha = \frac{8}{17}$ $\left(= \frac{k-5}{17}\right)$ $\tan\alpha = \frac{k-5}{8}$ $\left(= \frac{8}{k-5}\right)$ cao
	Total for Question 9	6	

Q10	Solution	Mark	Notes
a)	Tension $= 7 \cdot 5g$ or $73 \cdot 5$ (N)	B1 **[1]**	
b)			$4 \cdot 5g = 44 \cdot 1$ $7 \cdot 5g = 73 \cdot 5$
i)	N2L applied to **both** A and B	M1	Dimensionally correct eqn. for at least 1 object T and $4 \cdot 5g / 7 \cdot 5g$ opposing
	$T - 4 \cdot 5g = 4 \cdot 5a$ $\quad (T - 44 \cdot 1 = 4 \cdot 5a)$	B1	1st correct equation
	$7 \cdot 5g - T = 7 \cdot 5a$ $\quad (73 \cdot 5 - T = 7 \cdot 5a)$	A1	2nd correct equation
	Eliminating T $3g = 12a$ $\quad (29 \cdot 4 = 12a)$	m1	Note $T = 55 \cdot 125$ (N)
	$a = \frac{1}{4}g = 2 \cdot 45$ (ms^{-2})	A1	cao
ii)	$s = ut + \frac{1}{2}at^2$, with $s = \pm 0 \cdot 9$ (or $\pm 1 \cdot 8$), $u = 0, a = \pm 2 \cdot 45$	M1	
	$0 \cdot 9 = \frac{1}{2}(2 \cdot 45)t^2$	A1	
	$t = \frac{6}{7} = 0 \cdot 85(7142 \ldots)$ s	A1	cao
iii)	Time (answer (ii)) will be greater	E1 **[9]**	oe
	Total for Question 10	**10**	

Q11	Solution	Mark	Notes
a)	At $t = 1$, acceleration = gradient $= \frac{5-2}{4-0}$ $= \frac{3}{4}$ or $0 \cdot 75$ (ms^{-2})	M1 A1 **[2]**	cao
	Alternative Solution $v = u + at,\ u = 2, v = 5, t = 4$ $5 = 2 + a(4)$ $a = \frac{3}{4}$ or $0 \cdot 75$ (ms^{-1})	(M1) (A1) **([2])**	Used cao
b)	 Distance = area $= \frac{1}{2}(2+5)(4) + \frac{1}{2}(8)(5)$ $= 34$ (m)	M1 A1 **[2]**	Attempt to find relevant area cao
c)	area below t – axis $= 34 - 14$ $\pm\frac{1}{2}(10V) = \pm(34-14)$ $V = -4$	M1 m1 A1 **[3]**	FT distance in b) cao
d)	Correct statement, for example • $12 < t < 18$ • 12 to 18 • $12 \rightarrow 18$	B1 **[1]**	Statement (mathematical or otherwise) to the effect that interval is between the given boundaries.
	Total for Question 11	**8**	