OCR
Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\cot x = 9\,\mathrm{cosec}\,x$$

to find y in terms of x subject to the condition $y = \pi$ when $x = \frac{1}{6}\pi$. **[8]**

2 The group G consists of the set $\{1, 5, 7, 11\}$ combined under multiplication modulo 12.

(i) Draw the group table for G. **[2]**

The group H consists of the set $\{1, 3, 5, 7\}$ combined under multiplication modulo 8.

(ii) Determine whether G and H are isomorphic. **[3]**

3 Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 25\sin x.$$ **[8]**

4 A plane Π_1 passes through the points $(1, 2, -1)$, $(2, -3, 1)$ and $(-1, 0, 2)$.

(i) Show that the plane Π_1 has equation $11x + 7y + 12z = 13$. **[4]**

The plane Π_2 has equation $3x + y + z = 4$.

(ii) Find a vector equation of the line of intersection of Π_1 and Π_2. **[4]**

(iii) Find the acute angle between Π_1 and Π_2. **[2]**

5 In an Argand diagram the points O, A and B are represented by the complex numbers 0, z and $2\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}z$ respectively, where z is a complex number with modulus 5.

(i) Calculate the exact area of the triangle OAB. **[3]**

The numbers $-1 + \mathrm{i}$ and $3 + 3\mathrm{i}$ are represented by the points P and Q respectively. The complex number w is represented by the point R, such that $PQ = PR$ and angle $QPR = \frac{1}{4}\pi$.

(ii) Sketch an Argand diagram showing P, Q and the two possible positions of R. Calculate the possible values of w, giving your answers in the form $a + b\mathrm{i}$. **[5]**

6 The plane Π and the line l have equations

$$\mathbf{r}.\begin{pmatrix}2\\-3\\5\end{pmatrix} = 7 \text{ and } \mathbf{r} = \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$$

respectively. The point A has coordinates $(1, 2, -4)$.

(i) Find the shortest distance from the point A to the plane Π. **[3]**

(ii) Find the acute angle between Π and l. **[3]**

(iii) Find the point where the line parallel to l passing through A intersects the plane Π. **[4]**

7 **(i)** By expressing $\cos\theta$ in terms of $e^{i\theta}$ show that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$ **[4]**

(ii) Hence solve, for $0 \leqslant \theta \leqslant \pi$,

$$\cos 6\theta + 6\cos 4\theta + 2\cos 2\theta = 3.$$ **[5]**

8 A group G has the elements $\begin{pmatrix}a & 0\\0 & b\end{pmatrix}$ where $a, b \in \{1, -1, \mathrm{i}, -\mathrm{i}\}$. The group operation is matrix multiplication. The subset H consists of the matrices with $a = 1$.

(i) State the order of G. **[1]**

(ii) Show that H is a subgroup of G. **[3]**

K is a proper subgroup of G such that H is a proper subgroup of K.

(iii) Show that K must have order 8. **[4]**

(iv) Show that there is only one such subgroup K and identify its elements. **[6]**

END OF QUESTION PAPER