OCR
Oxford Cambridge and RSA

# Tuesday 21 June 2016 – Morning

## A2 GCE MATHEMATICS

**4723/01** Core Mathematics 3

**QUESTION PAPER**



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4723/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions

**1** Find the equation of the tangent to the curve

$$y = 3x^2(x+2)^6$$

at the point $(-1, 3)$, giving your answer in the form $y = mx + c$. **[5]**

**2** Find

**(i)** $\int \left(2 - \frac{1}{x}\right)^2 \mathrm{d}x$,

**(ii)** $\int (4x+1)^{\frac{1}{3}} \mathrm{d}x$.

**[5]**

**3** The mass of a substance is decreasing exponentially. Its mass is $m$ grams at time $t$ years. The following table shows certain values of $t$ and $m$.

| $t$ | 0 | 5 | 10 | 25 |
|---|---|---|---|---|
| $m$ | 200 | 160 | | |

**(i)** Find the values missing from the table. **[2]**

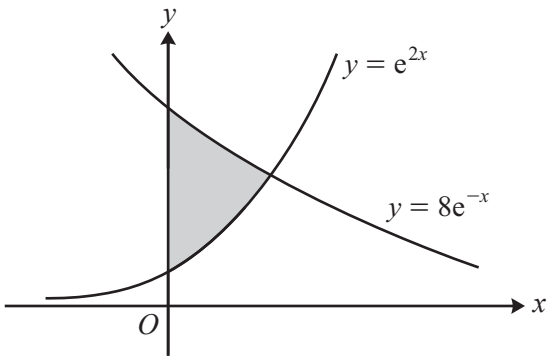
**(ii)** Determine the value of $t$, correct to the nearest integer, for which the mass is 50 grams. **[4]**

**4** It is given that $A$ and $B$ are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of $\tan(A - B)$. **[8]**

**5**




The diagram shows the curves $y = \mathrm{e}^{2x}$ and $y = 8\mathrm{e}^{-x}$. The shaded region is bounded by the curves and the $y$-axis. Without using a calculator,

**(i)** solve an appropriate equation to show that the curves intersect at a point for which $x = \ln 2$, **[2]**

**(ii)** find the area of the shaded region, giving your answer in simplified form. **[5]**

**6** The curves $C_1$ and $C_2$ have equations

$$y = \ln(4x-7)+18 \quad \text{and} \quad y = a(x^2+b)^{\frac{1}{2}}$$

respectively, where $a$ and $b$ are positive constants. The point $P$ lies on both curves and has $x$-coordinate 2. It is given that the gradient of $C_1$ at $P$ is equal to the gradient of $C_2$ at $P$. Find the values of $a$ and $b$. **[8]**

**7** **(i)** By sketching the curves $y = x(2x+5)$ and $y = \cos^{-1}x$ (where $y$ is in radians) in a single diagram, show that the equation $x(2x+5) = \cos^{-1}x$ has exactly one real root. **[3]**

**(ii)** Use the iterative formula

$$x_{n+1} = \frac{\cos^{-1}x_n}{2x_n+5} \text{ with } x_1 = 0.25$$

to find the root correct to 3 significant figures. Show the result of each iteration correct to at least 4 significant figures. **[4]**

**(iii)** Two new curves are obtained by transforming each of the curves $y = x(2x+5)$ and $y = \cos^{-1}x$ by the pair of transformations:

reflection in the $x$-axis followed by reflection in the $y$-axis.

State an equation of each of the new curves and determine the coordinates of their point of intersection, giving each coordinate correct to 3 significant figures. **[4]**

**8** The functions f and g are defined for all real values of $x$ by

$$\mathrm{f}(x) = |2x+a|+3a \quad \text{and} \quad \mathrm{g}(x) = 5x-4a,$$

where $a$ is a positive constant.

**(i)** State the range of f and the range of g. **[2]**

**(ii)** State why f has no inverse, and find an expression for $\mathrm{g}^{-1}(x)$. **[3]**

**(iii)** Solve for $x$ the equation $\mathrm{gf}(x) = 31a$. **[5]**

**9** **(i)** Show that $\sin 2\theta(\tan\theta+\cot\theta) \equiv 2$. **[4]**

**(ii)** Hence

**(a)** find the exact value of $\tan\frac{1}{12}\pi+\tan\frac{1}{8}\pi+\cot\frac{1}{12}\pi+\cot\frac{1}{8}\pi$, **[3]**

**(b)** solve the equation $\sin 4\theta(\tan\theta+\cot\theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$, **[3]**

**(c)** express $(1-\cos 2\theta)^2\left(\tan\frac{1}{2}\theta+\cot\frac{1}{2}\theta\right)^3$ in terms of $\sin\theta$. **[2]**

**END OF QUESTION PAPER**