Mathematics

Paper 4 (Extended)

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Geometrical instruments
Tracing paper (optional).

Read These Instructions First

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Do not write in any barcodes.

Answer all questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.
1 12 000 vehicles drive through a road toll on one day. The ratio cars : trucks : motorcycles = 13 : 8 : 3.

(a) (i) Show that 6500 cars drive through the road toll on that day.

*Answer (a)(i)*

(b) The toll charges in 2014 are shown in the table.

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cars</td>
<td>$2</td>
</tr>
<tr>
<td>Trucks</td>
<td>$5</td>
</tr>
<tr>
<td>Motorcycles</td>
<td>$1</td>
</tr>
</tbody>
</table>

Show that the total amount paid in tolls on that day is $34,500.

*Answer (b)*
(c) This total amount is a decrease of 8% on the total amount paid on the same day in 2013.

Calculate the total amount paid on that day in 2013.

Answer(c) $........................................... [3]

(d) 2750 of the 6500 car drivers pay their toll using a credit card.

Write down, in its simplest terms, the fraction of car drivers who pay using a credit card.

Answer(d) ........................................... [2]

(e) To the nearest thousand, 90 000 cars drive through the road toll in one week.

Write down the lower bound for this number of cars.

Answer(e) ........................................... [1]
2 The table shows some values for \( y = x^2 - \frac{1}{2x}, \ x \neq 0 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>( -1.5 )</th>
<th>( -1 )</th>
<th>( -0.5 )</th>
<th>( -0.25 )</th>
<th>( -0.2 )</th>
<th>( 0.2 )</th>
<th>( 0.25 )</th>
<th>( 0.5 )</th>
<th>( 1 )</th>
<th>( 1.5 )</th>
<th>( 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.25</td>
<td>2.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table of values. \[4\]

(b) On the grid, draw the graph of \( y = x^2 - \frac{1}{2x} \) for \(-2 \leq x \leq -0.2\) and \(0.2 \leq x \leq 2\).

(c) By drawing a suitable line, use your graph to solve the equation \( x^2 - \frac{1}{2x} = 2 \).

Answer(c) \( x = \) ................. or \( x = \) ................. or \( x = \) ................. \[3\]
(d) The equation \( x^2 - \frac{1}{2x} = k \) has only one solution.

Write down the range of values of \( k \) for which this is possible.

Answer(d) ................................................... [2]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where \( x = -1 \).

Answer(e) ................................................... [3]
(a) Draw the image of

(i) shape $A$ after a translation by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$, [2]

(ii) shape $A$ after a rotation through $180^\circ$ about the point $(0, 0)$, [2]

(iii) shape $A$ after the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. [3]

(b) Describe fully the single transformation that maps shape $A$ onto shape $B$.

Answer(b) ...........................................................................................................................................................................[3]

(c) Find the matrix which represents the transformation that maps shape $A$ onto shape $C$.

Answer(c) $\begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix}$ [2]
30 students were asked if they had a bicycle ($B$), a mobile phone ($M$) and a computer ($C$). The results are shown in the Venn diagram.

(a) Work out the value of $x$.

Answer(a) $x =$ ........................................... [1]

(b) Use set notation to describe the shaded region in the Venn diagram.

Answer(b) ........................................... [1]

(c) Find $n(C \cap (M \cup B)')$.

Answer(c) ........................................... [1]

(d) A student is chosen at random.

(i) Write down the probability that the student is a member of the set $M'$.

Answer(d)(i) ........................................... [1]

(ii) Write down the probability that the student has a bicycle.

Answer(d)(ii) ........................................... [1]

(e) Two students are chosen at random from the students who have computers.

Find the probability that each of these students has a mobile phone but no bicycle.

Answer(e) ........................................... [3]
(a) Andrei stands on level horizontal ground, 294 m from the foot of a vertical tower which is 55 m high.

(i) Calculate the angle of elevation of the top of the tower.

Answer (a)(i) .................................................. [2]

(ii) Andrei walks a distance $x$ metres directly towards the tower. The angle of elevation of the top of the tower is now $24.8^\circ$.

Calculate the value of $x$.

Answer (a)(ii) $x =$ .................................................. [4]
(b) The diagram shows a pyramid with a horizontal rectangular base.

The rectangular base has length 4.8 m and width 3 m and the height of the pyramid is 4 m.

Calculate

(i) \( y \), the length of a sloping edge of the pyramid,

\[ Answer(b)(i) \ y = \text{........................................} \text{ m} \ 4 \]

(ii) the angle between a sloping edge and the rectangular base of the pyramid.

\[ Answer(b)(ii) \ \text{........................................} \ 2 \]
The table shows the time, \( t \) minutes, that 400 people take to complete a test.

<table>
<thead>
<tr>
<th>Time taken (( t ) mins)</th>
<th>( 0 &lt; t \leq 10 )</th>
<th>( 10 &lt; t \leq 24 )</th>
<th>( 24 &lt; t \leq 30 )</th>
<th>( 30 &lt; t \leq 40 )</th>
<th>( 40 &lt; t \leq 60 )</th>
<th>( 60 &lt; t \leq 70 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>10</td>
<td>90</td>
<td>135</td>
<td>85</td>
<td>70</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) (i) Write down the modal time interval.

\[
Answer(a)(i) \text{ ........................................} \text{ min [1]}
\]

(ii) Calculate an estimate of the mean time taken to complete the test.

\[
Answer(a)(ii) \text{ ........................................} \text{ min [4]}
\]

(b) (i) Complete the table of cumulative frequencies.

<table>
<thead>
<tr>
<th>Time taken (( t ) mins)</th>
<th>( t \leq 10 )</th>
<th>( t \leq 24 )</th>
<th>( t \leq 30 )</th>
<th>( t \leq 40 )</th>
<th>( t \leq 60 )</th>
<th>( t \leq 70 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td>10</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td>400</td>
</tr>
</tbody>
</table>

(ii) On the grid opposite, draw a cumulative frequency diagram to show this information.
(c) Use your graph to estimate

(i) the median time,

Answer(c)(i) ........................................... min [1]

(ii) the inter-quartile range,

Answer(c)(ii) ........................................... min [2]

(iii) the 15th percentile,

Answer(c)(iii) ........................................... min [2]

(iv) the number of people who took more than 50 minutes.

Answer(c)(iv) ........................................... [2]
In the triangle $PQR$, $QR = 7.6$ cm and $PR = 8.4$ cm. Angle $QRP = 62^\circ$.

Calculate

(i) $PQ$.

(ii) the area of triangle $PQR$.

\[ \text{Answer (a)i } PQ = \ldots \ldots \ldots \ldots \ldots \ldots \text{ cm} \quad [4] \]

\[ \text{Answer (a)ii } \text{ cm}^2 \quad [2] \]
The diagram shows the positions of three small islands $G$, $H$ and $J$.
The bearing of $H$ from $G$ is $045^\circ$.
The bearing of $J$ from $G$ is $126^\circ$.
The bearing of $J$ from $H$ is $164^\circ$.
The distance $HJ$ is $63$ km.

Calculate the distance $GJ$.

\[ \text{Answer}(b) \quad GJ = \ldots \ldots \ldots \ldots \ldots \text{km} \]
Jamil, Kiera and Luther collect badges.
Jamil has $x$ badges.
Kiera has 12 badges more than Jamil.
Luther has 3 times as many badges as Kiera.
Altogether they have 123 badges.

Form an equation and solve it to find the value of $x$.

Answer(a) $x =$ .................................................  [3]

(b) Find the integer values of $t$ which satisfy the inequalities.

$$4t + 7 < 39 \leq 7t + 2$$

Answer(b) ..................................................  [3]

(c) Solve the following equations.

(i) \( \frac{21 - x}{x + 5} = 4 \)

Answer(c)(i) $x =$ ..................................................  [3]
(ii) \[ 3x^2 + 7x - 5 = 0 \]

Show all your working and give your answers correct to 2 decimal places.

\[ \text{Answer (c)(ii)} \quad x = \ldots \quad \text{or} \quad x = \ldots \quad [4] \]
9  (a) The points $A$, $B$, $C$ and $D$ lie on a circle.
$AC$ is a diameter of the circle.
$ST$ is the tangent to the circle at $A$.

Find the value of

(i) $x$,

(ii) $y$.

Answer (a)(i) $x =$ .................................................. [2]

Answer (a)(ii) $y =$ .................................................. [2]
(b) The diagram shows a shaded shape formed by three semi-circular arcs. The radius of each semi-circle is shown in the diagram.

(i) Calculate the perimeter of the shaded shape.

(ii) The shaded shape is made from metal 1.6 mm thick.

Calculate the volume of metal used to make this shape.
Give your answer in cubic millimetres.

Answer (b) (i) ........................................... cm [2]

Answer (b) (ii) ........................................... mm$^3$ [5]
10 The diagram is a scale drawing of three straight roads, $AB$, $BC$ and $CD$. The scale is $1:5000$.

(a) Find the actual length of the road $BC$. Give your answer in metres.

Answer (a) .............................................. m [2]

(b) Another straight road starts at $M$, the midpoint of $AB$. This road is perpendicular to $AB$ and it meets the road $CD$ at $X$.

Using a straight edge and compasses only, construct $MX$. [2]
(c) There is a park in the area enclosed by the four roads.

The park is

- less than 290 m from $B$
- nearer to $CD$ than to $CB$.

**Using a ruler and compasses only**, construct the boundaries of the park.

Leave in all your construction arcs and label the park $P$.  

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**Question 11 is printed on the next page.**
11  (a) Make \( x \) the subject of the formula.

\[ A - x = \frac{xf}{l} \]

Answer(a) \( x = \) .................................................. [4]

(b) Find the value of \( a \) and the value of \( b \) when \( x^2 - 16x + a = (x + b)^2 \).

Answer(b) \( a = \) .................................................. \\
\( b = \) .................................................. [3]

(c) Write as a single fraction in its simplest form.

\[ \frac{6}{x-4} - \frac{5}{3x-2} \]

Answer(c) .................................................. [3]