GCSE MARKING SCHEME

SUMMER 2023

GCSE
MATHEMATICS
UNIT 1 – HIGHER TIER
3300U50-1

INTRODUCTION

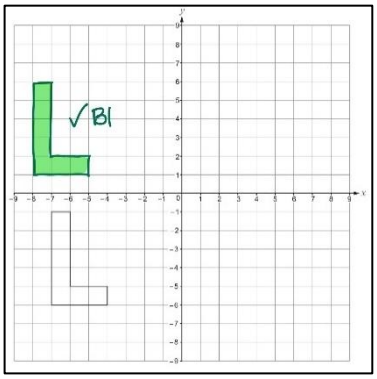
This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

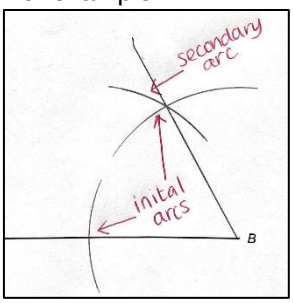
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCSE MATHEMATICS

SUMMER 2023 MARK SCHEME

Unit 1: Higher Tier	Mark	Comments
1.(a) 	B1	
1.(b) $\begin{pmatrix}1\\-7\end{pmatrix}$	B1	Award B0 for • $\begin{matrix}1\\-7\end{matrix}$ (missing brackets) • $\begin{pmatrix}-7\\1\end{pmatrix}$ • (1,−7) • $\frac{1}{-7}$ with or without brackets. • $-\begin{pmatrix}-1\\7\end{pmatrix}$.
2.(a) For a single method that produces 2 prime factors from the set {3, 3, 3, 5, 5} before the 2nd error. 3, 3, 3, 5, 5 $3^3 \times 5^2$	M1 A1 B1	Must be a method that involves only division. Check for errors in the method before checking the 2 prime factors from the set. (Note 675 = 5 × 135 675 = 3 × 225 135 = 5 × 27 135 = 3 × 45) CAO. For sight of the five correct factors (Ignore 1s) Do not FT non-primes. FT 'their primes' provided at least one index form used with at least a square. Allow $(3^3)(5^2)$ and $3^3.5^2$ and 3^35^2 Do not allow $3^3,5^2$ Inclusion of 1 as a factor gets B0.
2.(b) 10	B1	Do not accept 2 × 5.
3.(a)(i) m^7	B1	
3.(a)(ii) m^{10}	B1	
3.(b) $7n-3$	B2	Mark final answer. B1 for sight of $7n$. Allow notation of $n7$ or $7\times n$ or $n\times 7$ for $7n$. Allow N for n, but penalise –1 for use of a different letter.

3.(c) 7, 8 and 9	B2	Answer line takes precedence. Award B2 for all three integers and no extras. Award B1 for one of the following indicated as a final answer: • 7, 8, 9 and only **one** other incorrect value • for two correct with no incorrect value • 7 to 9 • 7, 7·5, 8, 8·5, 9 • sight of $6.5 < n < 9.5$ or equivalent • 14,16,18 • 14,15,16,17,18. Allow B2 for correct embedded answers of 7, 8 and 9 (e.g. sight of only $2 \times 7 = 14$, $2 \times 8 = 16$, $2 \times 9 = 18$ with no other calculations) BUT only B1 if contradicted on answer line (e.g. 14, 16, 18 for the example above).
4.(a) Correct construction of 60°	B1	Must be at point *B*. Correct construction arcs (two or three) must be seen (initial and secondary). B0 if 60° and 30° drawn. Ignore additional lines provided intended 60° is clear (e.g any triangle, including equilateral *ABC*). For example: 
4.(b) Correct construction of 90°	B1	Must be at point *R* above or below *LM*. Correct construction arcs (initial and secondary) **must** be seen.
4.(c) <u>All</u> correct construction arcs shown	M1	Arc, <u>centre P</u>, intersecting *XY* at two points. (*X* may be one of the points with no arc seen at point *X*.) [Note to markers: These arcs may be identified by the fact that they will 'cross the line *XY* at an acute angle'. Arcs 'crossing the line at 90°' is evidence of an inappropriate method.] **AND** Intersecting arcs (equal radii) using the above two points as centres. Ignore line extended above *XY* for M1.
Line drawn	A1	Ignore line extended above *XY* for M1A1.
4.(c) *<u>Alternative method</u>* *(Using the properties of a kite.)* *<u>All</u> correct construction arcs shown.*	*M1*	*Intersecting arcs whose centres are **any** two points on the line XY and respective radii equal in length to the distance from the points to the point P.* *[Note to markers: The arcs will always intersect at a point that is a 'reflection of point P' in the line XY.]*
Line drawn	*A1*	

5. $(AC^2 =)\ \ 8^2 + 6^2$ $(AC =)\ \sqrt{8^2 + 6^2}$ or equivalent $(AC =)$ 10 (cm)	 M1 m1 A1	Check diagram. Note: $(AC^2 =)\ \ 64 + 36$ FT from $\sqrt{'their\ 8^{2'} + 'their\ 6^{2'}}$ CAO. Final answer of AC = 100 is M1m0A0.
(Curved length =) • $\frac{6 \times 3{\cdot}14}{2}$ • $\frac{2 \times 3 \times 3{\cdot}14}{2}$ • $3 \times 3{\cdot}14$ • 3π • 9·42 or equivalent	B1	Do not ignore subsequent working e.g. 3 × 3·14 = 9·42, then 9·42 × 2 or 9·42 ÷ 2 would gain B0.
(Perimeter of shape = 8 + 10 + 9·42 =) 27·42(cm)	A1	Allow 27·4(cm). Award A0 for 18 + 3π. FT 'their *AC*' **only** if M1 gained. FT 'their curved length' **only** if B0 awarded **and** for one of the following: • 3·14 used to find the circumference of a circle • area of a semicircle used. For example, M1m1A1 awarded for 10 (cm), B1 for 3 × 3·14 = 9 (cm) A0 for 8 + 10 + 9 = 27 (cm) as B1 previously awarded. Note, if a final answer of: • 33·4(2)(cm) is given (6cm also included) award M1m1A1B1A0 • 36·8(4)(cm) is given (full circumference used) award M1m1A1B0A1 • 32·1(3) (cm) is given (area semicircle used) award M1m1A1B0A1.
Organisation and Communication.	OC1	For OC1, candidates will be expected to: • present their response in a structured way • explain to the reader what they are doing at each step of their response • lay out their explanation and working in a way that is clear and logical • write a conclusion that draws together their results and explains what their answer means
Accuracy of writing.	W1	For W1, candidates will be expected to: • show all their working • make few, if any, errors in spelling, punctuation and grammar • use correct mathematical form in their working • use appropriate terminology, units, etc

6. Sight of 4(hr) 35(min) AND 2(hr) 45(min) OR Sight of 275(min) AND 165(min) OR sight of 2 × 5(min) in an appropriate calculation. Valid method e.g. • 4(hr) 35(min) + 2(hr) 45(min)(=6(hr) 80(min)) 275(min) + 165(min) (= 440 (min)) • 6 (hr) 90 (min) – 10 (min) • 7 (hr) 30 (min) – 10 (min) • 4(hr) 40(min) + 2(hr) 50(min) – 10 (min) • 280 (min) + 170 (min) – 10(min) 7 (hr) 20 (min)	B1 M1 A1	Allow incorrect notation for time (e.g. 4:35 for 4(hr) 35(min)). If B0, FT provided unambiguously chosen: '4h 30m ≤ t_1 < 4h 40m' and '2h 40m ≤ t_2 < 2h 50m' OR '270m ≤ t_1 < 280m' and '160m ≤ t_2 < 170m' Allow incorrect notation for time (e.g. 4:35 for 4(hr) 35(min)). CAO. If units are given they must be correct. Award B1M1A0 for a final answer of 6hrs 80min, 6:80 or 7:20.
7.(a) P(Bus =) 1 − 0·25 − 0·45 = 0·3 AND shown on relevant branch. 0·96 shown on <u>all</u> three branches.	M1 A1 B1	Award M1A0 for 0·3 in working space and not on diagram.
7.(b) 0·25 × 0·04 or equivalent = 0·01 or equivalent	M1 A1	CAO
8. (Length) Area None Length Volume Length	B3	*Must use the terminology given in the question.* B3 for all 5 correct. B2 for 3 or 4 correct. B1 for 2 correct. B0 otherwise.
9.(a) $7{\cdot}6 \times 10^{-3}$	B1	
9.(b) 6×10^{5}	B1	
9.(c) $2{\cdot}8 \times 10^{4}$	B2	Mark final answer. Award B1 for one of the following: • sight of 28×10^{3} • sight of 28 000 • equivalent correct value but not in standard form • sight of 23 000 AND 5000 • 'their 28000' is written correctly in standard form, following one place value error in one of the numbers from work seen.

10.(a) (*AOY* =) 36(°) (% shaded =) $\frac{36}{360}$ (× 100) or equivalent = 10(%)	B1 M1 A1	Check diagram. FT 'their derived or stated angle *AOY*' provided not 54°. Award M0A0 for $\frac{360(°)}{36(°)}$ = 10, but award M1A1 if a final answer of 10% is seen. If no marks awarded, award: • SC2 for unsupported 10% (*AOY* not shown or stated to be 36(°)) • SC1 for a final answer of 15% (from using 54(°)).
10.(b) Statement explaining that, 'The **tangent** at any point on a circle is **perpendicular** (or equivalent) to the **radius** at that point'.	E1	Accept unambiguous similar wording. e.g. 'Radius and tangent 90(°)'. Diameter could be used in place of radius. Must refer to tangent and radius by name (not simply *AY* and *OA* or description).
11. (a) $y \propto \frac{1}{x}$ OR $y = \frac{k}{x}$ $0{\cdot}2 = \frac{k}{160}$ OR $k = 32$ $y = \frac{32}{x}$	B1 M1 A1	Allow $y \propto \frac{k}{x}$. M1 implies B1. FT from $y \propto \frac{1}{x^n}$ with $n > 1$ and n = integer. (Use of $n = 2$ leads to $k = 5120$.) May be seen in part (b).
11. (b) 	x	160	128	**40**
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y	0·2	**0·25**	0·8			B1 B1	Check working space if table is empty. Table takes precedence over working space. For y-value, accept equivalents to 0·25 including $\frac{32}{128}$. Mark final answer. For x-value, do not accept $\frac{32}{0{\cdot}8}$ or $\frac{320}{8}$. Mark final answer. FT from 'their k' (using $y = \frac{k}{x}$) or FT from $y \propto \frac{1}{x^n}$, with $n > 1$ and n = integer. Use of $n = 2$ leads to answers of 0·3125 and 80. No FT from continued use of direct proportion from part (a). If no marks in part (a), allow B1 B1 for answers of $y = 0{\cdot}25$, $x = 40$ in part (b).

12. (Volume of sphere =) 36π (cm^3)	B1	Allow 113·04 (from using π = 3·14).
(Volume of cone =) 300π (cm^3)	B1	Allow 942 (from using π = 3·14). Accept absence of π (in <u>both</u> expressions). If neither volume has been correctly evaluated, award SC1 for at least one correct volume calculation: $\frac{4}{3}\times\pi\times3^3$ OR $\frac{1}{3}\times\pi\times10^2\times9$ (allowing the use of π = 3·14). Calculations may be seen in stages.
3 : 25	B2	B1 for partially simplified ratio in the form a : b where a, b are integers (with π eliminated and fractions or decimals cleared) or B1 for $3\pi : 25\pi$. FT 'their volumes' if possible (provided at least B1 or SC1 already awarded). An answer of 24 : 25 (from $288\pi : 300\pi$, using 6 cm for radius of sphere) is awarded B1 FT B2.
12. *<u>Alternative method</u> (without explicitly calculating volumes):*		
. $\frac{4}{3}\times\pi\times3^3$ *OR* $\frac{1}{3}\times\pi\times10^2\times9$ *or equivalent*	*B1*	*Calculation for at least one volume (allowing the use of π = 3·14). Calculations may be seen in stages.* *Accept absence of π (in <u>both</u> expressions).*
. $4\times\pi\times3^3$: $\pi\times10^2\times9$ *or equivalent* *OR* $\frac{4}{3}\times3^3$: $\frac{1}{3}\times10^2\times9$ *or equivalent*	*B1*	*Both calculations correct AND one further step (clearing fractions OR eliminating π). Ratio may be implied.*
3 : 25	*B2*	*B1B0 for partially simplified ratio in the form a : b where a, b are integers (with π eliminated and fractions or decimals cleared)* *or B1B0 for $3\pi : 25\pi$* *or B0B1 for one error in calculation of integers followed by correct simplification e.g. 12:300 = 1:25.* *FT 'their volumes' if possible (provided at least B1 already awarded).* *An answer of 24 : 25 (from $288\pi : 300\pi$, using 6 cm for radius of sphere) is awarded B0 B1 FT B2.*

13.(a) (Area of triangle =) $\frac{1}{2} \times 4x \times (2x - 1)$ $\frac{8x^2-4x}{2} = \frac{3}{4}$ or equivalent $16x^2 - 8x - 3 = 0$	M1 m1 A1	Allow award of M1 if brackets omitted. Accept equivalent e.g. using area of rectangle = 1·5. Expanding brackets and equating. Clearing fractions and equating to zero. Must be convincing.
13.(b)(i) $(4x - 3)(4x + 1)$ [= 0] $x = \frac{3}{4}$ AND $x = -\frac{1}{4}$	M2 A1	Solution may be seen in part (a). If seen in both, the work in the answer space for part (b)(i) takes precedence. M1 for $(4x \ldots 3)(4x \ldots 1)$ M1 for two brackets which multiply to give $16x^2 - 8x + k$ or $16x^2 + mx - 3$. Strict FT from M1 <u>*Using quadratic formula*</u> *(x =)* $\frac{-(-8) \pm \sqrt{(-8)^2-4(16)(-3)}}{2(16)}$ *M1* *For M1, allow one error, in sign or substitution, but not in formula.* $x = \frac{8 \pm \sqrt{256}}{32}$ *A1* $x = \frac{3}{4}$ *or* $x = -\frac{1}{4}$ *(both answers required) A1* Do not allow a trial and improvement method.
13.(b)(ii) ($BC = 2 \times \frac{3}{4} - 1 =$) 0·5 (m) AND a valid statement e.g. length cannot be negative; length must be positive.	E1	Solution may be seen in part (a) or (b)(i). If seen in both, the work in the answer space for part (b)(ii) takes precedence. FT 'their x' provided an equivalent decision is required i.e. one value of x is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$.
14.(a)(i) 2·5	B1	
14.(a)(ii) At least 6 correct plots and no incorrect plot. A smooth <u>curve</u> drawn through their plots.	P1 C1	FT 'their (0·5, 2·5)'. Allow '±½ a small square'. FT 'their 7 plots' OR a curve through the 6 given plots and through (0·5, 2·5). Allow for the intention to pass through their plots (± 1 small square horizontal OR vertical). The curve should NOT intercept the y-axis.
14.(b) Two correct readings from their graph.	B2	For reference, x = 0·4 AND 2·6 to 1d.p. B1 for one correct reading. Strict FT 'their graph'. If more than two points of intersection, award B1 for one correct answer, B2 for a complete set of answers. If no marks, award SC1 for drawing the line $y = 3$.

15. (a) P(BBB or YYY) = $\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$ or equivalent $\frac{66}{336}$ (= $\frac{11}{56}$) or equivalent	M2 A1	M1 for sight of one correct product. M0 for use of an incorrect total e.g. 9. Must show intention to <u>add</u> for second M mark. ISW If no marks awarded, SC1 for an answer of $\frac{152}{512}$ or equivalent (from a method 'with replacement').
15. (b) P(BYY or YBY or YYB) $\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} + \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}$ or $\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times 3$ or equivalent $\frac{90}{336}$ (= $\frac{15}{56}$) or equivalent	 S1 M1 A1	FT from part *(a)* consistent use of a wrongly calculated denominator (8 × 7 × 6) OR use of an incorrect total e.g. 9. Consideration of 3 permutations (may be implied) M1 implies S1. Must show intention to <u>add</u> for second mark. ISW If no marks awarded, SC1 for sight of one correct product or SC1 for an answer of $\frac{135}{512}$ or equivalent (from a method 'with replacement'.
15.(b) *<u>Alternative method</u>:* *1 – P(YYY or YBB or BYB or BBY or BBB)* *= 1 – [* $\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} \times 3 + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$ *]* *or equivalent* *= 1 – [* $\frac{6}{336} + \frac{180}{336} + \frac{60}{336}$ *]* $\frac{90}{336}$ (= $\frac{15}{56}$) or equivalent	*S1* *M1* *A1*	*A complete method.* *FT 'their P(BBB)' or 'their P(YYY)' or both from (a).* *M1 implies S1.* *ISW* *If no marks awarded,* *SC1 for 1 – [* $\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} + \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$ *] (=* $\frac{5}{8}$*) from not considering 3 different permutations of BBY or* *SC1 for an answer of* $\frac{135}{512}$ *or equivalent (from a method 'with replacement'.*
16. (a) 242° and 298° with no other values	B2	B1 for either angle. Check diagram. Penalise –1 for each extra value within range (beyond 2 attempts). Ignore extra (correct or incorrect) values outside the required range. If only two angles offered and no marks gained, award SC1 for sight of both 180° + 62° and 360° – 62°.

16.(b)(i) Reflection in x-axis with maximum at (270°, +1), minimum at (90°, –1)	B2	Mark clear intention. Must be the correct shape, i.e. a single cycle of a negative sine curve, with x-intercepts at $x = 0$, $x = 180°$ and $x = 360°$, minimum at $x = 90°$, maximum at $x = 270°$. Accept any clear indication of y-coordinates. If not B2, award B1 for one of the following: • Fully correct shape and position (both for $0 \le x \le 180°$ and for $180° \le x \le 360°$) without correct coordinates indicated OR • Correct shape and position either for $0 \le x \le 180°$ or for $180° \le x \le 360°$ (a sine curve entirely between $y = –1$ and $y = 1$) AND indication of $y = –1$ and $y = 1$. SC1 for a graph which is fully correct (including labelling) other than having pointed minimum and maximum (formed from straight lines).
16.(b)(ii) Vertical translation +1 with maximum at (90°, +2), minimum at (270°, 0), with y-intercept at +1.	B2	Mark clear intention. Must be the correct shape, i.e. a single cycle of a sine curve, with consistent y values at $x = 0$, $x = 180°$ and $x = 360°$, maximum at $x = 90°$, minimum at $x = 270°$. Accept any clear indication of y-coordinates. Must have correct points for $x = 180°$ and $x = 360°$. 1 and 2 indicated on the y-axis. If not B2, award B1 for one of the following: • Fully correct shape and position (both for $0 \le x \le 180°$ and for $180° \le x \le 360°$) without correct coordinates indicated OR • Correct shape and position either for $0 \le x \le 180°$ or for $180° \le x \le 360°$ (a sine curve entirely between $y = 0$ and $y = 2$) AND indication of $y = 1$ and $y = 2$. SC1 for a graph which is fully correct (including labelling) other than having pointed minimum and maximum (formed from straight lines).
17.(a) $4 + 4 \times \sqrt{6} – (1 \times) \sqrt{6} – 6$ or equivalent $= 3\sqrt{6} – 2$ or $– 2 + 3\sqrt{6}$	M1 A1	$(\sqrt{6})^2$ or $\sqrt{6}\sqrt{6}$ is insufficient for 6. Mark final answer. If no marks awarded, SC1 for 3 of the 4 terms correct.
17.(b)(i) Any square number greater than 5 e.g. 9, 16, 25, 36, …	B1	
17.(b)(ii) Any cube number greater than 5 e.g. 8, 27, 64, 125, 216, ...1000, …	B1	
17.(b)(iii) Any value of n^6 where n is an integer > 1. e.g 64 , 729, … 1 000 000, …	B1	Allow 2^6 or 3^6 or …