GCE AS MARKING SCHEME

SUMMER 2025

AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

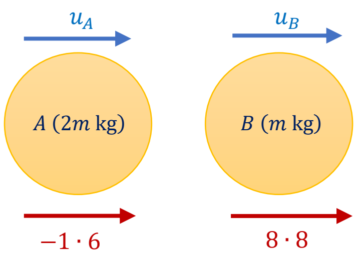
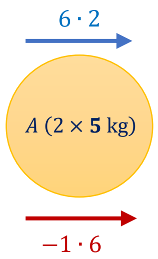
Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

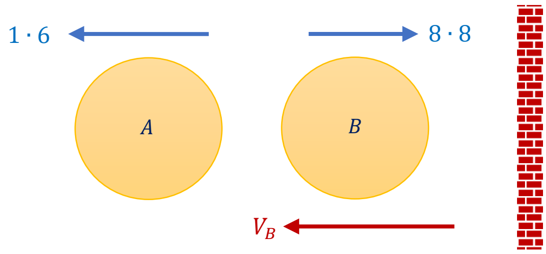
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

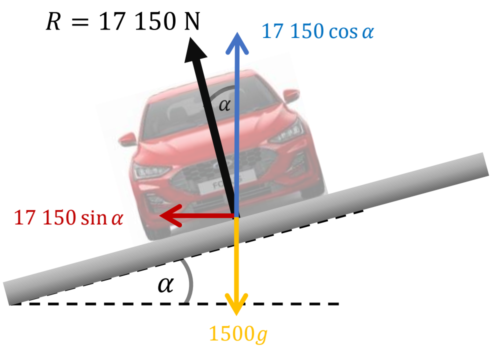
SUMMER 2025 MARK SCHEME

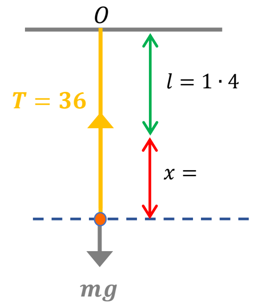
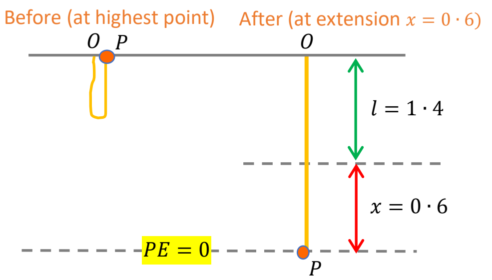
Q1	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	M1	Correct differentiation of at least one term
	$\mathbf{a} = 6t^2\mathbf{i} + 4\mathbf{j} - 2t\mathbf{k}$	A1	All correct
	$\mathbf{F} = m\mathbf{a} = 1\cdot5(6t^2\mathbf{i} + 4\mathbf{j} - 2t\mathbf{k})$	B1	FT their **a**, isw
	$\mathbf{F} = 9t^2\mathbf{i} + 6\mathbf{j} - 3t\mathbf{k}$	**[3]**	
(b)	$\mathbf{F}.\mathbf{v} = (9t^2 \times 2t^3) + (6 \times 4t) + (-3t \times -t^2)$	M1	Correct method for dot product Must lose **i, j, k** FT F from part (a)
	$\mathbf{F}.\mathbf{v} = 18t^5 + 24t + 3t^3$		
	At $t = 2$, $\mathbf{F}.\mathbf{v} = 18(2)^5 + 24(2) + 3(2)^3$	m1	Use of $t = 2$
	$\mathbf{F}.\mathbf{v} = 648\ \mathrm{W}$	A1	cao, **including** units (oe)
		[3]	For example, • $\mathrm{J\,s^{-1}}$ • $\mathrm{kg\,m^2s^{-3}}$ • $\mathrm{N\,m\,s^{-1}}$
	<u>Alternative Solution</u>		
	At $t = 2$, $\mathbf{v} = 16\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$, $\mathbf{F} = 36\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$	(M1)	Use of $t = 2$ for **both** FT F from part (a)
	$\mathbf{F}.\mathbf{v} = (36 \times 16) + (6 \times 8) + (-6 \times -4)$	(m1)	Correct method for dot product Must lose **i, j, k**
	$\mathbf{F}.\mathbf{v} = 648\ \mathrm{W}$	(A1)	cao, **including** units (oe)
		([3])	
	Total for Question 1	**6**	

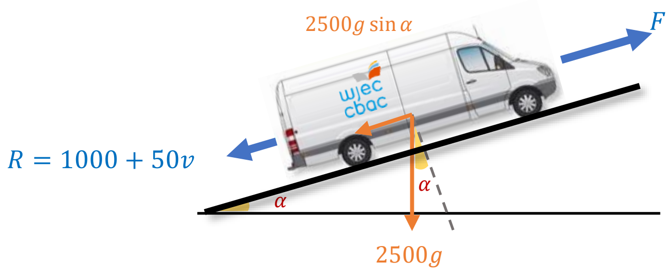
Q2	Solution	Mark	Notes
(a)			$e = 0\cdot8$
	Conservation of momentum $(2m)(u_A) + (m)(u_B) = (2m)(-1\cdot6) + (m)(8\cdot8)$ $2u_A + u_B = 5\cdot6$ Restitution $8\cdot8 - (-1\cdot6) = -0\cdot8(u_B - u_A)$ $u_A - u_B = 13$ Elimination of one variable $u_A = \ \ 6\cdot2 \Rightarrow$ speed of A = $6\cdot2$ (ms^{-1}) $u_B = -6\cdot8 \Rightarrow$ speed of B = $6\cdot8$ (ms^{-1})	M1 A1 M1 A1 m1 A1 A1 **[7]**	Attempted. Allow 1 sign error All correct Attempted. Allow 1 sign error All correct, oe (Ratio) cao cao
(b)			
	Impulse on B (Impulse on A) $I_B = m_Bv_B - m_Bu_B$ $\quad -I_A = m_Av_A - m_Au_A$ $= (5)(8\cdot8 - -6\cdot8)$ $\quad = (10)(-1\cdot6 - 6\cdot2)$ $= 78$ $\quad = -78$ $\lvert I_B\rvert = 78$ (Ns) **Opposite** direction to original motion of B	M1 A1 B1 **[3]**	Used, allow 1 sign error cao cao Accept • 'to the right' • $\longrightarrow$ • Towards the wall

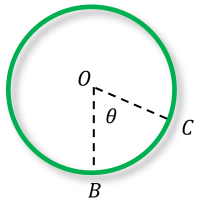
Q2	Solution	Mark	Notes
(c)			
	$V_B = \pm 8 \cdot 8e$ To collide with A again, $V_B > 1 \cdot 6$ $8 \cdot 8e > 1 \cdot 6$ $e > \frac{2}{11}$ $(= 0 \cdot 1818 \ldots)$ Range: $\frac{2}{11} < e \leq 1$	B1 M1 A1 **[3]**	si ANY comparison of $\pm 8 \cdot 8e$ with $\pm 1 \cdot 6$ cao
	Total for Question 2	**13**	

Q3	Solution	Mark	Notes
(a)	Position vector of the form $\mathbf{r} = \mathbf{p} + \mathbf{v}t$	M1	oe
	$\mathbf{r}_A = \begin{cases} 3\mathbf{i} - 10\mathbf{j} + 20\mathbf{k} + (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})t \\ \begin{pmatrix} 3 - t \\ -10 + 2t \\ 20 + t \end{pmatrix} \\ (3 - t)\mathbf{i} + (-10 + 2t)\mathbf{j} + (20 + t)\mathbf{k} \end{cases}$	A1	isw when correct result seen
	$\mathbf{r}_B = \begin{cases} -2\mathbf{i} + 30\mathbf{j} + 15\mathbf{k} + (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})t \\ \begin{pmatrix} -2 + 3t \\ 30 - 4t \\ 15 + 2t \end{pmatrix} \\ (-2 + 3t)\mathbf{i} + (30 - 4t)\mathbf{j} + (15 + 2t)\mathbf{k} \end{cases}$	A1	isw when correct result seen
		[3]	
(b)	$AB = \mathbf{r}_B - \mathbf{r}_A$	M1	$\pm(\mathbf{r}_B - \mathbf{r}_A)$ Attempted difference for all 3 terms
	$= (-5 + 4t)\mathbf{i} + (40 - 6t)\mathbf{j} + (-5 + t)\mathbf{k}$	A1	oe, at least 2 correct terms
	$AB^2 = (-5 + 4t)^2 + (40 - 6t)^2 + (-5 + t)^2$	m1	oe. Must lose **i, j, k** and be linear expressions
	$AB^2 = 53t^2 - 530t + 1650$	A1	Convincing, intermediate line required
		[4]	
(c)	$\frac{\mathrm{d}}{\mathrm{d}t}AB^2 = \frac{\mathrm{d}}{\mathrm{d}t}(53t^2 - 530t + 1650)$ $(= 106t - 530)$	M1	First derivative attempted No FT as quadratic given in (b)
	$\frac{\mathrm{d}}{\mathrm{d}t}AB^2 = 0$ $(106t - 530 = 0)$	m1	Used
	$t = 5$	A1	cao
	When $t = 5$, $AB^2 = 53(5)^2 - 530(5) + 1650$ $(= 325)$		
	Least distance is $\sqrt{325} = 5\sqrt{13} = 18 \cdot 02(775638)$ (m)	A1	cao, any form
		[4]	
	<u>Alternative solution</u>		
	$AB^2 = 53(t^2 - 10t) + 1650$ $AB^2 = 53(t - 5)^2 + 325$	(M1) (m1)	Attempt to 'complete the square' $53(t \pm a)^2 + b$, allow $b = 0$
	Minimum value occurs when $t = 5$	(A1)	cao
	When $t = 5$, $AB^2 = 325$		
	Least distance is $\sqrt{325} = 5\sqrt{13} = 18 \cdot 02(775638)$ (m)	(A1)	cao, any form
		([4])	
	Total for Question 3	**11**	

Q4	Solution	Mark	Notes
			$\cos\alpha = \frac{6}{7}$ $\sin\theta = \frac{\sqrt{13}}{7}$ $1500g = 14\ 700$
(a)	Resolve vertically $R\cos\alpha = mg$ $17\ 150\cos\alpha = 1500g$ $\cos\alpha = \frac{6}{7} = \frac{14700}{17150}$ $\alpha = 31 \cdot 00271913 \ldots (\approx 31°)$	M1 A1 A1 **[3]**	Dimensionally correct eqn. Accept $R\sin\alpha = mg$ Convincing
(b)	N2L towards centre $R\sin\alpha = ma$ $17\ 150\sin\alpha = 1500a$ $17\ 150\sin\alpha = 1500\left(\frac{v^2}{r}\right)$ $17\ 150\left(\frac{\sqrt{13}}{7}\right) = 1500\left(\frac{v^2}{150}\right)$ $v^2 = 883 \cdot 3600625$ $v = 29 \cdot 72(137383 \ldots) \quad (\text{ms}^{-1})$ **SC1** Award SC1 for $v = 29 \cdot 72(137383 \ldots)$ via $v^2 = rg\tan\alpha$	M1 A1 m1 A1 **[4]**	Dimensionally correct eqn. Accept $R\cos\alpha = ma$ Used with $a = \frac{v^2}{r}$ $v^2 = \frac{1715\sqrt{13}}{7} = 245\sqrt{13}$ Note. Using $\alpha = 31°$ $v^2 = 883 \cdot 2902985$ $v = 29 \cdot 72(020018 \ldots) \quad (\text{ms}^{-1})$
	Total for Question 4	7	

Q5	Solution	Mark	Notes
(a)	Use of Hooke's Law $36 = \frac{156\cdot 8x}{1\cdot 4}$	M1	Used 
	$x = \frac{9}{28}$ or $0\cdot 32(142857\ldots)$ (m)	A1	
		[2]	
(b)			
	Using expression for $\mathrm{EE} = \frac{\lambda x^2}{2l}$	M1	Used
	$\mathrm{EE} = \frac{156\cdot 8(0\cdot 6)^2}{2(1\cdot 4)} = \frac{504}{25} = 20\cdot 16$	A1	A correct expression
	Using expression for $\mathrm{PE} = mgh$	M1	Used
	$\mathrm{PE} = mg(\pm 2) = \frac{98}{5}m = 19\cdot 6m = 2mg$	A1	A correct exp. relative to their $PE = 0$. For example, $mg(0\cdot 32) = 3\cdot 136m$ $mg(0\cdot 28) = 2\cdot 744m$
	$\mathrm{KE} = 15\cdot 12$ (provided in question)		
	Conservation of Energy (before = after)	M1	KE $(15\cdot 12)$, EE and PE all present Any combination of signs
	$mg(2) = 15\cdot 12 + \frac{156\cdot 8(0\cdot 6)^2}{2(1\cdot 4)}$	A1	All correct, oe
	$19\cdot 6m = 15\cdot 12 + 20\cdot 16$ $19\cdot 6m = 35\cdot 28$ $2gm = 35\cdot 28$ $m = 1\cdot 8$	A1	Convincing, intermediate line required
		[7]	
	Total for Question 5	9	

Q6	Solution	Mark	Notes
(a)			
	$F = \frac{P}{20}$	B1	Used, oe, si
	At constant speed $F = R$ (N2L with $a = 0$)	M1	$F - R = ma$ with $a = 0$ $1000 + 50v = F$
	$1000 + 50(20) = \frac{P}{20}$ $(= F)$ $\quad 2000 = \frac{P}{20}(= F)$	A1	Correct equation using $v = 20$
	$P = 40\,000$ (W)	A1	cao, oe
		[4]	
(b)	 $2500g\left(\frac{2}{49}\right) = 1000$		
	$F = \frac{100 \times 1000}{v} = \frac{100000}{v}$	B1	si
	Using N2L up slope	M1	All forces, dim. correct. Allow sign errors and use of $mg\cos\alpha$
	$F - R - mg\sin\alpha = ma$	A1	Correct equation
	$\frac{100000}{v} - (1000 + 50v) - 2500g\left(\frac{2}{49}\right) = 2500(0 \cdot 3)$	A1	oe, equation in v.
	$\frac{100000}{v} - (1000 + 50v) - 1000 = 750$		
	Forming **and** solving a quadratic $100000 - 2750v - 50v^2 = 0$	m1	
	$v^2 + 55v - 2000 = 0$ $(v - 25)(v + 80) = 0$		
	$v = 25$ (ms^{-1}) (or $v = -80$)	A1	cao $v = -80$ must be clearly discarded (as it leads to $R = -3000$)
		[6]	
	Total for Question 6	**10**	

Q7	Solution	Mark	Notes
			$r = 0 \cdot 5$ $m = 0 \cdot 04$
(a)	(i) Conservation of energy	M1	KE and PE in dim. correct equation $\pm\frac{1}{2}mu^2 = \pm\frac{1}{2}mv^2 \pm mgr(trig)$
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$	A1 A1	KE, both terms PE, $mgr(1-\cos\theta)$
	$v^2 = u^2 - 2g(0 \cdot 5)(1-\cos\theta)$		
	$v^2 = \begin{cases} u^2 - g(1-\cos\theta) = u^2 - 9 \cdot 8(1-\cos\theta) \\ u^2 - g + g\cos\theta \quad = u^2 - 9 \cdot 8 + 9 \cdot 8\cos\theta \end{cases}$	A1	Allow in terms of g
	(ii) N2L towards centre $\pm(R - mg\cos\theta) = ma$	M1	Dim. correct equation $mg\cos\theta$, R opposing. Allow $mg\sin\theta$
	$R - mg\cos\theta = \dfrac{mv^2}{r}$	A1	Correct equation with $a = \frac{v^2}{r}$
	$R = \frac{0 \cdot 04}{0 \cdot 5}(u^2 - 9 \cdot 8 + 9 \cdot 8\cos\theta) + (0 \cdot 04)(9 \cdot 8)\cos\theta$	m1	Substitute for v^2
	$R = 0 \cdot 08u^2 - 0 \cdot 784 + 1 \cdot 176\cos\theta$	A1	Convincing, intermediate line required
	(iii) Let $R = 0$ and $\theta = 120^\circ$	M1	Used
	$u^2 = 17 \cdot 15 = \dfrac{343}{20}$		
	$u = 4 \cdot 14(12558\ldots)$ or $= \dfrac{7\sqrt{35}}{10}$ (ms^{-1})	A1 **[10]**	cao
(b)	Work done overcoming resistance $WD = 0 \cdot 03 \times 10$ $(= 0 \cdot 3)$	B1	Used, si, $F \times d = E$
	Work-energy principle $\frac{1}{2}(0 \cdot 04)(5)^2 - 0 \cdot 3 = \frac{1}{2}(0 \cdot 04)\,v^2$	M1 A1	All terms included, oe FT their WD
	$0 \cdot 5 - 0 \cdot 3 = 0 \cdot 02v^2$		
	$v^2 = 10$		
	$v = \sqrt{10} = 3 \cdot 1(6227766\ldots)$ (ms^{-1})		
	$v = 3 \cdot 1(62277\ldots) < 4$, so the ball will drop into the hole.	A1 **[4]**	oe, i.e. comparison of squares Convincing
	Total for Question 7	**14**	