OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

3248700211

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=\sin x\,.$$ **[8]**

2 The elements of a group G are polynomials of the form $a+bx+cx^2$, where $a,b,c \in \{0,1,2,3,4\}$. The group operation is addition, where the coefficients are added modulo 5.

(i) State the identity element. **[1]**

(ii) State the inverse of $3+2x+x^2$. **[2]**

(iii) State the order of G. **[1]**

The proper subgroup H contains $2+x$ and $1+x$.

(iv) Find the order of H, justifying your answer. **[4]**

3 The plane Π passes through the points $(1,2,1)$, $(2,3,6)$ and $(4,-1,2)$.

(i) Find a cartesian equation of the plane Π. **[5]**

The line l has equation $\mathbf{r}=\begin{pmatrix}-1\\-2\\6\end{pmatrix}+\lambda\begin{pmatrix}4\\3\\-2\end{pmatrix}$.

(ii) Find the coordinates of the point of intersection of Π and l. **[3]**

(iii) Find the acute angle between Π and l. **[3]**

4 In an Argand diagram, the complex numbers $0, z$ and $ze^{\frac{1}{6}i\pi}$ are represented by the points O, A and B respectively.

(i) Sketch a possible Argand diagram showing the triangle OAB. Show that the triangle is isosceles and state the size of angle AOB. **[4]**

The complex numbers $1+i$ and $5+2i$ are represented by the points C and D respectively. The complex number w is represented by the point E, such that $CD = CE$ and angle $DCE=\frac{1}{6}\pi$.

(ii) Calculate the possible values of w, giving your answers exactly in the form $a+bi$. **[5]**

5 Find the particular solution of the differential equation

$$x\frac{dy}{dx}+3y=x^2+x$$

for which $y=1$ when $x=1$, giving y in terms of x. **[8]**

6 Find the shortest distance between the lines with equations

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4}=\frac{y-1}{-2}=\frac{z+1}{3}.$$ **[7]**

7 **(i)** Use de Moivre's theorem to show that $\tan 4\theta \equiv \dfrac{4\tan\theta-4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$. **[4]**

(ii) Hence find the exact roots of $t^4+4\sqrt{3}t^3-6t^2-4\sqrt{3}t+1=0$. **[5]**

8 Let G be any multiplicative group. H is a subset of G. H consists of all elements h such that $hg=gh$ for every element g in G.

(i) Prove that H is a subgroup of G. **[8]**

Now consider the case where G is given by the following table:

	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	s	t	r
q	q	e	p	t	r	s
r	r	t	s	e	q	p
s	s	r	t	p	e	q
t	t	s	r	q	p	e

(ii) Show that H consists of just the identity element. **[4]**

END OF QUESTION PAPER