GCE A LEVEL MARKING SCHEME

SUMMER 2025

A LEVEL
MATHEMATICS
UNIT 3 PURE MATHEMATICS B
1300U30-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL MATHEMATICS

UNIT 3 PURE MATHEMATICS B

SUMMER 2025 MARK SCHEME

Q	Solution	Mark	Notes
1(a)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3(\mathrm{e}^{-x} + x^2)^2 f(x)$	M1	$f(x) \neq k$ (constant)
	$= 3(\mathrm{e}^{-x} + x^2)^2(-\mathrm{e}^{-x} + 2x)$	A1	ISW
	<u>Alternative solution</u>		
	$y = \mathrm{e}^{-3x} + 3x^2\mathrm{e}^{-2x} + 3x^4\mathrm{e}^{-x} + x^6$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3\mathrm{e}^{-3x} + (6x\mathrm{e}^{-2x} - 6x^2\mathrm{e}^{-2x})$	M1	2 terms correctly differentiated.
	$+ (12x^3\mathrm{e}^{-x} - 3x^4\mathrm{e}^{-x}) + 6x^5$	A1	all correct, ISW

Q	Solution	Mark	Notes
1(a)(ii)	$\frac{dy}{dx} = f(x)\sin 3x + e^{2x}g(x)$	M1	$f(x), g(x) \neq 0, \pm 1$
	$= 2e^{2x}\sin 3x + 3e^{2x}\cos 3x$	A1	$f(x) = 2e^{2x}$
		A1	$g(x) = 3\cos 3x$ ISW
1(a)(iii)	$\frac{dy}{dx} = \frac{(x^2+1)f(x) - g(x)\cos 2x}{(x^2+1)^2}$	M1	$f(x), g(x) \neq 0, 1$, condone brackets missing around $x^2 + 1$ in numerator
	$= \frac{(x^2+1)(-2\sin 2x) - (2x)\cos 2x}{(x^2+1)^2}$	A1	$f(x) = -2\sin 2x$
		A1	$g(x) = 2x$ ISW
	<u>Alternative solution</u>		
	$y = \cos 2x(x^2+1)^{-1}$		
	$\frac{dy}{dx} = f(x)\cos 2x + (x^2+1)^{-1}g(x)$	M1	$f(x), g(x) \neq 0, 1$
	$= -(x^2+1)^{-2}(2x)\cos 2x - (x^2+1)^{-1}2\sin 2x$	A1	$f(x) = -(x^2+1)^{-2}(2x)$
		A1	$g(x) = -2\sin 2x$ ISW

Q	Solution	Mark	Notes
1(b)	$2 - (6xy^3 + 3x^2 \times 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}) = 0$	B1	$2 \pm 6xy^3$
		B1	$(\pm)\ 3x^2 \times 3y^2 \frac{\mathrm{d}y}{\mathrm{d}x}$
	$9x^2y^2 \frac{\mathrm{d}y}{\mathrm{d}x} = 2 - 6xy^3$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 - 6xy^3}{9x^2y^2}$	B1	must be simplified, oe

Q	Solution	Mark	Notes
2(a)	$\cos 2x + \sin x = 0$.		
	$1 - 2\sin^2 x + \sin x = 0$	M1	use of $\cos 2x = 1 - 2\sin^2 x$
			Or use of $\cos 2x = \cos^2 x - \sin^2 x$
			AND $\cos^2 x + \sin^2 x = 1$
	$2\sin^2 x - \sin x - 1 = 0$		
	$(2\sin x + 1)(\sin x - 1) = 0$	m1	$a\sin^2 x + b\sin x + c = (p\sin x + q)(r\sin x + s)$
			$pr = a$, $qs = c$
	$\sin x = -\frac{1}{2}, 1$	A1	both values, cao
	$\sin x = -\frac{1}{2}$		
	$x = \frac{7\pi}{6}$	B1	
	$x = \frac{11\pi}{6}$	B1	
	$\sin x = 1$		
	$x = \frac{\pi}{2}$	B1	
	Note: M0 no workings		

Notes

Do not FT for other trig functions

FT for different values of $\sin x$.

Correct angles (both) from +ve sign, B1

Correct angles from -ve sign B1 B1

Ignore repeated roots and roots outside range $0 \le x \le 2\pi$

-1 for each extra root within range, i.e. 3rd and 4th root, or 2nd root if there is only 1.

Q	Solution	Mark	Notes
2(b)	$2(1 + \tan^2\theta) - 5 = \tan\theta$	M1	$1 + \tan^2\theta = \sec^2\theta$ Or $2(\sec^2\theta - 1) - 3 = \tan\theta$ $2\tan^2\theta - 3 = \tan\theta$
	$2\tan^2\theta - \tan\theta - 3 = 0$	m1	quadratic (= 0) obtained Allow 1 slip
	$(2\tan\theta - 3)(\tan\theta + 1) = 0$		
	$\tan\theta = -1, \frac{3}{2}$	A1	both values, cao
	$\tan\theta = \frac{3}{2}$		
	$\theta = 56.31°, 236.31°$	B1	both values, accept 56°, 236°, 0.9828^c, 4.1244^c
	$\tan\theta = -1$		
	$\theta = 135°$	B1	$\frac{3\pi}{4}$ rad or 2.3562^c
	$\theta = 315°$	B1	$\frac{7\pi}{4}$ rad or 5.4978^c

<u>Note</u>: M0 no workings

<u>Notes</u>

Do not FT for trig functions other than tan.

FT for different values of $\tan\theta$.

Correct angles (both) from +ve sign, B1

Correct angles from -ve sign B1 B1

Ignore roots outside range $0° \le \theta \le 360°$

-1 for each extra root within range, ie 3rd and 4th root, or 2nd root if there is only 1.

Q	Solution	Mark	Notes
3	$\frac{\cos\theta}{1+\sin 2\theta} \cong \frac{1-\frac{\theta^2}{2}}{1+2\theta} = 0.8$	M1	use of $\cos\theta \cong 1-\frac{\theta^2}{2}$
		M1	use of $\sin 2\theta \cong 2\theta$
	$1-\frac{\theta^2}{2} = 0.8 + 1.6\theta$	m1	Attempt to form a quadratic provided at least M1 awarded
	$0.5\theta^2 + 1.6\theta - 0.2 = 0$		
	$\theta = \frac{-1.6 \pm \sqrt{1.6^2 - 4\times 0.5\times(-0.2)}}{2\times 0.5}$	m1	attempt to solve si
	$\theta = 0.12(04650534), -3.32(0465053)$	A1	$\frac{-8\pm\sqrt{74}}{5}$, $-1.6 \pm \sqrt{2.96}$, cao
	Since θ is small, $\theta = 0.12$ (discard -3.32)	A1	A0 if incorrect reason

<u>Alternative solution</u>

Q	Solution	Mark	Notes
3	$\frac{\cos\theta}{1+\sin 2\theta} = \frac{\cos\theta}{1+2\sin\theta\cos\theta}$		
	$\cong \frac{1-\frac{\theta^2}{2}}{1+2\theta\left(1-\frac{\theta^2}{2}\right)} = 0.8$	M1	$\cos\theta \cong 1-\frac{\theta^2}{2}$
		M1	$\sin\theta \cong \theta$
	$\cong \frac{1-\frac{\theta^2}{2}}{1+2\theta-\theta^3} = 0.8$		
	$1-\frac{\theta^2}{2} = 0.8(1 + 2\theta - \theta^3)$	m1	Attempt to form cubic equation provided at least M1 awarded
	$1.6\theta^3 - \theta^2 - 3.2\theta + 0.4 = 0$	m1	attempt to solve si
	$8\theta^3 - 5\theta^2 - 16\theta + 2 = 0$		
	$\theta \cong -1.21,\ 0.12,\ 1.71$	A1	cao
	Since θ is small, $\theta = 0.12$	A1	discard $-1.21,\ 1.71$, A0 if incorrect reason

Q	Solution	Mark	Notes
4(a)	x y		
	0 1		
	$\frac{\pi}{12}$ $8-4\sqrt{3} = 1.071(79677\ldots)$		
	$\frac{\pi}{6}$ $\frac{4}{3} = 1.333(33333\ldots)$		
	$\frac{\pi}{4}$ 2		
	$\frac{\pi}{3}$ 4	B2	all y values correct
			B1 if any 2 y values correct
	$\int_0^{\frac{\pi}{3}} \sec^2 x \,\mathrm{d}x = \frac{\frac{\pi}{12}}{2}[(1+4)$		
	$+ 2(1.0718 + 1.3333 + 2)]$	M1	correct use of formula, si
			Allow y_k's
	$\int_0^{\frac{\pi}{3}} \sec^2 x \,\mathrm{d}x = \frac{\pi}{12\times 2} \times 13.810(2602\ldots)$		
	$\int_0^{\frac{\pi}{3}} \sec^2 x \,\mathrm{d}x$ (= 1.807(758833…)) = 1.808	A1	cao
4(b)	$\int_0^{\frac{\pi}{3}} (\tan^2 x + 2)\mathrm{d}x = \int_0^{\frac{\pi}{3}} (\sec^2 x + 1)\mathrm{d}x$	M1	oe, $\tan^2 x + 2 = \sec^2 x + 1$
	$= 1.808 + \frac{\pi}{3}$		
	$= 2.855$	A1	ft their (a) if possible

Q	Solution	Mark	Notes

Solution with 6 ordinates

4(a)	x	y		
	0	1		
	$\frac{\pi}{15}$	1.04518029191		
	$\frac{2\pi}{15}$	1.19822858222		
	$\frac{3\pi}{15}$	1.527864045		
	$\frac{4\pi}{15}$	2.2334601581		
	$\frac{5\pi}{15}$	4	B0B1	any 4 y values correct

	$\int_0^{\frac{\pi}{3}} \sec^2 x \, \mathrm{d}x = \frac{\frac{\pi}{15}}{2}[(1 + 4)$		
	$+ 2(1.0452 + 1.1982 + 1.5279 + 2.2335)]$	M1	correct use of formula, si
	$\int_0^{\frac{\pi}{3}} \sec^2 x \, \mathrm{d}x = \frac{\pi}{15 \times 2} \times 17.0096$		
	$\int_0^{\frac{\pi}{3}} \sec^2 x \, \mathrm{d}x$ (= 1.78124114668) = 1.781	A1	cao
4(b)	$\int_0^{\frac{\pi}{3}} (\tan^2 x + 2) \mathrm{d}x = \int_0^{\frac{\pi}{3}} (\sec^2 x + 1) \mathrm{d}x$	M1	oe, $\tan^2 x + 2 = \sec^2 x + 1$
	$= 1.781 + \frac{\pi}{3}$		
	$= 2.828$	A1	ft their (a) if possible

Q	Solution	Mark	Notes
5(a)	$f(x) = x^3 - 6x^2 + 12x - 5$		
	$f'(x) = 3x^2 - 12x + 12$	B1	
	(At stationary point,) $f'(x) = 0$	M1	used
	$x^2 - 4x + 4 = 0$		
	$(x - 2)^2 = 0$		
	$x = 2$	A1	cao
	$f(2) = 2^3 - 6 \times 2^2 + 12 \times 2 - 5 = 3$	A1	cao
	Stationary point is (2, 3)		
	Since $f'(x) = 3(x - 2)^2 \geq 0$ for all x,	E1	Any correct method, eg $f''(2) = 0$ and $f'''(2) = 6 \neq 0$. Cubic with only 1 stationary point, so point of inflection Sketch. Do not award for $f''(x) = 0$ only
	(2, 3) is a point of inflection	B1	
	OR $f''(x) = 6x - 12 = 0$ when $x = 2$ and		
	< 0 when $x < 2$, > 0 when $x > 2$	(E1)	
	(2, 3) is a point of inflection	(B1)	
5(b)	(Curve is convex when) $f''(x) > 0$	B1	used, $f''(x)$ may be seen in (a), si
	$6x - 12 > 0$		
	$x > 2$	B1	$(2, \infty)$

Q	Solution	Mark	Notes
6(a)	$x_{n+1} = \frac{1}{3}e^{x_n}$		
	$x_0 = 0.62$		
	$x_1 = (0.61964268061 =)0.620$	B1	B0 for 0.62
	$x_2 = (0.61942130982 =)0.619$		
	$x_3 = (0.61928420321 =)0.619$	B1	x_2, x_3 correct, Mark final answers
			If answers more than 3dp, allow B1 for all correct.
6(b)(i)	$f(x) = e^x - 3x$,		
	$f'(x) = e^x - 3$	B1	si
	$x_{n+1} = x_n - \frac{e^{x_n} - 3x_n}{e^{x_n} - 3}$	M1	seen or used, FT their $f'(x)$
	$x_0 = 1.5$		
	x_1 (= 1.512(358146)) = 1.512	A1	cao
6(b)(ii)	Consider $f(x) = e^x - 3x$		
	$f(1.5115) = e^{1.5115} - 3\times1.5115 = -0.00097 < 0$	M1	ft their x_1 for M1 only
	$f(1.5125) = e^{1.5125} - 3\times1.5125 = 0.00056 > 0$	(M1)	if not awarded previously
	Sign change (and continuity of $f(x)$)		
	shows 1.512 is a root to 3 dp	A1	
6(b)(iii)	If $x = \ln 3$,		
	denominator $f'(\ln 3) = e^{\ln 3} - 3 = 3 - 3 = 0$		
	Hence the Newton-Raphson process will fail.	E1	convincing

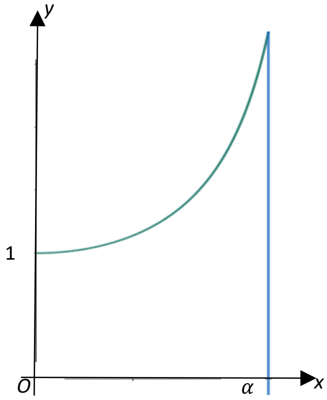
Q	Solution	Mark	Notes
7(a)	$\frac{n}{2}(-3 + 61) = 957$	M1	oe
	$n = 33$	A1	cao
	OR		
	$\frac{n}{2}[2(-3) + (n - 1)d] = 957$		
	$-3 + (n - 1)d = 61$	M1	
	$(n - 1)d = 64$		
	$\frac{n}{2}[-6 + 64] = 957$		
	$n = 33$	A1	cao
7(b)	$-3 + (33 - 1)d = 61$	M1	oe
	$d = 2$	A1	ft their positive integer n
	OR		
	$\frac{33}{2}[2(-3) + 32d] = 957$	M1	
	$d = 2$	A1	ft their positive integer n
7(c)	Middle term = 17^{th} term.		
	17^{th} term $= -3 + (17 - 1) \times 2$		
	$= 29$	B1	$\frac{-3 + 61}{2} = 29$, cao Mark final answer

Q	Solution	Mark	Notes
8	Correct method for a factor of the numerator	M1	Long division by $(x + 5)$, $(x + 3)$, $(x - 1)$ or $(x - 2)$, no remainder. Or Factor theorem. Or Correct cubic seen: $(x - 1)(x^3 + 6x^2 - x - 30)$ $(x + 5)(x^3 - 7x + 6)$ $(x + 3)(x^3 + 2x^2 - 13x + 10)$ $(x - 2)(x^3 + 7x^2 + 7x - 15)$
	First correct factor	A1	
	Second correct factor	A1	
	Third correct factor	A1	
	$\frac{x^4 + 5x^3 - 7x^2 - 29x + 30}{3(x+5)(x+3)(x-2)} = \frac{1}{3}(x - 1)$	A1	cao, ISW

<u>Notes</u>

If no method shown or is unclear M0

Q	Solution	Mark	Notes
9	Assume that there are integers x and y such that $3x + 15y = 2$.	B1	
	$3(x + 5y) = 2$		
	$(x + 5y) = \frac{2}{3}$	B1	oe fraction required.
			e.g. $x = \frac{2}{3} - 5y$
	Since x and y are integers,		
	$x + 5y$ is also an integer.	B1	oe
	This contradicts $x + 5y = \frac{2}{3}$.		
	Therefore, there are no integers x and y		
	for which $3x + 15y = 2$.	B1	cso

Q	Solution	Mark	Notes
10(a)		G1	condone domain (0^c, $\frac{\pi^c}{2}$), ($0°$, $90°$), ignore graph outside of range, condone no labels if graph correct shape and starts above 0 on y axis. G0 if label is incorrect.
10(b)	The rectangle with sides 1 and α which is below the graph has area α.	E1	
	Since $\int_0^\alpha \sec x \, dx$ is the area under the graph, $\int_0^\alpha \sec x \, dx > \alpha$.	E1	

Q	Solution	Mark	Notes
11	$\frac{5x+2}{(x+1)^2(2x-1)} \equiv \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}$	M1	
	$5x + 2 \equiv A(x+1)(2x-1) + B(2x-1) + C(x+1)^2$	m1	
	$x = -1$, $-3 = -3B$		
	$B = 1$	A1	one correct coefficient
	$x = \frac{1}{2}$, $\frac{9}{2} = \frac{9}{4}C$		
	$C = 2$		
	Coefficient of x^2, $0 = 2A + C$		
	$A = -1$	A1	all correct
	$\int_1^2 \frac{5x+2}{(x+1)^2(2x-1)}\mathrm{d}x$		
	$= \int_1^2 \left(\frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{2x-1}\right)\mathrm{d}x$	M1	ft their A, B, C and at least 2 non-zero values for A,B,C
	$= \left[-\ln\|x+1\| - \frac{1}{x+1} + \ln\|2x-1\|\right]_1^2$	A1	$A \times \ln\|x+1\|$,
		A1	$-\frac{B}{x+1}$
		A1	$C \times \frac{1}{2} \times \ln\|2x-1\|$,
	$= (-\ln 3 - \frac{1}{3} + \ln 3) - (-\ln 2 - \frac{1}{2} + \ln 1)$	m1	correct use of limits
	$= \ln 2 + \frac{1}{6}$	A1	cao, allow $-\ln\frac{1}{2} + \frac{1}{6}$

Q	Solution	Mark	Notes
12	$(1-2x)^{-\frac{1}{3}}$		
	$= 1 + \left(-\frac{1}{3}\right)(-2x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2}(-2x)^2$		
	$+ \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3\times 2}(-2x)^3 + \ldots$	B2	-1 each error term
	$= 1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3 + \cdots$		
	$(2-x)(1-2x)^{-\frac{1}{3}}$		
	$= (2-x)[1 + \frac{2}{3}x + \frac{8}{9}x^2 + \frac{112}{81}x^3 + \cdots]$		
	$= (2 + \frac{4}{3}x + \frac{16}{9}x^2 + \frac{224}{81}x^3) - (x + \frac{2}{3}x^2 + \frac{8}{9}x^3) + \ldots$	M1	correct method (3 correct terms)
	$= 2 + \frac{1}{3}x + \frac{10}{9}x^2 + \frac{152}{81}x^3 + \ldots$	A1	$2 + \frac{1}{3}x$, cao
		A1	$+\frac{10}{9}x^2$, cao
		A1	$+\frac{152}{81}x^3$, cao
			Ignore further terms, isw
	Expansion is valid for $\lvert x\rvert < \frac{1}{2}$.	B1	$-\frac{1}{2} < x < \frac{1}{2}$, mark final answer

Q	Solution	Mark	Notes

Note 1

12	$(1-2x)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)(-2x) + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)}{2}(-2x)^2 + \frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3\times2}(-2x)^3 + \ldots..$		
	$= 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3 + \ldots.$	(B0 B0)	
	$(2-x)(1-2x)^{\frac{1}{3}}$		
	$= (2 - \frac{4}{3}x - \frac{8}{9}x^2 - \frac{80}{81}x^3) - (x - \frac{2}{3}x^2 - \frac{4}{9}x^3) + \ldots$	(M1)	correct method (3 correct terms)
	$= 2 - \frac{7}{3}x - \frac{2}{9}x^2 - \frac{44}{81}x^3 + \ldots.$	(A1)	$2 - \frac{7}{3}x$, cao
		(A1)	$-\frac{2}{9}x^2$, cao
		(A1)	$-\frac{44}{81}x^3$, cao
			Ignore further terms, ISW
	Expansion is valid for $\lvert x\rvert < \frac{1}{2}$.	(B1)	$-\frac{1}{2} < x < \frac{1}{2}$, mark final answer

Note 2

12	$(1+2x)^{-\frac{1}{3}} = 1 + \left(-\frac{1}{3}\right)(2x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)}{2}(2x)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)}{3\times2}(2x)^3 + \ldots..$		
	$= 1 - \frac{2}{3}x + \frac{8}{9}x^2 - \frac{112}{81}x^3 + \ldots.$	(B0 B0)	
	$(2-x)(1+2x)^{-\frac{1}{3}}$		
	$= (2 - \frac{4}{3}x + \frac{16}{9}x^2 - \frac{224}{81}x^3) - (x - \frac{2}{3}x^2 + \frac{8}{9}x^3) + \ldots$	(M1)	correct method (3 correct terms)
	$= 2 - \frac{7}{3}x + \frac{22}{9}x^2 - \frac{296}{81}x^3 + \ldots$	(A1)	$2 - \frac{7}{3}x$, cao
		(A1)	$+\frac{22}{9}x^2$, cao
		(A1)	$-\frac{296}{81}x^3$, cao
			Ignore further terms, ISW
	Expansion is valid for $\lvert x\rvert < \frac{1}{2}$.	(B1)	$-\frac{1}{2} < x < \frac{1}{2}$, mark final answer

Q	Solution	Mark	Notes
13(a)	$\frac{dy}{dt} = 6t$	B1	
	$\frac{dx}{dt} = 6t^2$	B1	
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1	used
	$\frac{dy}{dx} = \frac{6t}{6t^2} = \frac{1}{t}$		
	Gradient of curve is $\frac{1}{p}$	A1	convincing
13(b)	Where $t = 1$, point is (3, 3)	B1	si, or $(2t^3 + 1, 3t^2)$ used in equation
	Gradient of tangent = 1	B1	si, allow if $-t$ used in equation
	Gradient of normal = -1	B1	si, $-t$ used in equation
	Equation of normal is		
	$y - 3 = -1(x - 3)$	B1	oe, cao ISW
	$y + x = 6$		
13(c)	Where $t = 0$,		
	Gradient of tangent = $\frac{1}{0}$, undefined	B1	si
	When $t = 0$, $x = 1$	B1	si
	Equation of tangent is $x = 1$	B1	

Q	Solution	Mark	Notes
14(a)	$\int_0^{\frac{\pi}{2}} x^2\cos x \, \mathrm{d}x$		
	$= [\pm x^2\sin x\,]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \pm 2x\sin x \, \mathrm{d}x$	M1	2 terms, $x^2\sin x$ and $\int 2x\sin x$ seen No limits needed
	$= [x^2\sin x\,]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x\sin x \, \mathrm{d}x$	A1	all correct
	$= [x^2\sin x\,]_0^{\frac{\pi}{2}} - [2x(-\cos x)\,]_0^{\frac{\pi}{2}}$		
	$+\int_0^{\frac{\pi}{2}} 2(-\cos x) \, \mathrm{d}x$	m1	integration of $\pm 2x\sin x$, $[2x(-\cos x)\,] - \int 2(-\cos x)\,\mathrm{d}x$, $2x\cos x$ and $\int 2\cos x$ seen
	$= [x^2\sin x\,]_0^{\frac{\pi}{2}} + [2x\cos x]_0^{\frac{\pi}{2}} - [2\sin x\,]_0^{\frac{\pi}{2}}$	A1	all correct
	$= (\frac{\pi^2}{4} - 0) + (0 - 0) - (2 - 0)$	m1	correct use of limits, 3 terms si by correct answer
	$= \frac{\pi^2}{4} - 2$ or $\frac{\pi^2-8}{4}$	A1	cao, ISW A0 for 0.467401… condone $\left(\frac{\pi}{2}\right)^2 - 2$

Note 1: No workings, no marks

Note 2: $\int_0^{\frac{\pi}{2}} x^2\cos x \, \mathrm{d}x$

	$= \left[\frac{x^3}{3}\cos x\,\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{x^3}{3}(\pm\sin x)\,\mathrm{d}x$	M1	2 terms, $\frac{x^3}{3}\cos x$ and $\int \frac{x^3}{3}\sin x$ seen No limits needed
	$= \left[\frac{x^3}{3}\cos x\,\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{x^3}{3}(-\sin x)\,\mathrm{d}x$	A1	all correct

Q	Solution	Mark	Notes
14(b)	$u = 1 + \ln x$		
	$\mathrm{d}u = \frac{\mathrm{d}x}{x}$	B1	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$
	$\int \frac{\ln x}{x(1 + \ln x)^2}\,\mathrm{d}x = \int \frac{u-1}{u^2}\,\mathrm{d}u$	M1	attempt to change $f(x)$ to $g(u)$ **and** dx to du
	$= \int \left(\frac{1}{u} - \frac{1}{u^2}\right)\mathrm{d}u$	A1	integrable form
	$= \ln\lvert u\rvert + \frac{1}{u}\ (+\ C)$	A1	correct integration, condone no modulus sign
	$= \ln\lvert 1 + \ln x\rvert + \frac{1}{1 + \ln x} + C$	A1	cao, Mark final answer

Q	Solution	Mark	Notes
15(a)	Range of f is $(0, \infty)$.	B1	(0,
		B1	, ∞)
			Accept equivalent notation
OR			
	$f(x) > 0$	B2	
	$f(x) \geq 0$	B1	
	$y > 0$	B1	
	$y \geq 0$	B0	
	$x > 0$	B0	

Q	Solution	Mark	Notes
15(b)	$y = \frac{9}{(x-3)^2}$	M1	
	$(x-3)^2 = \frac{9}{y}$	m1	Attempt to make x the subject
	$(x-3) = \pm\frac{3}{\sqrt{y}}$		
	$x = 3 \pm \frac{3}{\sqrt{y}}$	A1	condone + and not $\pm$
	Since $x > 3$, $x = 3 + \frac{3}{\sqrt{y}}$	A1	correct justification for +.
	$f^{-1}(x) = 3 + \frac{3}{\sqrt{x}}$	A1	cao, oe, ISW
OR			
	$y = \frac{9}{(x-3)^2}$	(M1)	
	$yx^2 - 6yx + 9(y-1) = 0$	(m1)	quadratic in x (or y)
	$x = \frac{6y \pm \sqrt{(-6y)^2 - 4 \times y \times 9(y-1)}}{2y}$	(A1)	condone + and not $\pm$
	$x = \frac{3(y \pm \sqrt{y})}{y}$		
	Since $x > 3$, $x = \frac{3(y + \sqrt{y})}{y}$	(A1)	correct justification for +
	$f^{-1}(x) = \frac{3(x + \sqrt{x})}{x}$	(A1)	cao, ISW

Q	Solution	Mark	Notes
16	At the end of 20 years,		
	Loan = $50000(1.05)^{20}$	B1	seen, or implied by correct working
	$= 132\,664.885257$		
	Total repayment		
	$= R[(1.05)^{19} + (1.05)^{18} + \ldots .. + (1.05)^2 + (1.05) + 1]$		
		M1	GP $r = 1.05$
			3 or more terms or statement or attempt to sum
		A1	20 terms a = any R including 1.
			Condone 19 or 21 terms.
	$= R\left(\frac{1-1.05^{20}}{1-1.05}\right)$	A1	$R(33.06595)$ award for any R,
			e.g. 2500 to give 82 664.875
	R is such that		
	$R\left(\frac{1-1.05^{20}}{1-1.05}\right) = 50000(1.05)^{20}$	M1	e.g. $\frac{132\,664.885257 \times 2500}{82\,664.875}$
	$R = 4012.12(936)$	A1	cao
	Annual instalment = £4012.13		