



GCE A LEVEL MARKING SCHEME

SUMMER 2025

**A LEVEL
FURTHER MATHEMATICS
UNIT 4 FURTHER PURE MATHEMATICS B
1305U40-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 4 FURTHER PURE MATHEMATICS B

SUMMER 2025 MARK SCHEME

Q	Solution	Mark	Notes
1. a)	Use of formula booklet expansion for e^x with x^2 : $e^{x^2} = 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \dots$ $e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \dots$	M1 A1 [2]	
b)	Approximate $\int_0^1 e^{x^2} dx = \int_0^1 \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}\right) dx$ $= \left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42} \right]_0^1$ $= \left(1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42} \right) - 0$ $\left(= \frac{51}{35} \right) = 1.457 \text{ correct to 3dp}$	M1 A1 A1 [3]	FT (a), attempt to integrate cao Final A0 for exact answer 1.463 to 3dp
		[5]	

Q	Solution	Mark	Notes																				
2. c)i)	METHOD 1: $8 \sin^4 \theta - 8 \sin^2 \theta + 1 = 2 \sin^2 \theta - 1$ $8 \sin^4 \theta - 10 \sin^2 \theta + 2 = 0$ $4 \sin^4 \theta - 5 \sin^2 \theta + 1 = 0$ $(4 \sin^2 \theta - 1)(\sin^2 \theta - 1) = 0$ Solving, $\sin \theta = \pm \frac{1}{2} \quad \text{or} \quad \sin \theta = \pm 1$ $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ \dots \quad \text{or} \quad \theta = 90^\circ, 270^\circ \dots$ $\theta = 360n^\circ + 30^\circ, \theta = 360n^\circ + 150^\circ,$ $\theta = 360n^\circ + 210^\circ, \theta = 360n^\circ + 330^\circ$ $\theta = 360n^\circ + 90^\circ, \theta = 360n^\circ + 270^\circ$ METHOD 2: $\cos 4\theta = 2 \sin^2 \theta - 1 = -\cos 2\theta$ $\cos 4\theta + \cos 2\theta = 0$ Use of summation formulae, $2 \cos \frac{4\theta+2\theta}{2} \cos \frac{4\theta-2\theta}{2} = 0$ $2 \cos 3\theta \cos \theta = 0$ Therefore, $\cos 3\theta = 0 \quad \text{or} \quad \cos \theta = 0$ $3\theta = 180n^\circ + 90^\circ$ $\theta = 60n^\circ + 30^\circ \quad \theta = 180n^\circ + 90^\circ$	M1 m1 A2 A1 A1 (B1) (M1) (A1) (m1) (A2)	Accept degrees or radian throughout Solvable form A1 for 2 solutions Accept $\pm\theta$ values For all 4 solns, oe For both solns, oe Identifying identity Either A1 each set oe																				
ii)	Therefore, the general solution is $\theta = 60n^\circ + 30^\circ$	B1 [7]																					
	Radian values: <table border="1"> <tr> <td>30°</td><td>150°</td><td>210°</td><td>330°</td></tr> <tr> <td>$\frac{\pi}{6}$</td><td>$\frac{5\pi}{6}$</td><td>$\frac{7\pi}{6}$</td><td>$\frac{11\pi}{6}$</td></tr> <tr> <td>90°</td><td>270°</td><td>180°</td><td>360°</td></tr> <tr> <td>$\frac{\pi}{2}$</td><td>$\frac{3\pi}{2}$</td><td>π</td><td>2π</td></tr> <tr> <td colspan="2">$60n^\circ + 30^\circ$</td><td colspan="2">$\frac{n\pi}{3} + \frac{\pi}{6}$</td></tr> </table>	30°	150°	210°	330°	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$	90°	270°	180°	360°	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	π	2π	$60n^\circ + 30^\circ$		$\frac{n\pi}{3} + \frac{\pi}{6}$			
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$60n^\circ + 30^\circ$		$\frac{n\pi}{3} + \frac{\pi}{6}$																					
		[20]																					

Q	Solution	Mark	Notes
3. a)	$\cosh x^2 = \frac{e^{x^2} + e^{-x^2}}{2}$ $\frac{d}{dx}(\cosh x^2)$ $= \frac{2xe^{x^2} - 2xe^{-x^2}}{2}$ $= 2x \left(\frac{e^{x^2} - e^{-x^2}}{2} \right)$ $= 2x \sinh x^2$	M1 A1 A1 [3]	 Differentiating Convincing, answer given
b)	METHOD 1: $\int 2x^3 \sinh x^2 dx$ Integrating by parts with $u = x^2, \frac{dv}{dx} = 2x \sinh x^2$ $= x^2 \cosh x^2 - \int 2x \cosh x^2 dx$ $= x^2 \cosh x^2 - \sinh x^2 + c$ METHOD 2: $\int 2x^3 \sinh x^2 dx$ Integration by substitution with $u = x^2$ $du = 2x dx$ leading to $\int u \sinh u du$ Integration by parts with $u = u, \frac{dv}{dx} = \sinh u$ $= u \cosh u - \int \cosh u du$ $= u \cosh u - \sinh u + c$ $= x^2 \cosh x^2 - \sinh x^2 + c$	 M1 m1 A1 A1 (M1) (A1) (m1) (A1) [4]	 ISW ISW
		[7]	

Q	Solution	Mark	Notes
5.	METHOD 1: Volume = $\pi \int_0^1 \sinh^2 2y \, dy$ Use of $\cosh 4y = 1 + 2 \sinh^2 2y$ $\pi \int_0^1 \left(\frac{\cosh 4y - 1}{2} \right) dy$ $\pi \left[\frac{1}{2} \times \frac{\sinh 4y}{4} - \frac{1}{2} y \right]_0^1$ $\pi \left[\left(3.411 \dots - \frac{1}{2} \right) - (0) \right]$ Volume = 9.146 METHOD 2: Volume = $\pi \int_0^1 \sinh^2 2y \, dy$ Use of $\sinh 2y = \frac{e^{2y} - e^{-2y}}{2}$ $\frac{\pi}{4} \int_0^1 (e^{4y} - 2 + e^{-4y}) dy$ $\frac{\pi}{4} \left[\frac{e^{4y}}{4} - 2y - \frac{e^{-4y}}{4} \right]_0^1$ $\frac{\pi}{4} \left[\left(\frac{e^4}{4} - 2 - \frac{e^{-4}}{4} \right) - \left(\frac{1}{4} - 0 - \frac{1}{4} \right) \right]$ Volume = 9.146	B1 M1 A1 A1 m1 A1 (B1) (M1) (A1) (A1) (m1) (A1) [6]	Correct notation required M0 no workings Integrable form with no more than 1 slip oe cao integration Substitute in correct limits cao Correct notation required M0 no workings Integrable form with no more than 1 slip oe cao integration Substitute in correct limits cao If M0, award SC1 for 9.1... unsupported
		[6]	

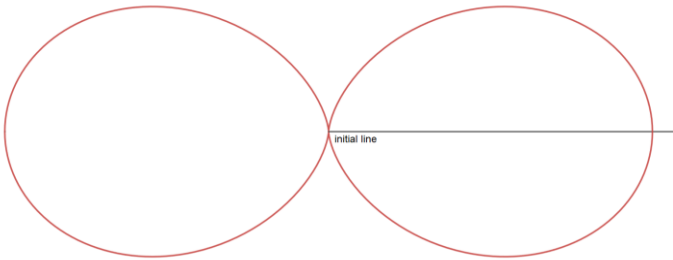
Q	Solution	Mark	Notes
6.	Dividing both sides by $\cos x$: $\frac{dy}{dx} + \frac{y}{\cos x} = \tan x$ Integrating factor: $e^{\int \frac{1}{\cos x} dx} = e^{\int \sec x dx}$ $= e^{\ln(\sec x + \tan x)} = \sec x + \tan x$ Multiplying both sides: $(\sec x + \tan x) \frac{dy}{dx} + y \frac{(\sec x + \tan x)}{\cos x} = (\sec x + \tan x) \tan x$ RHS: $\sec x \tan x + \tan^2 x \rightarrow \sec x \tan x + \sec^2 x - 1$ Integrating both sides: $y(\sec x + \tan x) = \sec x + \tan x - x + c$ Substituting $3 = 1 + 0 - 0 + c \rightarrow c = 2$ Solution: $y(\sec x + \tan x) = \sec x + \tan x - x + 2$ $y = \frac{\sec x + \tan x - x + 2}{\sec x + \tan x}$	M1 M1 A1 M1 A1 m1 A1 B1 [8]	Right-side integrable form oe
		[8]	

Q	Solution	Mark	Notes
7. a) i)	METHOD 1: $\frac{dy}{dx} = -\frac{1}{\sqrt{1-(2x-1)^2}} \times 2$ $\frac{dy}{dx} = -\frac{2}{\sqrt{1-4x^2+4x-1}}$ $\frac{dy}{dx} = -\frac{1}{\sqrt{x-x^2}}$ METHOD 2: $\cos y = 2x - 1$ $-\sin y \frac{dy}{dx} = 2$ $\frac{dy}{dx} = -\frac{2}{\sin y}$ $\frac{dy}{dx} = -\frac{2}{\sqrt{1-\cos^2 y}}$ $\frac{dy}{dx} = -\frac{2}{\sqrt{1-(2x-1)^2}}$ $\frac{dy}{dx} = -\frac{2}{\sqrt{1-4x^2+4x-1}}$ $\frac{dy}{dx} = -\frac{1}{\sqrt{x-x^2}}$	M1 A1 A1 (M1) (A1) (A1) [3]	Must be chain rule oe oe simplified
ii)	$\frac{dy}{dx} = (3x \times \cosh 4x \times 4) + (\sinh 4x \times 3)$ $\frac{dy}{dx} = 12x \cosh 4x + 3 \sinh 4x \quad \text{or} \quad 3(4x \cosh 4x + \sinh 4x)$	M1 A1 A1 [3]	Product rule One term correct Fully correct and simplified
b)	$\frac{dy}{dx} = \frac{(\cosh x \times -\sin x) - (\cos x \times \sinh x)}{(\cosh x)^2}$ At $x = \frac{\pi}{2}$, $\frac{dy}{dx} = -\frac{1}{\cosh \frac{\pi}{2}} = -0.3985 \dots$ and $y = 0$ Gradient of normal: $\cosh \frac{\pi}{2} = \frac{-1}{-0.3985 \dots} = 2.509 \dots$ Equation: $y - 0 = 2.509 \dots \left(x - \frac{\pi}{2}\right)$ $y = 2.51x - 3.94$	M1 A1 B1 B1 m1 A1 [6]	Quotient rule oe Fully correct Both dy/dx and y FT their dy/dx Dependant on M1 Must be 2d.p.
		[12]	

Q	Solution	Mark	Notes
9.	Solve auxiliary $m^2 + 16 = 0$ $m^2 = -16$ $m = \pm 4i$ Complementary function: $y = A \sin 4x + B \cos 4x$ Use particular integral of the form $C \sin x + D \cos x$ $\frac{dy}{dx} = C \cos x - D \sin x$ $\frac{d^2y}{dx^2} = -C \sin x - D \cos x$ Therefore, $-C \sin x - D \cos x + 16(C \sin x + D \cos x) = 3 \sin x$ $-C + 16C = 3 \rightarrow C = \frac{1}{5}$ $-D + 16D = 0 \rightarrow D = 0$ General Solution: $y = A \sin 4x + B \cos 4x + \frac{1}{5} \sin x$ $\frac{dy}{dx} = 4A \cos 4x - 4B \sin 4x + \frac{1}{5} \cos x$ When $x = \pi$, $y = 3$, $\frac{dy}{dx} = 0$ $B = 3$ and $\frac{dy}{dx} = 4A - \frac{1}{5} = 0 \rightarrow A = \frac{1}{20}$ Therefore, $y = \frac{1}{20} \sin 4x + 3 \cos 4x + \frac{1}{5} \sin x$	M1 A1 M1 A1 A1 A1 M1 A1 M1 A1 B1 [11]	Substitution Both values <i>FT C,D for M1A1M1A1</i> Sub and differentiate Substituting π and one other value into either y or $\frac{dy}{dx}$ Both A and B cao
		[11]	

Q	Solution	Mark	Notes
10. a)i)	$\frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = \frac{2(x^3-2x^2-3x+6)}{x^3-2x^2-3x+6} + \frac{5x^2+6x}{x^3-2x^2-3x+6}$ $\therefore \frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = 2 + \frac{5x^2+6x}{x^3-2x^2-3x+6}$	M1 A1 [2]	Or polynomial division convincing
ii)	METHOD 1: $(x-2)$ is a linear factor of $x^3 - 2x^2 - 3x + 6$ $\therefore \frac{2x^3+x^2+12}{x^3-2x^2-3x+6} = 2 + \frac{5x^2+6x}{(x-2)(x^2-3)}$ $\frac{5x^2+6x}{(x-2)(x^2-3)} = \frac{A}{x-2} + \frac{Bx+C}{x^2-3}$ $5x^2 + 6x = A(x^2 - 3) + (Bx + C)(x - 2)$ When $x = 2$, $32 = A$ When $x = 0$, $0 = -3A - 2C \rightarrow C = -48$ Comparing x^2 coefficients: $5 = A + B \rightarrow B = -27$ Therefore, $\int \left(\frac{2x^3+x^2+12}{x^3-2x^2-3x+6} \right) dx = \int \left(2 + \frac{32}{x-2} + \frac{-27x-48}{x^2-3} \right) dx$ $= \int \left(2 + \frac{32}{x-2} + \frac{-27x}{x^2-3} + \frac{-48}{x^2-3} \right) dx$ $= 2x + 32\ln x-2 - \frac{27}{2}\ln x^2-3 - \frac{48}{2\sqrt{3}}\ln\left \frac{x-\sqrt{3}}{x+\sqrt{3}}\right + c \quad \text{oe}$	B1 M1 m1 A2 m1 A3	si Partial fractions All values A1 two values Integration FT their partial fractions Fully correct ISW A2 for 4 correct terms A1 for 2 correct terms Note: $\frac{48}{2\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$

Q	Solution	Mark	Notes
10. a) ii)	METHOD 2: $(x - 2)$ is a linear factor of $x^3 - 2x^2 - 3x + 6$ $\therefore \frac{2x^3 + x^2 + 12}{x^3 - 2x^2 - 3x + 6} = 2 + \frac{5x^2 + 6x}{(x-2)(x^2-3)}$ $\frac{5x^2 + 6x}{(x-2)(x^2-3)} = \frac{A}{x-2} + \frac{B}{x+\sqrt{3}} + \frac{C}{x-\sqrt{3}}$ $5x^2 + 6x = A(x + \sqrt{3})(x - \sqrt{3}) + B(x - \sqrt{3})(x - 2) + C(x + \sqrt{3})(x - 2)$ When $x = 2$, $32 = A$ When $x = \sqrt{3}$, $15 + 6\sqrt{3} = C(\sqrt{3} + \sqrt{3})(\sqrt{3} - 2)$ $\frac{15+6\sqrt{3}}{6-4\sqrt{3}} = C \quad \left(= \frac{-16\sqrt{3}-27}{2} \right)$ When $x = -\sqrt{3}$, $15 - 6\sqrt{3} = B(-\sqrt{3} - \sqrt{3})(-\sqrt{3} - 2)$ $\frac{15-6\sqrt{3}}{6+4\sqrt{3}} = B \quad \left(= \frac{16\sqrt{3}-27}{2} \right)$ Therefore, $\int \left(\frac{2x^3 + x^2 + 12}{x^3 - 2x^2 - 3x + 6} \right) dx = \int \left(2 + \frac{32}{x-2} + \frac{\frac{15-6\sqrt{3}}{6+4\sqrt{3}}}{x+\sqrt{3}} + \frac{\frac{15+6\sqrt{3}}{6-4\sqrt{3}}}{x-\sqrt{3}} \right) dx$ $= 2x + 32\ln x-2 + \frac{15-6\sqrt{3}}{6+4\sqrt{3}} \ln x+\sqrt{3} + \frac{15+6\sqrt{3}}{6-4\sqrt{3}} \ln x-\sqrt{3} + c \quad \text{oe}$ e.g. with rationalised coefficients: $= 2x + 32\ln x-2 + \frac{16\sqrt{3}-27}{2} \ln x+\sqrt{3} + \frac{-16\sqrt{3}-27}{2} \ln x-\sqrt{3} + c$	(B1) (M1) (m1) (A2) (m1) (A3) [9]	si Partial fractions All values A1 two values Integration FT their partial fractions Fully correct ISW A2 for 4 correct terms A1 for 2 correct terms

Q	Solution	Mark	Notes
11. a)		G2 [2]	Fully correct graph Allow G1 for - two circles - half graph
b)	$y = r \sin \theta = \cos^2 \theta \sin \theta$ Differentiating $\frac{dy}{d\theta} = \cos^3 \theta - 2 \cos \theta \sin^2 \theta$ When parallel to initial line $\frac{dy}{d\theta} = 0$ $\cos^3 \theta - 2 \cos \theta \sin^2 \theta = 0$ $\cos \theta (\cos^2 \theta - 2 \sin^2 \theta) = 0$ $\cos \theta = 0$ (no tangent parallel to initial line) or $\cos^2 \theta - 2 \sin^2 \theta = 0$ $\cos^2 \theta - 2(1 - \cos^2 \theta) = 0$ $3 \cos^2 \theta - 2 = 0$ $\cos \theta = \pm \sqrt{\frac{2}{3}}$ ($\sin \theta = \pm \frac{1}{\sqrt{3}}, \tan \theta = \pm \frac{1}{\sqrt{2}}$) $r = \frac{2}{3}$ Then, $x = r \cos \theta = \frac{2}{3} \times \sqrt{\frac{2}{3}} = \frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{6}}{9}$ $y = r \sin \theta = \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$ Area = $2x \times 2y$ $= 2 \times \frac{2\sqrt{2}}{3\sqrt{3}} \times 2 \times \frac{2}{3\sqrt{3}} = \frac{16\sqrt{2}}{27}$ oe Alternative possible derivatives: • $y = \left(\frac{\cos 2\theta + 1}{2}\right) \sin \theta$ $\frac{dy}{d\theta} = \left(\frac{\cos 2\theta + 1}{2}\right) \cos \theta - \sin \theta \sin 2\theta$ • $y = \left(\frac{\cos 2\theta + 1}{2}\right) \sin \theta = \frac{1}{2} \cos 2\theta \sin \theta + \frac{1}{2} \sin \theta$ $\frac{dy}{d\theta} = \frac{1}{2} \cos 2\theta \sin \theta - \sin 2\theta \sin \theta + \frac{1}{2} \cos \theta$ • $y = (1 - \sin^2 \theta) \sin \theta = \sin \theta - \sin^3 \theta$ $\frac{dy}{d\theta} = \cos \theta - 3 \cos \theta \sin^2 \theta$	M1 m1 A1 m1 A1 A1 m1 A1 [10]	Must be product rule Any value Attempt to find x, y for any point Must be exact
		[12]	