

**Tuesday 21 June 2016 – Morning**

**A2 GCE MATHEMATICS**

**4723/01** Core Mathematics 3

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

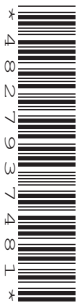
**OCR supplied materials:**

- Printed Answer Book 4723/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions

- 1 Find the equation of the tangent to the curve

$$y = 3x^2(x+2)^6$$

at the point  $(-1, 3)$ , giving your answer in the form  $y = mx + c$ . [5]

- 2 Find

(i)  $\int \left(2 - \frac{1}{x}\right)^2 dx$ ,

(ii)  $\int (4x+1)^{\frac{1}{3}} dx$ .

[5]

- 3 The mass of a substance is decreasing exponentially. Its mass is  $m$  grams at time  $t$  years. The following table shows certain values of  $t$  and  $m$ .

$t$	0	5	10	25
$m$	200	160		

- (i) Find the values missing from the table. [2]

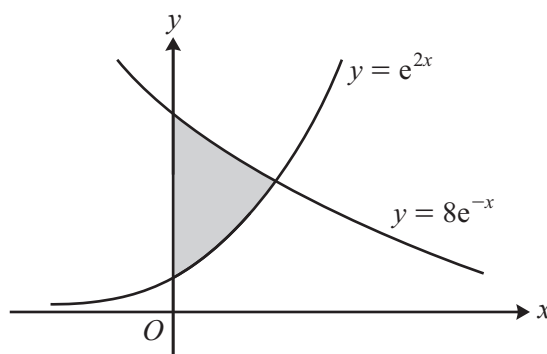
- (ii) Determine the value of  $t$ , correct to the nearest integer, for which the mass is 50 grams. [4]

- 4 It is given that  $A$  and  $B$  are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of  $\tan(A - B)$ . [8]

- 5



The diagram shows the curves  $y = e^{2x}$  and  $y = 8e^{-x}$ . The shaded region is bounded by the curves and the  $y$ -axis. Without using a calculator,

- (i) solve an appropriate equation to show that the curves intersect at a point for which  $x = \ln 2$ , [2]

- (ii) find the area of the shaded region, giving your answer in simplified form. [5]

- 6 The curves  $C_1$  and  $C_2$  have equations

$$y = \ln(4x - 7) + 18 \quad \text{and} \quad y = a(x^2 + b)^{\frac{1}{2}}$$

respectively, where  $a$  and  $b$  are positive constants. The point  $P$  lies on both curves and has  $x$ -coordinate 2. It is given that the gradient of  $C_1$  at  $P$  is equal to the gradient of  $C_2$  at  $P$ . Find the values of  $a$  and  $b$ . [8]

- 7 (i) By sketching the curves  $y = x(2x + 5)$  and  $y = \cos^{-1}x$  (where  $y$  is in radians) in a single diagram, show that the equation  $x(2x + 5) = \cos^{-1}x$  has exactly one real root. [3]

- (ii) Use the iterative formula

$$x_{n+1} = \frac{\cos^{-1}x_n}{2x_n + 5} \quad \text{with} \quad x_1 = 0.25$$

to find the root correct to 3 significant figures. Show the result of each iteration correct to at least 4 significant figures. [4]

- (iii) Two new curves are obtained by transforming each of the curves  $y = x(2x + 5)$  and  $y = \cos^{-1}x$  by the pair of transformations:

reflection in the  $x$ -axis followed by reflection in the  $y$ -axis.

State an equation of each of the new curves and determine the coordinates of their point of intersection, giving each coordinate correct to 3 significant figures. [4]

- 8 The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = |2x + a| + 3a \quad \text{and} \quad g(x) = 5x - 4a,$$

where  $a$  is a positive constant.

- (i) State the range of  $f$  and the range of  $g$ . [2]

- (ii) State why  $f$  has no inverse, and find an expression for  $g^{-1}(x)$ . [3]

- (iii) Solve for  $x$  the equation  $gf(x) = 31a$ . [5]

- 9 (i) Show that  $\sin 2\theta(\tan \theta + \cot \theta) \equiv 2$ . [4]

- (ii) Hence

- (a) find the exact value of  $\tan \frac{1}{12}\pi + \tan \frac{1}{8}\pi + \cot \frac{1}{12}\pi + \cot \frac{1}{8}\pi$ , [3]

- (b) solve the equation  $\sin 4\theta(\tan \theta + \cot \theta) = 1$  for  $0 < \theta < \frac{1}{2}\pi$ , [3]

- (c) express  $(1 - \cos 2\theta)^2 \left(\tan \frac{1}{2}\theta + \cot \frac{1}{2}\theta\right)^3$  in terms of  $\sin \theta$ . [2]

**END OF QUESTION PAPER**

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.