Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Use an algebraic method to find the square roots of $-77 - 36\text{i}$. **[6]**

2 P, Q and T are three transformations in 2-D.

P is a reflection in the x-axis. **A** is the matrix that represents P.

(a) Write down the matrix **A**. **[1]**

Q is a shear in which the y-axis is invariant and the point $\begin{pmatrix}1\\0\end{pmatrix}$ is transformed to the point $\begin{pmatrix}1\\2\end{pmatrix}$. **B** is the matrix that represents Q.

(b) Find the matrix **B**. **[2]**

T is P followed by Q. **C** is the matrix that represents T.

(c) Determine the matrix **C**. **[2]**

L is the line whose equation is $y = x$.

(d) Explain whether or not L is a line of invariant points under T. **[2]**

An object parallelogram, M, is transformed under T to an image parallelogram, N.

(e) Explain what the value of the determinant of **C** means about

- the area of N compared to the area of M,
- the orientation of N compared to the orientation of M. **[3]**

3 **In this question you must show detailed reasoning.**

The complex number $7 - 4\text{i}$ is denoted by z.

(a) Giving your answers in the form $a + b\text{i}$, where a and b are rational numbers, find the following.

(i) $3z - 4z^*$ **[2]**

(ii) $(z+1-3\text{i})^2$ **[2]**

(iii) $\dfrac{z+1}{z-1}$ **[2]**

(b) Express z in modulus-argument form giving the modulus exactly and the argument correct to 3 significant figures. **[3]**

(c) The complex number ω is such that $z\omega = \sqrt{585}(\cos(0.5) + \text{i}\sin(0.5))$.

Find the following.

- $|\omega|$
- $\arg(\omega)$, giving your answer correct to 3 significant figures **[3]**

4 You are given the system of equations

$$a^2x - 2y = 1$$
$$x + b^2y = 3$$

where a and b are real numbers.

(a) Use a matrix method to find x and y in terms of a and b. **[4]**

(b) Explain why the method used in part **(a)** works for all values of a and b. **[2]**

5 **In this question you must show detailed reasoning.**

The cubic equation $5x^3 + 3x^2 - 4x + 7 = 0$ has roots α, β and γ.

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. **[7]**

6 Prove that $n! > 2^{2n}$ for all integers $n \geqslant 9$. **[5]**

7 The equations of two **intersecting** lines are

$$\mathbf{r} = \begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$$

where a is a constant.

(a) Find a vector, **b**, which is perpendicular to both lines. **[2]**

(b) Show that $\mathbf{b}.\begin{pmatrix} -12 \\ a \\ -1 \end{pmatrix} = \mathbf{b}.\begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$. **[2]**

(c) Hence, or otherwise, find the value of a. **[2]**

8 Two loci, C_1 and C_2, are defined by

$C_1 = \left\{z : |z| = |z - 4d^2 - 36|\right\}$

$C_2 = \left\{z : \arg(z - 12d - 3\mathrm{i}) = \frac{1}{4}\pi\right\}$

where d is a real number.

(a) Find, in terms of d, the complex number which is represented on an Argand diagram by the point of intersection of C_1 and C_2.

[You may assume that $C_1 \cap C_2 \neq \varnothing$.] **[6]**

(b) Explain why the solution found in part **(a)** is not valid when $d = 3$. **[2]**

END OF QUESTION PAPER