

# OCR

Oxford Cambridge and RSA

## Friday 16 June 2017 – Afternoon

### A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4727/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Solve the differential equation

$$\frac{dy}{dx} + y \cot x = 9 \operatorname{cosec} x$$

to find  $y$  in terms of  $x$  subject to the condition  $y = \pi$  when  $x = \frac{1}{6}\pi$ . [8]

- 2 The group  $G$  consists of the set  $\{1, 5, 7, 11\}$  combined under multiplication modulo 12.

(i) Draw the group table for  $G$ . [2]

The group  $H$  consists of the set  $\{1, 3, 5, 7\}$  combined under multiplication modulo 8.

(ii) Determine whether  $G$  and  $H$  are isomorphic. [3]

- 3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 25 \sin x. \quad [8]$$

- 4 A plane  $\Pi_1$  passes through the points  $(1, 2, -1)$ ,  $(2, -3, 1)$  and  $(-1, 0, 2)$ .

(i) Show that the plane  $\Pi_1$  has equation  $11x + 7y + 12z = 13$ . [4]

The plane  $\Pi_2$  has equation  $3x + y + z = 4$ .

(ii) Find a vector equation of the line of intersection of  $\Pi_1$  and  $\Pi_2$ . [4]

(iii) Find the acute angle between  $\Pi_1$  and  $\Pi_2$ . [2]

- 5 In an Argand diagram the points  $O$ ,  $A$  and  $B$  are represented by the complex numbers  $0$ ,  $z$  and  $2e^{\frac{1}{3}\pi i}z$  respectively, where  $z$  is a complex number with modulus 5.

(i) Calculate the exact area of the triangle  $OAB$ . [3]

The numbers  $-1 + i$  and  $3 + 3i$  are represented by the points  $P$  and  $Q$  respectively. The complex number  $w$  is represented by the point  $R$ , such that  $PQ = PR$  and angle  $QPR = \frac{1}{4}\pi$ .

(ii) Sketch an Argand diagram showing  $P$ ,  $Q$  and the two possible positions of  $R$ . Calculate the possible values of  $w$ , giving your answers in the form  $a + bi$ . [5]

6 The plane  $\Pi$  and the line  $l$  have equations

$$\mathbf{r} \cdot \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = 7 \text{ and } \mathbf{r} = \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

respectively. The point  $A$  has coordinates  $(1, 2, -4)$ .

(i) Find the shortest distance from the point  $A$  to the plane  $\Pi$ . [3]

(ii) Find the acute angle between  $\Pi$  and  $l$ . [3]

(iii) Find the point where the line parallel to  $l$  passing through  $A$  intersects the plane  $\Pi$ . [4]

7 (i) By expressing  $\cos \theta$  in terms of  $e^{i\theta}$  show that

$$\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10). \quad [4]$$

(ii) Hence solve, for  $0 \leq \theta \leq \pi$ ,

$$\cos 6\theta + 6 \cos 4\theta + 2 \cos 2\theta = 3. \quad [5]$$

8 A group  $G$  has the elements  $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$  where  $a, b \in \{1, -1, i, -i\}$ . The group operation is matrix multiplication. The subset  $H$  consists of the matrices with  $a = 1$ .

(i) State the order of  $G$ . [1]

(ii) Show that  $H$  is a subgroup of  $G$ . [3]

$K$  is a proper subgroup of  $G$  such that  $H$  is a proper subgroup of  $K$ .

(iii) Show that  $K$  must have order 8. [4]

(iv) Show that there is only one such subgroup  $K$  and identify its elements. [6]

**END OF QUESTION PAPER**

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