



Oxford Cambridge and RSA

**Monday 4 October 2021 – Afternoon**

**A Level Further Mathematics A**

**Y540/01 Pure Core 1**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

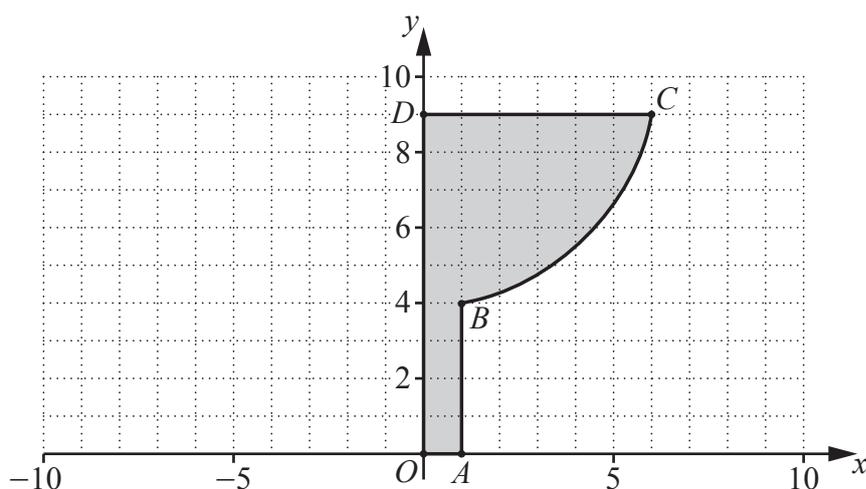
- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Sketch on a single Argand diagram the loci given by
- (i)  $|z - 1 + 2i| = 3$ , [2]
- (ii)  $|z + 1| = |z - 2|$ . [2]
- (b) Indicate, by shading, the region of the Argand diagram for which  $|z - 1 + 2i| \leq 3$  and  $|z + 1| \leq |z - 2|$ . [2]
- 2 You are given that  $f(x) = \tan^{-1}(1 + x)$ .
- (a) (i) Find the value of  $f(0)$ . [1]
- (ii) Determine the value of  $f'(0)$ . [2]
- (iii) Show that  $f''(0) = -\frac{1}{2}$ . [3]
- (b) Hence find the Maclaurin series for  $f(x)$  up to and including the term in  $x^2$ . [2]
- 3 A function  $f(z)$  is defined on all complex numbers  $z$  by  $f(z) = z^3 - 3z^2 + kz - 5$  where  $k$  is a real constant. The roots of the equation  $f(z) = 0$  are  $\alpha$ ,  $\beta$  and  $\gamma$ . You are given that  $\alpha^2 + \beta^2 + \gamma^2 = -5$ .
- (a) Explain why  $f(z) = 0$  has only one real root. [3]
- (b) Find the value of  $k$ . [3]
- (c) Find a cubic equation with integer coefficients that has roots  $\frac{1}{\alpha}$ ,  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$ . [2]
- 4 Points  $A$ ,  $B$  and  $C$  have coordinates  $(4, 2, 0)$ ,  $(1, 5, 3)$  and  $(1, 4, -2)$  respectively. The line  $l$  passes through  $A$  and  $B$ .
- (a) Find a cartesian equation for  $l$ . [3]
- $M$  is the point on  $l$  that is closest to  $C$ .
- (b) Find the coordinates of  $M$ . [4]
- (c) Find the exact area of the triangle  $ABC$ . [4]

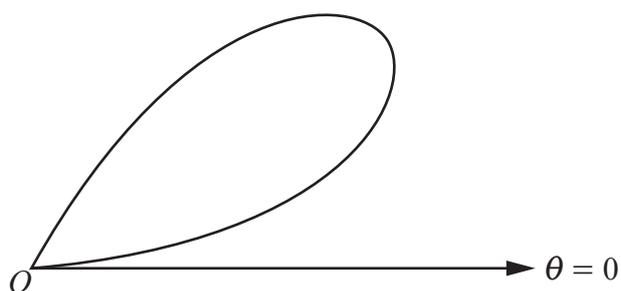
- 5 Use de Moivre's theorem to find the constants  $A$ ,  $B$  and  $C$  in the identity  $\sin^5 \theta \equiv A \sin \theta + B \sin 3\theta + C \sin 5\theta$ . [4]

- 6  $O$  is the origin of a coordinate system whose units are cm. The points  $A$ ,  $B$ ,  $C$  and  $D$  have coordinates  $(1, 0)$ ,  $(1, 4)$ ,  $(6, 9)$  and  $(0, 9)$  respectively. The arc  $BC$  is part of the curve with equation  $x^2 + (y - 10)^2 = 37$ . The closed shape  $OABCD$  is formed, in turn, from the line segments  $OA$  and  $AB$ , the arc  $BC$  and the line segments  $CD$  and  $DO$  (see diagram). A funnel can be modelled by rotating  $OABCD$  by  $2\pi$  radians about the  $y$ -axis.



- Find the volume of the funnel according to the model. [3]

- 7 The diagram below shows the curve with polar equation  $r = \sin 3\theta$  for  $0 \leq \theta \leq \frac{1}{3}\pi$ .



- (a) Find the values of  $\theta$  at the pole. [1]
- (b) Find the polar coordinates of the point on the curve where  $r$  takes its maximum value. [2]
- (c) **In this question you must show detailed reasoning.**  
Find the exact area enclosed by the curve. [4]
- (d) Given that  $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ , find a cartesian equation for the curve. [2]

8 You are given that  $f(x) = 4 \sinh x + 3 \cosh x$ .

(a) Show that the curve  $y = f(x)$  has no turning points. [3]

(b) Determine the exact solution of the equation  $f(x) = 5$ . [5]

9 You are given that the matrix  $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$  represents a transformation T.

(a) You are given that the line with equation  $y = kx$  is invariant under T.

Determine the value of  $k$ . [4]

(b) Determine whether the line with equation  $y = kx$  in part (a) is a line of invariant points under T. [1]

10 Using an algebraic method, determine the least value of  $n$  for which  $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} \geq 0.49$ . [8]

- 11 The displacement of a door from its equilibrium (closed) position is measured by the angle,  $\theta$  radians, which the door makes with its closed position. The door can swing either side of the equilibrium position so that  $\theta$  can take positive and negative values. The door is released from rest from an open position at time  $t = 0$ .

A proposed differential equation to model the motion of the door for  $t \geq 0$  is

$$\frac{d^2\theta}{dt^2} + \lambda \frac{d\theta}{dt} + 3\theta = 0 \text{ where } \lambda \text{ is a constant and } \lambda \geq 0.$$

- (a) (i) According to the model, for what value of  $\lambda$  will the motion of the door be simple harmonic? [1]
- (ii) Explain briefly why modelling the motion of the door as simple harmonic is unlikely to be realistic. [1]
- (b) Find the range of values of  $\lambda$  for which the model predicts that the door will never pass through the equilibrium position. [2]
- (c) Sketch a possible graph of  $\theta$  against  $t$  when  $\lambda$  lies **outside** the range found in part (b) but the motion is not simple harmonic. [1]

**END OF QUESTION PAPER**

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