



GCE AS MARKING SCHEME

SUMMER 2025

**AS
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

WJEC GCE AS FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
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Q	Solution	Mark	Notes
1. a)	Substituting $3 - 2i$, $(3 - 2i)^3 = 27 - 54i - 36 + 8i (= -9 - 46i)$ OR $15 - 10i - 36i - 24$ $(3 - 2i)^2 = 9 - 12i - 4 (= 5 - 12i)$ $f(3 - 2i) = (-9 - 46i) - 8(5 - 12i) + 25(3 - 2i) - 26$ $= 0$	M1 B1 A1 [3]	 For x^3 with working Convincing
b)	METHOD 1: If $x = 3 - 2i$ is a root then $x = 3 + 2i$ is a root $(x - 3 + 2i)(x - 3 - 2i)$ $x^2 - 6x + 13$ is the quadratic factor $x^3 - 8x^2 + 25x - 26 = 0$ $(x^2 - 6x + 13)(x - 2) = 0$ The roots are $x = 3 - 2i, x = 3 + 2i, x = 2$ METHOD 2: If $x = 3 - 2i$ is a root then $x = 3 + 2i$ is a root Let other root be α $\therefore 3 - 2i + 3 + 2i + \alpha = 8$ $\alpha = 2$ The roots are $x = 3 - 2i, x = 3 + 2i, x = 2$ METHOD 3: If $x = 3 - 2i$ is a root then $x = 3 + 2i$ is a root Let other root be α $\therefore (3 - 2i)(3 + 2i)\alpha = 26$ $13\alpha = 26$ $\alpha = 2$ The roots are $x = 3 - 2i, x = 3 + 2i, x = 2$ METHOD 4: If $x = 3 - 2i$ is a root then $x = 3 + 2i$ is a root Use of Factor Theorem $f(2) = 0$ The roots are $x = 3 - 2i, x = 3 + 2i, x = 2$	B1 M1 A1 A1 (B1) (M1) (A1) (A1) (B1) (M1) (A1) (A1) [4] (B1) (M1) (A1) (A1) [4]	 M0 no working For $x - 2$ M0 no working M0 no working

Q	Solution	Mark	Notes
c)	METHOD 1: If $\alpha = 3 - 2i$, $\beta = 3 + 2i$, $\gamma = 2$ Then, $\frac{13}{\alpha} = 3 + 2i$, $\frac{13}{\beta} = 3 - 2i$, $\frac{13}{\gamma} = \frac{13}{2}$ New equation: Sum of roots: $(3 + 2i) + (3 - 2i) + \frac{13}{2} = \frac{25}{2}$ Sum of pairs: $(3 + 2i)(3 - 2i) + (3 + 2i)\frac{13}{2} + (3 - 2i)\frac{13}{2} = 52$ Product: $(3 + 2i)(3 - 2i)\frac{13}{2} = \frac{169}{2}$ Therefore, $\frac{-b}{a} = \frac{25}{2}$, $\frac{c}{a} = 52$, $\frac{-d}{a} = \frac{169}{2}$ New equation: e.g. $x^3 - \frac{25}{2}x^2 + 52x - \frac{169}{2} = 0$ e.g. $2x^3 - 25x^2 + 104x - 169 = 0$ METHOD 2: $\alpha + \beta + \gamma = 8$ $\alpha\beta + \beta\gamma + \gamma\alpha = 25$ $\alpha\beta\gamma = 26$ New equation roots: Sum of roots: $\frac{13}{\alpha} + \frac{13}{\beta} + \frac{13}{\gamma} = \frac{13(\beta\gamma + \alpha\gamma + \alpha\beta)}{\alpha\beta\gamma} = \frac{13 \times 25}{26} = \frac{25}{2}$ Sum of pairs: $\left(\frac{13}{\alpha} \times \frac{13}{\beta}\right) + \left(\frac{13}{\beta} \times \frac{13}{\gamma}\right) + \left(\frac{13}{\gamma} \times \frac{13}{\alpha}\right)$ $= \frac{169}{\alpha\beta} + \frac{169}{\beta\gamma} + \frac{169}{\gamma\alpha} = \frac{169(\gamma + \alpha + \beta)}{\alpha\beta\gamma} = \frac{169 \times 8}{26} = 52$ Product: $\frac{13}{\alpha} \times \frac{13}{\beta} \times \frac{13}{\gamma} = \frac{2197}{\alpha\beta\gamma} = \frac{2197}{26} = \frac{169}{2}$ Therefore, $\frac{-b}{a} = \frac{25}{2}$, $\frac{c}{a} = 52$, $\frac{-d}{a} = \frac{169}{2}$ New equation: e.g. $x^3 - \frac{25}{2}x^2 + 52x - \frac{169}{2} = 0$ METHOD 3: Substitute $\frac{13}{x}$ in place of x $\left(\frac{13}{x}\right)^3 - 8\left(\frac{13}{x}\right)^2 + 25\left(\frac{13}{x}\right) - 26 = 0$ $\frac{2197}{x^3} - \frac{1352}{x^2} + \frac{325}{x} - 26 = 0$ Multiplying throughout by x^3 $2197 - 1352x + 325x^2 - 26x^3 = 0$ Therefore, $x^3 - \frac{25}{2}x^2 + 52x - \frac{169}{2} = 0$	B2 B2 B1 B1 (B1) (B1) (B1) (B1) (B1) (B1) (M1) (A1) (A1) (m1) (A1) (A1) [6]	FT for real root only B1 for any 2 values B1 for any 2 values FT previous values for B1B1 oe ISW Must =0 FT for real root only any two correct FT previous values for B1B1 oe ISW Must =0 At least 2 correct terms oe ISW Must =0

Q	Solution	Mark	Notes
1. c)	METHOD 4: $\frac{13}{3-2i} = \frac{13(3+2i)}{(3-2i)(3+2i)} \quad \text{OR} \quad \frac{13}{3+2i} = \frac{13(3-2i)}{(3-2i)(3+2i)}$ leading to $3 + 2i$ AND $3 - 2i$ Therefore, new roots are $3 + 2i$, $3 - 2i$, $\frac{13}{2}$ Quadratic factor is $(x - 3 - 2i)(x - 3 + 2i)$ leading to $x^2 - 6x + 13$ Multiplying by $\left(x - \frac{13}{2}\right)$: $x^3 - \frac{25}{2}x^2 + 52x - \frac{169}{2} = 0$	(B2) (M1) (A1) (m1) (A1) [6]	B1 for any two values FT quadratic factor from (b) for sign errors only Or other pairing $x^2 - \frac{19}{2}x + \frac{39}{2} \mp 2ix \pm 13i$ oe ISW Must =0
		[13]	

Q	Solution	Mark	Notes
3. a)	$\left(\frac{5}{1} - \frac{4}{2} - \frac{1}{3}\right) + \left(\frac{5}{2} - \frac{4}{3} - \frac{1}{4}\right) + \left(\frac{5}{3} - \frac{4}{4} - \frac{1}{5}\right) + \dots$ $+ \left(\frac{5}{n-3} - \frac{4}{n-2} - \frac{1}{n-1}\right) + \left(\frac{5}{n-2} - \frac{4}{n-1} - \frac{1}{n}\right)$ $+ \left(\frac{5}{n-1} - \frac{4}{n} - \frac{1}{n+1}\right)$ $= \frac{5}{1} - \frac{4}{2} + \frac{5}{2}$ $- \frac{1}{n} - \frac{4}{n} - \frac{1}{n+1}$ $= \frac{11}{2} - \frac{5}{n} - \frac{1}{n+1}$ $a = 11, b = -5, c = -1$	M1 A1 A1 A1 A1 [5]	Substituting values At least three correct expansions of which at least 1 must be algebraic A0 if any other terms Accept 5.5 for 1 st term ISW
b)	METHOD 1: $\frac{11}{2} - \frac{5}{n} - \frac{1}{n+1} > 5$ $-\frac{5}{n} - \frac{1}{n+1} > -0.5$ $\frac{5}{n} + \frac{1}{n+1} < 0.5$ $5(n+1) + n < 0.5n(n+1)$ $0.5n^2 - 5.5n - 5 > 0$ $n^2 - 11n - 10 > 0$ Solving, Critical values = 11.8..., -0.8... $n = 12$ METHOD 2: $\frac{11}{2} - \frac{5}{n} - \frac{1}{n+1} > 5$ $\frac{11n(n+1) - 10(n+1) - 2n}{2n(n+1)} > 5$ $\frac{11n^2 + 11n - 10n - 10 - 2n}{2n(n+1)} > 5$ $\frac{11n^2 - n - 10}{2n(n+1)} > 5$ $11n^2 - n - 10 > 10n(n+1)$ $n^2 - 11n - 10 > 0$ Solving, Critical values = 11.8..., -0.8... $n = 12$	M1 A1 A1 m1 A1 (M1) (A1) (A1) (m1) (A1) [5]	FT (a) for non-zero a, b, c Removing algebraic fractions Solvable form cao Accept $n \geq 12$ FT (a) for non-zero a, b, c Removing algebraic fractions Solvable form cao Accept $n \geq 12$
		[10]	

Q	Solution	Mark	Notes
4.	Let $z = (a + ib)^2 = 7 - 24i$ $a^2 + 2abi - b^2 = 7 - 24i$ Therefore, $a^2 - b^2 = 7$ and $2ab = -24$ Solving simultaneous equations e.g. $b = \frac{-12}{a} \rightarrow a^2 - \frac{144}{a^2} = 7$ $a^4 - 144 = 7a^2$ $a^4 - 7a^2 - 144 = 0$ $(a^2 - 16)(a^2 + 9) = 0$ $a^2 = 16$ or $a^2 = -9$ (no real solutions) $\therefore a = \pm 4$ AND $b = \mp 3$ $\sqrt{z} = 4 - 3i$ or $\sqrt{z} = -4 + 3i$	M1 A1 m1 A1 A1 [5]	Both Solvable form Both
		[5]	

5.	METHOD 1:		
a)	Multiplying matrices $\mathbf{M} \times \mathbf{M} = \mathbf{I}$ leading to $a^2 + bc = 1$ $(ab + bd = 0)$ $(ac + dc = 0)$ $bc + d^2 = 1$ Rearranging, or equivalent, $a^2 = 1 - bc = d^2$ Therefore, $a^2 = d^2$	M1 A1 m1 A1	Must have 1st and 4th equation Convincing
	METHOD 2: Multiplying matrices $\mathbf{M} \times \mathbf{M} = \mathbf{I}$ leading to $a^2 + bc = 1$ $(ab + bd = 0)$ $(ac + dc = 0)$ $bc + d^2 = 1$ Using 2nd equation AND at least one other, $b(a + d) = 0$ $b = 0$ or $a + d = 0$ $a = -d$ $a^2 = d^2$ when $b = 0$, $a^2 = 1$ (from 1st eqn) and $d^2 = 1$ (from 4th eqn) Therefore, $a^2 = d^2$	(M1) (A1) (m1) (A1) [4]	Must have 1st and 4th equation Accept equivalent reasoning with c from 3rd equation Convincing

Q	Solution	Mark	Notes
5. b)	When $n = 1$, LHS = $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ and RHS = $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ Therefore, proposition is valid for $n = 1$. Assume result is true for $n = k$ i.e. $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}^k = \begin{bmatrix} \frac{1}{2} + \frac{(-1)^k}{2} & 1 - (-1)^k \\ \frac{1}{4} - \frac{(-1)^k}{4} & \frac{1}{2} + \frac{(-1)^k}{2} \end{bmatrix}$ Consider $n = k + 1$ $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}^{k+1} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} + \frac{(-1)^k}{2} & 1 - (-1)^k \\ \frac{1}{4} - \frac{(-1)^k}{4} & \frac{1}{2} + \frac{(-1)^k}{2} \end{bmatrix}$ $= \begin{bmatrix} 2\left(\frac{1}{4} - \frac{(-1)^k}{4}\right) & 2\left(\frac{1}{2} + \frac{(-1)^k}{2}\right) \\ \frac{1}{2}\left(\frac{1}{2} + \frac{(-1)^k}{2}\right) & \frac{1}{2}(1 - (-1)^k) \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{2} - \frac{(-1)^k}{2} & 1 + (-1)^k \\ \frac{1}{4} + \frac{(-1)^k}{4} & \frac{1}{2} - \frac{(-1)^k}{2} \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{2} + \frac{(-1)(-1)^k}{2} & 1 - (-1)(-1)^k \\ \frac{1}{4} - \frac{(-1)(-1)^k}{4} & \frac{1}{2} + \frac{(-1)(-1)^k}{2} \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{2} + \frac{(-1)^{k+1}}{2} & 1 - (-1)^{k+1} \\ \frac{1}{4} - \frac{(-1)^{k+1}}{4} & \frac{1}{2} + \frac{(-1)^{k+1}}{2} \end{bmatrix}$ If proposition is true for $n = k$, it is also true for $n = k + 1$. As it is true for $n = 1$, by mathematical induction, it is true for all positive integers n.	B1 M1 M1 A1 A1 A1 E1 [7]	oe Award for a perfect solution including the last line.
		[11]	

Q	Solution	Mark	Notes
6.	$ z - 2 + 5i = 3 z + q - i $ $ x + iy - 2 + 5i = 3 x + iy + q - i $ $ (x - 2) + i(y + 5) = 3 (x + q) + i(y - 1) $ $\sqrt{(x - 2)^2 + (y + 5)^2} = 3\sqrt{(x + q)^2 + (y - 1)^2}$ $x^2 - 4x + 4 + y^2 + 10y + 25$ $\quad = 9x^2 + 18qx + 9q^2 + 9y^2 - 18y + 9$ $8x^2 + 8y^2 + x(18q + 4) - 28y - 20 + 9q^2 = 0$ $x\text{-coordinate at centre point: } \frac{18q+4}{-2 \times 8}$ Therefore, $\frac{18q+4}{-2 \times 8} = -\frac{5}{2}$ leading to $q = 2$	M1 m1 A2 m1 A1 [6]	 Finding the modulus A1 each side of equation Finding x -coordinate from a circle equation cao
		[6]	

7. a)	Reflection matrix: $\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Rotation matrix: $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Translation matrix: $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $T = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $T = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ or $T = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $T = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 2 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$	B1 B1 B1 M1 A1 A1 [6]	Accept 2×2 matrix Accept 2×2 matrix either If M0, award SC1 for correct matrix following multiplication in reverse order
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Q	Solution	Mark	Notes
b)	METHOD 1: $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 2 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ Giving, $\frac{1}{2}x - \frac{\sqrt{3}}{2}y + 2 = x$ AND $-\frac{\sqrt{3}}{2}x - \frac{1}{2}y + 1 = y$ Solving simultaneous equations e.g. collecting like terms leads to $-\frac{\sqrt{3}}{2}y + 2 = \frac{1}{2}x$ and $-\frac{\sqrt{3}}{2}x + 1 = \frac{3}{2}y$ Substituting 1st equation into 2nd equation leads to $-\sqrt{3}\left(-\frac{\sqrt{3}}{2}y + 2\right) + 1 = \frac{3}{2}y$ $\frac{3}{2}y - 2\sqrt{3} + 1 = \frac{3}{2}y$ $-2\sqrt{3} + 1 = 0$ (contradiction) Therefore, there are no fixed points under T. METHOD 2: $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 2 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ Giving, $\frac{1}{2}x - \frac{\sqrt{3}}{2}y + 2 = x$ AND $-\frac{\sqrt{3}}{2}x - \frac{1}{2}y + 1 = y$ Forming a pair of linear equations with equal coefficients for x and for y OR two expressions for $x = \dots$ or $y = \dots$ ready to compare e.g. $\sqrt{3}x + 3y = 2$ and $\sqrt{3}x + 3y = 4\sqrt{3}$ These equations are inconsistent since it is not possible for $\sqrt{3}x + 3y$ to be 2 and $4\sqrt{3}$ simultaneously (contradiction), Therefore, there are no fixed points under T.	M1 A1 m1 A1 A1 (M1) (A1) (m1) (A1) (A1) [5]	FT their T from (a) Both equations Both equations
		[11]	