OCR

Tuesday 24 June 2014 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

3169569542

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Alan is running in a straight line on a bearing of 090° at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. Ben sees Alan when they are 50 m apart and Alan is on a bearing of 060° from Ben. Ben sets off immediately to intercept Alan by running at a constant speed of $6\,\text{m}\,\text{s}^{-1}$.

(i) Calculate the bearing on which Ben should run to intercept Alan. **[3]**

(ii) Calculate the magnitude of the velocity of Ben relative to Alan and find the time it takes, from the moment Ben sees Alan, for Ben to intercept Alan. **[4]**

2 A uniform solid circular cone has mass M and base radius R.

(i) Show by integration that the moment of inertia of the cone about its axis of symmetry is $\frac{3}{10}MR^2$. (You may assume the standard formula $\frac{1}{2}mr^2$ for the moment of inertia of a uniform disc about its axis and that the volume of a cone is $\frac{1}{3}\pi r^2 h$.) **[6]**

The axis of symmetry of the cone is fixed vertically and the cone is rotating about its axis at an angular speed of $6\,\text{rad}\,\text{s}^{-1}$. A frictional couple of constant moment 0.027 N m is applied to the cone bringing it to rest. Given that the mass of the cone is 2 kg and its base radius is 0.3 m, find

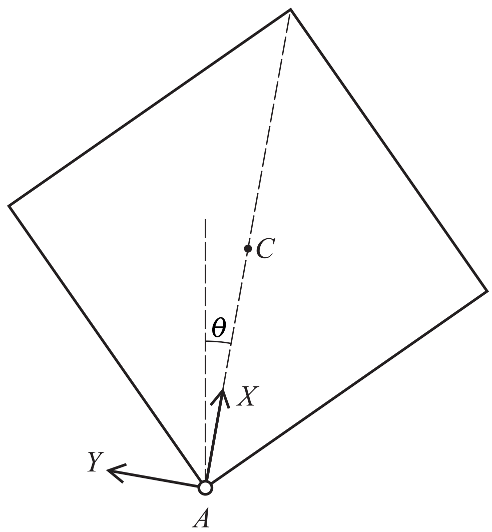
(ii) the constant angular deceleration of the cone, **[3]**

(iii) the time taken for the cone to come to rest from the instant that the couple is applied. **[2]**

3 The region bounded by the y-axis and the curves $y = \sin 2x$ and $y = \sqrt{2}\cos x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ is occupied by a uniform lamina. Find the exact value of the x-coordinate of the centre of mass of the lamina. **[8]**

4 A uniform square lamina has mass m and sides of length $2a$.

(i) Calculate the moment of inertia of the lamina about an axis through one of its corners perpendicular to its plane. **[3]**



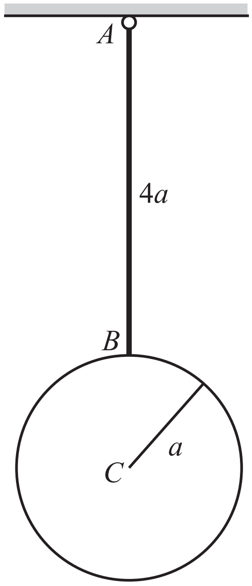
The uniform square lamina has centre C and is free to rotate in a vertical plane about a fixed horizontal axis passing through one of its corners A. The lamina is initially held such that AC is vertical with C above A. The lamina is slightly disturbed from rest from this initial position. When AC makes an angle θ with the upward vertical, the force exerted by the axis on the lamina has components X parallel to AC and Y perpendicular to AC (see diagram).

(ii) Show that the angular speed, ω, of the lamina satisfies $a\omega^2 = \frac{3}{4}g\sqrt{2}(1-\cos\theta)$. **[4]**

(iii) Find X and Y in terms of m, g and θ. **[6]**

Question 5 begins on page 4.

5



A pendulum consists of a uniform rod AB of length $4a$ and mass $4m$ and a spherical shell of radius a, mass m and centre C. The end B of the rod is rigidly attached to a point on the surface of the shell in such a way that ABC is a straight line. The pendulum is initially at rest with B vertically below A and it is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through A (see diagram).

(i) Show that the moment of inertia of the pendulum about the axis of rotation is $47ma^2$. **[4]**

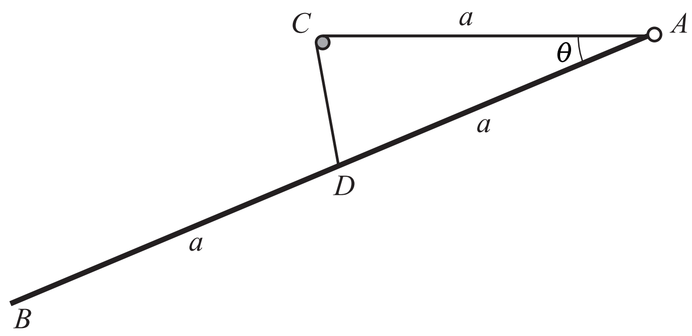
A particle of mass m is moving horizontally in the plane in which the pendulum is free to rotate. The particle has speed $\sqrt{kga}$, where k is a positive constant, and strikes the rod at a distance $3a$ from A. In the subsequent motion the particle adheres to the rod and the combined rigid body P starts to rotate.

(ii) Show that the initial angular speed of P is $\frac{3}{56}\sqrt{\frac{kg}{a}}$. **[4]**

(iii) For the case $k = 4$, find the angle that P has turned through when P first comes to instantaneous rest. **[4]**

(iv) Find the least value of k such that the rod reaches the horizontal. **[2]**

6



A uniform rod AB has mass m and length $2a$. The rod can rotate in a vertical plane about a smooth fixed horizontal axis passing through A. One end of a light elastic string of natural length a and modulus of elasticity $\sqrt{3}mg$ is attached to A. The string passes over a small smooth fixed pulley C, where AC is horizontal and $AC = a$. The other end of the string is attached to the rod at its mid-point D. The rod makes an angle θ below the horizontal (see diagram).

(i) Taking A as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = mga(\sqrt{3} - \sin\theta - \sqrt{3}\cos\theta).$$ **[4]**

(ii) Show that $\theta = \frac{1}{6}\pi$ is a position of stable equilibrium for the system. **[5]**

The system is making small oscillations about the equilibrium position.

(iii) By differentiating the energy equation with respect to time, show that

$$\frac{4}{3}a\ddot{\theta} = g(\cos\theta - \sqrt{3}\sin\theta).$$ **[4]**

(iv) Using the substitution $\theta = \phi + \frac{1}{6}\pi$, show that the motion is approximately simple harmonic, and find the approximate period of the oscillations. **[6]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE