GCE A LEVEL MARKING SCHEME

SUMMER 2025

A LEVEL
FURTHER MATHEMATICS
UNIT 5 FURTHER STATISTICS B
1305U50-1

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE A LEVEL FURTHER MATHEMATICS

UNIT 5 FURTHER STATISTICS B

SUMMER 2025 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	Standard error $= \frac{50}{\sqrt{100}}$ (=5) Use of $785 \pm z \times \mathrm{SE}$ $=785 \pm 1.96 \times \frac{50}{\sqrt{100}}$ $[775.2, 794.8]$	B1 M1 A1 A1 **[4]**	$\mathrm{SE}^2 = \frac{50^2}{100}$ (=25) FT their $\mathrm{SE} \neq 50$ for M1 only. cao 3sf or better.
(b)	Valid interpretation e.g. The confidence interval does not support the manufacturer's claim about the mean lifespan. e.g. If the claim about the standard deviation being 50 is correct then the claim about the average is too high. e.g. One (or both) of the claims appear(s) to be incorrect, i.e. either the claimed population mean of 800 hours is too high, or the claimed population standard deviation is too low (or both)	E1 **[1]**	FT their CI Must reference their confidence interval. E0 for 95% of samples will contain the true mean.
(c)	$1.96 \times \frac{50}{\sqrt{n}} < 5$ $\frac{1.96 \times 50}{5} < \sqrt{n}$ $n > 384.16$ $n = 385$	M1 A1 A1 **[3]**	Attempt at inequality/equation with 1.96, n and 5 oe e.g. $2 \times 1.96 \times \frac{50}{\sqrt{n}} < 10$ Correct inequality (or equation). cao
(d)	The lifespans of bulbs are not said to be normally distributed, but because n is large, the CLT allows us to use the normal distribution in our calculations.	E1 **[1]**	
	Total for Question 1	9	

Q2	Solution	Mark	Notes
(a)(i)	$\mathrm{E}(X) = \left(\frac{-\theta+\theta}{2}\right) = 0$	B1	Must be seen.
	$\mathrm{Var}(X) = \frac{1}{12}(\theta - -\theta)^2 = \frac{1}{3}\theta^2$	B1	oe
	$\mathrm{E}(X^2) = \mathrm{Var}(X) + \left(\mathrm{E}(X)\right)^2$	M1	Used
	$\mathrm{E}(X^2) = \frac{1}{3}\theta^2 + 0 = \frac{1}{3}\theta^2$	A1	
		[4]	
	OR		
	$\mathrm{E}(X^2) = \int_{-\theta}^{\theta} x^2 f(x)\mathrm{d}x$	(M1)	$f(x)$ must be a constant. Limits not required.
	$f(x) = \frac{1}{2\theta}$	(B1)	
	$\mathrm{E}(X^2) = \left[\frac{x^3}{3}\right]_{-\theta}^{\theta} \times \frac{1}{2\theta}$	(A1)	
	$\mathrm{E}(X^2) = \frac{\theta^2}{3}$	(A1)	
(b)	Unbiased estimator for θ^2 is $3X^2$	B1 **[1]**	B1 $3\bar{X}^2$ FT where possible B0 for $3\mathrm{E}(X^2)$ Mark final answer.
	Total for Question 2	5	

<table>
<tr><th>Q3</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr><td>(a)</td><td>H_0: The median difference in wellbeing is 0.
H_1: The median difference, when subtracting wellbeing before from wellbeing after is greater than 0.
OR $H_0: \eta_d = 0$ $H_1: \eta_d > 0$ where η_d is the median of the differences from subtracting wellbeing before from wellbeing after.
$\eta_d = \eta_A - \eta_B$</td><td>B1</td><td>Both.
oe
Do not condone median difference in levels of wellbeing before and after is greater than zero.</td></tr>
<tr><td></td><td><table><tr><td>Participant</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td></tr><tr><td>Difference</td><td>8</td><td>9</td><td>1</td><td>-3</td><td>10</td><td>-2</td><td>20</td><td>5</td><td>4</td></tr></table></td><td>B1</td><td>Accept differences with opposite signs.</td></tr>
<tr><td></td><td>Ranks<table><tr><td>Participant</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td></tr><tr><td>Ranks</td><td>6</td><td>7</td><td>1</td><td>3</td><td>8</td><td>2</td><td>9</td><td>5</td><td>4</td></tr></table></td><td>M1
A1</td><td>M1 either attempt at ranks. FT one slip in difference for A1</td></tr>
<tr><td></td><td>W^+ = Sum of ranks of positive differences (W$^-$ = Sum of ranks of negative differences
= 6 + 7 + 1 + 8 + 9 + 5 + 4 = 40 (= 3 + 2 = 5)</td><td>M1

A1</td><td>Must match direction of H_1</td></tr>
<tr><td></td><td>Upper CV = 37 (Lower CV = 8)</td><td>B1</td><td></td></tr>
<tr><td></td><td>Since 40 > 37 (OR 5 < 8) there is sufficient evidence to reject H_0.</td><td>B1</td><td>FT their W and CV.</td></tr>
<tr><td></td><td>There is sufficient evidence to suggest taking time off social media is beneficial for wellbeing.</td><td>E1

[9]</td><td>cso</td></tr>
<tr><td>(b)</td><td>Let μ be the population mean.
$H_0: \mu_X = \mu_Y$ $H_1: \mu_X \neq \mu_Y$</td><td>B1
[1]</td><td>Both</td></tr>
<tr><td>(c)</td><td>$\bar{x} = \frac{1560}{30} = 52$
$\bar{y} = \frac{940}{20} = 47$
$\bar{x} - \bar{y} = 5$</td><td>B1

B1</td><td>Either 52 or 47

si</td></tr>
<tr><td></td><td>SE of difference of means $= \sqrt{\frac{5.5^2}{30} + \frac{5.5^2}{20}} = \frac{11\sqrt{3}}{12}$
$= 1.5877 \ldots$</td><td>M1</td><td>Or SE^2</td></tr>
<tr><td></td><td>$\mathrm{P}(\bar{X} - \bar{Y} > 5) = 0.00081863 \ldots$</td><td>A1</td><td>awrt 0.00082</td></tr>
<tr><td></td><td>*p*-value $= 2 \times 0.00081863 \ldots$</td><td>m1</td><td></td></tr>
<tr><td></td><td>$= 0.001637$</td><td>A1</td><td>cao</td></tr>
<tr><td></td><td>ALTERNATIVE SOLUTION
$\mathrm{P}(-5 < \bar{X} - \bar{Y} < 5) = \mathrm{P}\left(-\frac{20\sqrt{3}}{11} < Z < \frac{20\sqrt{3}}{11}\right) = 0.99836$
p-value $= 1 - 0.99836$
$= 0.00164$</td><td>(A1)
(m1)
(A1)</td><td>

cao</td></tr>
<tr><td></td><td>Since 0.001637 < 0.01 there is sufficient evidence to reject H_0.
There is sufficient evidence to suggest that time off social media changes anxiety levels. (In this case it seems to suggest a decrease.)</td><td>m1

A1
[8]</td><td>Dep on M1 only.
Allow other significance levels.
cso</td></tr>
<tr><td></td><td>Total for Question 3</td><td>18</td><td></td></tr>
</table>

Q4	Solution	Mark	Notes
(a)	Let $T = X_1 + X_2 + X_3 + X_4 - X_5 - X_6 - X_7$ $\mathrm{E}(T) = 10$ $\mathrm{Var}(T) = 7\mathrm{Var}(X)$ $\mathrm{Var}(T) = 71.68$ $\mathrm{P}(T < 0) = 0.11877$	B1 M1 A1 A1 **[4]**	Or $\mathrm{E}(T) = -10$ 3sf or better. Or $\mathrm{P}(T > 0) = 0.11877$ cao
(b)	X_1 is the time taken by Ela. X_2 is the time taken by Mali. Consider $U = \frac{X_1}{2} - X_2$ or $U = X_1 - 2X_2$ $\mathrm{E}(U) = -5$ or $\mathrm{E}(U) = -10$ $\mathrm{Var}(U) = \frac{1}{4}\mathrm{Var}(X_1) + \mathrm{Var}(X_2)$ or $\mathrm{Var}(U) = \mathrm{Var}(X_1) + 4\mathrm{Var}(X_2)$ $\mathrm{Var}(U) = \frac{64}{5} = 12.8$ or $\mathrm{Var}(U) = \frac{256}{5} = 51.2$ $\mathrm{P}(U > 0)$ $= 0.08113$	M1 A1 M1 A1 m1 A1 **[6]**	oe Or $\mathrm{E}(U) = 5$ or $\mathrm{E}(U) = 10$. M1 for equation Dep on both M1. Or $\mathrm{P}(U < 0)$ 3sf or better. cao
(c)	Consider $V = X - Y$ $\mathrm{E}(V) = 2$ $\mathrm{Var}(V) = 2.8^2 + 3.2^2$ $\mathrm{Var}(V) = 18.08$ $\mathrm{P}(V < 0) = 0.3190$	M1 A1 M1 A1 **[4]**	 Or $\mathrm{E}(V) = -2$ 3sf or better. Or $\mathrm{P}(V > 0) = 0.3190$ cao
	Total for Question 4	**14**	

Q5	Solution	Mark	Notes
	H_0: The median numbers of litres of milk produced by cows on antibiotics and not on antibiotics are the same. H_1: The median number of litres of milk produced by cows on anitbiotics is less than the median number of litres of milk produced by cows not on antibiotics.	B1	Accept $H_0: \eta_A = \eta_B \qquad H_1: \eta_B < \eta_A$ Where η_A is the median numbers of litres of milk produced by cows on antibiotics and η_B is the median numbers of litres of milk produced by cows not on antibiotics.
	Use of the formula $U = \sum\sum z_{ij}$ $U = 9 + 4 + 0 + 6 + 9 + 1 + 4$ OR $U = 5 + 5 + 2 + 2 + 1 + 5 + 4 + 2 + 4$ $U = 33$ OR $U = 30$	M1 A1	Attempt to use Accept ranks with opposite signs ALTERNATIVE METHOD $t_A =$ sum of ranks for $A = 58$ $U_A = 58 - \frac{8\times7}{2} = 30$ $B =$ sum of ranks for $B = 78$ $U_A = 78 - \frac{9\times10}{2} = 33$
	Upper critical value is 48 OR Lower CV is 15 33 < 48 OR 30 > 15, there is insufficient evidence to reject H_0.	B1 B1	 FT their U and CV.
	There is insufficient evidence to suggest that cows produce less milk when they are on antibiotics	E1 **[6]**	cso Conclusion should not imply two-tailed test.
	Total for Question 5	**6**	

Q6	Solution	Mark	Notes
(a)	Valid assumption. e.g. The population heights are normally distributed.	E1	
	$\sum x = 2076 \quad \sum x^2 = 359340$ $\bar{x} = 173$	B1	All three attempted. At least two correct. si.
	$s^2 = \frac{1}{n-1}(\sum x^2 - n\bar{x}^2)$ $s^2 = \frac{1}{11}(359340 - 12 \times 173^2)$	M1	Use of. oe $\frac{12}{11} \times \left(\frac{359340}{12} - 173^2\right)$
	$= \frac{192}{11} = 17.4545 \ldots\ldots$ or $s = \frac{8\sqrt{33}}{11} = 4.17786 \ldots$	A1	
	DF = 11	B1	
	t-value = 1.796	B1	FT their DF
	Standard error = $\sqrt{\frac{17.4545\ldots}{12}} = \frac{4\sqrt{11}}{11} = 1.20604 \ldots$	B1	
	CL = $173 \pm 1.796 \times \sqrt{\frac{17.4545\ldots}{12}}$	M1	FT their t-value and $\bar{x}$ and SE, si Must be t-value.
	90%CI is [170.8, 175.2]	A1	cao, 3sf or better.
		[9]	
(b)	Unlikely Country A is The Netherlands because the mean height is outside the confidence interval.	E1 **[1]**	FT their CI
(c)	Valid comment e.g. It's reasonable because heights are often normally distributed.	E1 **[1]**	
	Total for Question 6	11	

Q7	Solution	Mark	Notes
(a)(i)	$\mathrm{E}(X) = -3\times\theta - 1\times 0.2 + 2\times\theta + 4\ \times(0.8-2\theta)$	M1	oe
	$\mathrm{E}(X) = 3 - 9\theta$	A1	Must be simplified.
(ii)	$\mathrm{Var}(X) = (-3)^2\ \times\theta + (-1)^2\times 0.2 + 2^2\times\theta$ $+4^2\ \times(0.8-2\theta) - (3-9\theta)^2$	M1	
	$\mathrm{Var}(X) = 9\theta + 0.2 + 4\theta + 12.8 - 32\theta$ $-(9 - 54\theta + 81\theta^2)$		
	$\mathrm{Var}(X) = 13 - 19\theta\ \ -9 + 54\theta - 81\theta^2)$ $\mathrm{Var}(X) = 4 + 35\theta - 81\theta^2$ ***ag**	A1 **[4]**	Convincing At least one line of working out between M1 and A1.
(b)(i)	$\mathrm{E}(T_1) = \mathrm{E}\left(\frac{3-\bar{X}}{9}\right)$		
	$E(T_1) = \frac{3-\mathrm{E}(\bar{X})}{9}$	M1	si
	$\mathrm{E}(T_1) = \frac{3-(3-9\theta)}{9}$ $\mathrm{E}(T_1) = \theta\ \therefore T_1$ is an unbiased estimator for θ.	A1	convincing.
(ii)	$\mathrm{Var}(T_1) = \frac{1}{81}\mathrm{Var}(\bar{X})$ $\mathrm{Var}(T_1) = \frac{1}{81}\frac{\mathrm{Var}(X)}{n}$	M1	
	$\mathrm{Var}(T_1) = \frac{4+35\theta-81\theta^2}{81n}$	A1 **[4]**	
(c)	$\mathrm{P}(X>0) = 0.8-\theta$	B1	si
	$Y\sim\mathrm{B}(n, 0.8-\theta)$ $\mathrm{E}(Y) = n(0.8-\theta)$	B1 M1	FT their $\mathrm{P}(X>0)$
	$\mathrm{E}(T_2) = \frac{4n-5\mathrm{E}(Y)}{5n}$ $\mathrm{E}(T_2) = \frac{4n-5(0.8-\theta)n}{5n}$ $\mathrm{E}(T_2) = \frac{4n-4n+5\theta n}{5n}$		
	$\mathrm{E}(T_2) = \theta\ \therefore T_2$ is an unbiased estimator for θ.	A1 **[4]**	convincing.
(d)	$\mathrm{Var}(Y) = n(0.8-\theta)(0.2+\theta)$	B1	No FT.
	$\mathrm{Var}(T_2) = \frac{(-5)^2 n(0.8-\theta)(\theta+0.2)}{5^2n^2}$	M1	FT their $\mathrm{Var}(Y)$ for M1 only.
	$\mathrm{Var}(T_2) = \frac{0.16+0.6\theta-\theta^2}{n}$	A1	
	$\mathrm{Var}(T_1) - \mathrm{Var}(T_2) = \frac{4+35\theta-81\theta^2}{81n} - \frac{0.16+0.6\theta-\theta^2}{n}$ $\mathrm{Var}(T_1) - \mathrm{Var}(T_2) = \frac{4+35\theta-81\theta^2}{81n} - \frac{12.96+48.6\theta-81\theta^2}{81n}$	m1	Dep on M1. Attempt to simplify FT their $\mathrm{Var}(T_1)$ and $\mathrm{Var}(T_2)$
	$\mathrm{Var}(T_1) - \mathrm{Var}(T_2) = \frac{-8.96-13.6\theta}{81n} < 0$ $\therefore T_1$ is the better estimator.	A1 **[5]**	convincing. cso
	Total for Question 7	**17**	