

OCR

Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = \sin x. \quad [8]$$

- 2 The elements of a group G are polynomials of the form $a + bx + cx^2$, where $a, b, c \in \{0, 1, 2, 3, 4\}$. The group operation is addition, where the coefficients are added modulo 5.

(i) State the identity element. [1]

(ii) State the inverse of $3 + 2x + x^2$. [2]

(iii) State the order of G . [1]

The proper subgroup H contains $2 + x$ and $1 + x$.

(iv) Find the order of H , justifying your answer. [4]

- 3 The plane Π passes through the points $(1, 2, 1)$, $(2, 3, 6)$ and $(4, -1, 2)$.

(i) Find a cartesian equation of the plane Π . [5]

The line l has equation $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of Π and l . [3]

(iii) Find the acute angle between Π and l . [3]

- 4 In an Argand diagram, the complex numbers 0 , z and $ze^{\frac{1}{6}i\pi}$ are represented by the points O , A and B respectively.

(i) Sketch a possible Argand diagram showing the triangle OAB . Show that the triangle is isosceles and state the size of angle AOB . [4]

The complex numbers $1 + i$ and $5 + 2i$ are represented by the points C and D respectively. The complex number w is represented by the point E , such that $CD = CE$ and angle $DCE = \frac{1}{6}\pi$.

(ii) Calculate the possible values of w , giving your answers exactly in the form $a + bi$. [5]

- 5 Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 3y = x^2 + x$$

for which $y = 1$ when $x = 1$, giving y in terms of x . [8]

- 6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4} = \frac{y-1}{-2} = \frac{z+1}{3}. \quad [7]$$

- 7 (i) Use de Moivre's theorem to show that $\tan 4\theta \equiv \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$. [4]

(ii) Hence find the exact roots of $t^4 + 4\sqrt{3}t^3 - 6t^2 - 4\sqrt{3}t + 1 = 0$. [5]

- 8 Let G be any multiplicative group. H is a subset of G . H consists of all elements h such that $hg = gh$ for every element g in G .

- (i) Prove that H is a subgroup of G . [8]

Now consider the case where G is given by the following table:

	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>e</i>	<i>e</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>
<i>p</i>	<i>p</i>	<i>q</i>	<i>e</i>	<i>s</i>	<i>t</i>	<i>r</i>
<i>q</i>	<i>q</i>	<i>e</i>	<i>p</i>	<i>t</i>	<i>r</i>	<i>s</i>
<i>r</i>	<i>r</i>	<i>t</i>	<i>s</i>	<i>e</i>	<i>q</i>	<i>p</i>
<i>s</i>	<i>s</i>	<i>r</i>	<i>t</i>	<i>p</i>	<i>e</i>	<i>q</i>
<i>t</i>	<i>t</i>	<i>s</i>	<i>r</i>	<i>q</i>	<i>p</i>	<i>e</i>

- (ii) Show that H consists of just the identity element. [4]

END OF QUESTION PAPER

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