



Oxford Cambridge and RSA

Monday 4 October 2021 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

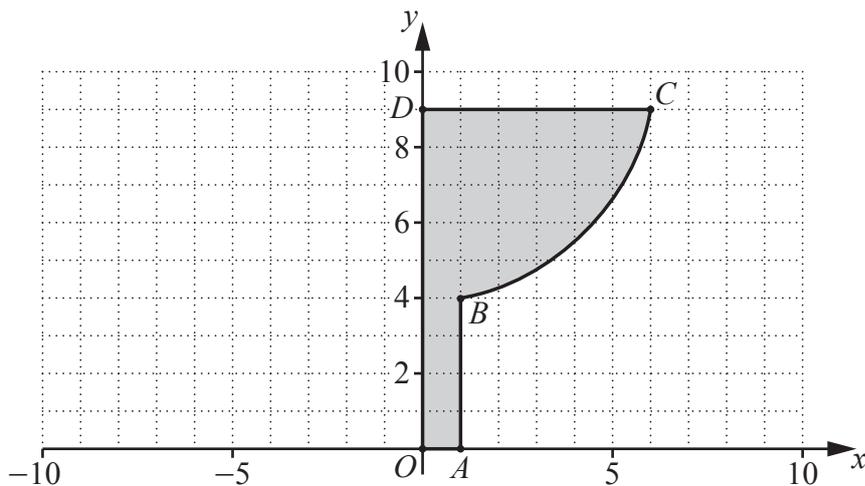
- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Sketch on a single Argand diagram the loci given by
- (i) $|z - 1 + 2i| = 3$, [2]
- (ii) $|z + 1| = |z - 2|$. [2]
- (b) Indicate, by shading, the region of the Argand diagram for which $|z - 1 + 2i| \leq 3$ and $|z + 1| \leq |z - 2|$. [2]
- 2 You are given that $f(x) = \tan^{-1}(1 + x)$.
- (a) (i) Find the value of $f(0)$. [1]
- (ii) Determine the value of $f'(0)$. [2]
- (iii) Show that $f''(0) = -\frac{1}{2}$. [3]
- (b) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2 . [2]
- 3 A function $f(z)$ is defined on all complex numbers z by $f(z) = z^3 - 3z^2 + kz - 5$ where k is a real constant. The roots of the equation $f(z) = 0$ are α , β and γ . You are given that $\alpha^2 + \beta^2 + \gamma^2 = -5$.
- (a) Explain why $f(z) = 0$ has only one real root. [3]
- (b) Find the value of k . [3]
- (c) Find a cubic equation with integer coefficients that has roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$. [2]
- 4 Points A , B and C have coordinates $(4, 2, 0)$, $(1, 5, 3)$ and $(1, 4, -2)$ respectively. The line l passes through A and B .
- (a) Find a cartesian equation for l . [3]
- M is the point on l that is closest to C .
- (b) Find the coordinates of M . [4]
- (c) Find the exact area of the triangle ABC . [4]

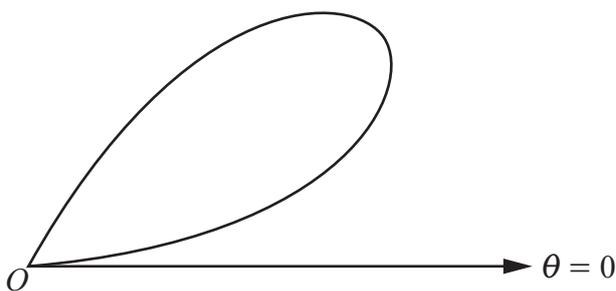
- 5 Use de Moivre's theorem to find the constants A , B and C in the identity $\sin^5 \theta \equiv A \sin \theta + B \sin 3\theta + C \sin 5\theta$. [4]

- 6 O is the origin of a coordinate system whose units are cm. The points A , B , C and D have coordinates $(1, 0)$, $(1, 4)$, $(6, 9)$ and $(0, 9)$ respectively. The arc BC is part of the curve with equation $x^2 + (y - 10)^2 = 37$. The closed shape $OABCD$ is formed, in turn, from the line segments OA and AB , the arc BC and the line segments CD and DO (see diagram). A funnel can be modelled by rotating $OABCD$ by 2π radians about the y -axis.



- Find the volume of the funnel according to the model. [3]

- 7 The diagram below shows the curve with polar equation $r = \sin 3\theta$ for $0 \leq \theta \leq \frac{1}{3}\pi$.



- (a) Find the values of θ at the pole. [1]
- (b) Find the polar coordinates of the point on the curve where r takes its maximum value. [2]
- (c) **In this question you must show detailed reasoning.**
Find the exact area enclosed by the curve. [4]
- (d) Given that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$, find a cartesian equation for the curve. [2]

8 You are given that $f(x) = 4 \sinh x + 3 \cosh x$.

(a) Show that the curve $y = f(x)$ has no turning points. [3]

(b) Determine the exact solution of the equation $f(x) = 5$. [5]

9 You are given that the matrix $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation T.

(a) You are given that the line with equation $y = kx$ is invariant under T.

Determine the value of k . [4]

(b) Determine whether the line with equation $y = kx$ in part (a) is a line of invariant points under T. [1]

10 Using an algebraic method, determine the least value of n for which $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} \geq 0.49$. [8]

- 11 The displacement of a door from its equilibrium (closed) position is measured by the angle, θ radians, which the door makes with its closed position. The door can swing either side of the equilibrium position so that θ can take positive and negative values. The door is released from rest from an open position at time $t = 0$.

A proposed differential equation to model the motion of the door for $t \geq 0$ is

$$\frac{d^2\theta}{dt^2} + \lambda \frac{d\theta}{dt} + 3\theta = 0 \text{ where } \lambda \text{ is a constant and } \lambda \geq 0.$$

- (a) (i) According to the model, for what value of λ will the motion of the door be simple harmonic? [1]
- (ii) Explain briefly why modelling the motion of the door as simple harmonic is unlikely to be realistic. [1]
- (b) Find the range of values of λ for which the model predicts that the door will never pass through the equilibrium position. [2]
- (c) Sketch a possible graph of θ against t when λ lies **outside** the range found in part (b) but the motion is not simple harmonic. [1]

END OF QUESTION PAPER

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