

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Use an algebraic method to find the square roots of $-77 - 36i$. [6]

2 P, Q and T are three transformations in 2-D.

P is a reflection in the x -axis. **A** is the matrix that represents P.

(a) Write down the matrix **A**. [1]

Q is a shear in which the y -axis is invariant and the point $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is transformed to the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. **B** is the matrix that represents Q.

(b) Find the matrix **B**. [2]

T is P followed by Q. **C** is the matrix that represents T.

(c) Determine the matrix **C**. [2]

L is the line whose equation is $y = x$.

(d) Explain whether or not L is a line of invariant points under T . [2]

An object parallelogram, M , is transformed under T to an image parallelogram, N .

(e) Explain what the value of the determinant of **C** means about

- the area of N compared to the area of M ,
- the orientation of N compared to the orientation of M .

[3]

3 In this question you must show detailed reasoning.

The complex number $7 - 4i$ is denoted by z .

(a) Giving your answers in the form $a + bi$, where a and b are rational numbers, find the following.

(i) $3z - 4z^*$ [2]

(ii) $(z + 1 - 3i)^2$ [2]

(iii) $\frac{z+1}{z-1}$ [2]

(b) Express z in modulus-argument form giving the modulus exactly and the argument correct to 3 significant figures. [3]

(c) The complex number ω is such that $z\omega = \sqrt{585}(\cos(0.5) + i\sin(0.5))$.

Find the following.

- $|\omega|$
- $\arg(\omega)$, giving your answer correct to 3 significant figures [3]

4 You are given the system of equations

$$a^2x - 2y = 1$$

$$x + b^2y = 3$$

where a and b are real numbers.

(a) Use a matrix method to find x and y in terms of a and b . [4]

(b) Explain why the method used in part (a) works for all values of a and b . [2]

5 In this question you must show detailed reasoning.

The cubic equation $5x^3 + 3x^2 - 4x + 7 = 0$ has roots α , β and γ .

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. [7]

6 Prove that $n! > 2^{2n}$ for all integers $n \geq 9$. [5]

7 The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where a is a constant.

(a) Find a vector, \mathbf{b} , which is perpendicular to both lines. [2]

(b) Show that $\mathbf{b} \cdot \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b} \cdot \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$. [2]

(c) Hence, or otherwise, find the value of a . [2]

8 Two loci, C_1 and C_2 , are defined by

$$C_1 = \{z: |z| = |z - 4d^2 - 36|\}$$

$$C_2 = \left\{z: \arg(z - 12d - 3i) = \frac{1}{4}\pi\right\}$$

where d is a real number.

(a) Find, in terms of d , the complex number which is represented on an Argand diagram by the point of intersection of C_1 and C_2 .

[You may assume that $C_1 \cap C_2 \neq \emptyset$.] [6]

(b) Explain why the solution found in part (a) is not valid when $d = 3$. [2]

END OF QUESTION PAPER

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