



Oxford Cambridge and RSA

**Monday 4 October 2021 – Afternoon**

**A Level Further Mathematics B (MEI)**

**Y420/01 Core Pure**

**Time allowed: 2 hours 40 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **144**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

## Section A (31 marks)

Answer **all** the questions.

1 (a) Express  $\frac{1}{(2r-1)(2r+1)}$  in partial fractions. [3]

(b) Hence find  $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)}$ , expressing the result as a single fraction. [4]

## 2 In this question you must show detailed reasoning.

Find the gradient of the curve  $y = 6 \arcsin(2x)$  at the point with  $x$ -coordinate  $\frac{1}{4}$ . Express the result in the form  $m\sqrt{n}$ , where  $m$  and  $n$  are integers. [4]

## 3 In this question you must show detailed reasoning.

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = -2 + 2i$  and  $z_2 = 2\left(\cos\frac{1}{6}\pi + i \sin\frac{1}{6}\pi\right)$ .

(a) Find the modulus and argument of  $z_1$ . [2]

(b) Hence express  $\frac{z_1}{z_2}$  in exact modulus-argument form. [4]

## 4 In this question you must show detailed reasoning.

Determine the mean value of  $\frac{1}{1+4x^2}$  between  $x = -1$  and  $x = 1$ . Give your answer to 3 significant figures. [4]

5 (a) Use a Maclaurin series to find a quadratic approximation for  $\ln(1+2x)$ . [1]

(b) Find the percentage error in using the approximation in part (a) to calculate  $\ln(1.2)$ . [3]

(c) Jane uses the Maclaurin series in part (a) to try to calculate an approximation for  $\ln 3$ .

Explain whether her method is valid. [2]

6 Given that  $y = mx$  is an invariant line of the transformation with matrix  $\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$ , determine the possible values of  $m$ . [4]

## Section B (113 marks)

Answer **all** the questions.

- 7 Prove that  $\sum_{r=1}^n \frac{r}{2^{r-1}} = 4 - \frac{n+2}{2^{n-1}}$  for all  $n \geq 1$ . [6]
- 8 The equation  $4x^4 - 4x^3 + px^2 + qx - 9 = 0$ , where  $p$  and  $q$  are constants, has roots  $\alpha$ ,  $-\alpha$ ,  $\beta$  and  $\frac{1}{\beta}$ .
- (a) Determine the exact roots of the equation. [5]
- (b) Determine the values of  $p$  and  $q$ . [4]
- 9 The transformation  $T$  of the plane has associated matrix  $\mathbf{M}$ , where  $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}$ .
- (a) On the grid in the Printed Answer Booklet, plot the image  $OA'B'C'$  of the unit square  $OABC$  under the transformation  $T$ . [2]
- (b) (i) Calculate the value of  $\det \mathbf{M}$ . [1]
- (ii) Explain the significance of the value of  $\det \mathbf{M}$  in relation to the image  $OA'B'C'$ . [2]
- (c)  $T$  is equivalent to a sequence of two transformations of the plane.
- (i) Specify fully **two** transformations equivalent to  $T$ . [3]
- (ii) Use matrices to verify your answer. [3]
- 10 (a) Show on an Argand diagram the points representing the three cube roots of unity. [2]
- (b) (i) Find the exact roots of the equation  $z^3 - 1 = \sqrt{3}i$ , expressing them in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta < \pi$ . [5]
- (ii) The points representing the cube roots of unity form a triangle  $\Delta_1$ . The points representing the roots of the equation  $z^3 - 1 = \sqrt{3}i$  form a triangle  $\Delta_2$ .
- State a sequence of two transformations that maps  $\Delta_1$  onto  $\Delta_2$ . [2]
- (iii) The three roots in part (b)(i) are  $z_1$ ,  $z_2$  and  $z_3$ .
- By simplifying  $z_1 + z_2 + z_3$ , verify that the sum of these roots is zero. [2]
- (iv) Hence show that  $\sin 20^\circ + \sin 140^\circ = \sin 100^\circ$ . [2]

- 11 (a) Given that  $\mathbf{u} = \lambda\mathbf{i} + \mathbf{j} - 3\mathbf{k}$  and  $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ , find the following, giving your answers in terms of  $\lambda$ .

(i)  $\mathbf{u} \cdot \mathbf{v}$  [1]

(ii)  $\mathbf{u} \times \mathbf{v}$  [2]

- (b) Hence determine

(i) the acute angle between the planes  $2x + y - 3z = 10$  and  $x + 2y - 2z = 10$ , [3]

(ii) the shortest distance between the lines  $\frac{x-3}{3} = \frac{y}{1} = \frac{z-2}{-3}$  and  $\frac{x}{1} = \frac{y-4}{2} = \frac{z+2}{-2}$ , giving your answer as a multiple of  $\sqrt{2}$ . [3]

- 12 Fig. 12 shows a rhombus OACB in an Argand diagram. The points A and B represent the complex numbers  $z$  and  $w$  respectively.

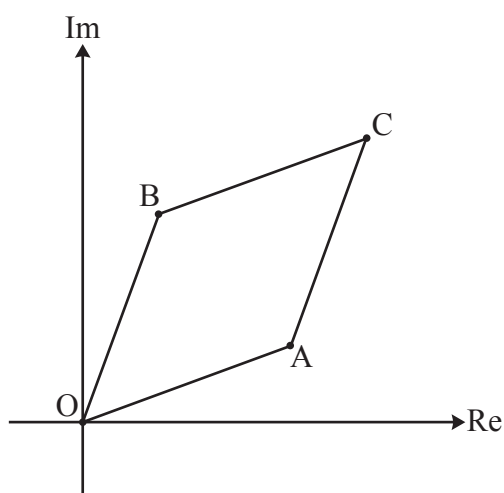


Fig. 12

Prove that  $\arg(z + w) = \frac{1}{2}(\arg z + \arg w)$ .

[A copy of Fig. 12 is provided in the Printed Answer Booklet.] [4]

- 13 Find the general solution of the differential equation  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 2e^x$ . [7]

- 14 A curve has polar equation  $r = a(\cos \theta + 2 \sin \theta)$ , where  $a$  is a positive constant and  $0 \leq \theta \leq \pi$ .
- (a) Determine the polar coordinates of the point on the curve which is furthest from the pole. [7]
- (b) (i) Show that the curve is a circle whose radius should be specified. [6]
- (ii) Write down the polar coordinates of the centre of the circle. [1]
- 15 The equations of three planes are
- $$-4x + ky + 7z = 4,$$
- $$x - 2y + 5z = l,$$
- $$2x + 3y + z = 2.$$
- Given that the planes form a sheaf, determine the values of  $k$  and  $l$ . [6]
- 16 (a) Show using exponentials that  $\cosh 2u = 1 + 2 \sinh^2 u$ . [4]
- (b) Show that  $\int_0^2 \frac{x^2}{\sqrt{4+x^2}} dx = 2\sqrt{2} - 2 \ln(1 + \sqrt{2})$ . [10]
- 17 In a chemical process, a vessel contains 1 litre of pure water. A liquid chemical is then passed into the top of the vessel at a constant rate of  $a$  litres per minute and thoroughly mixed with the water. At the same time, the resulting mixture is drawn from the bottom of the vessel at a constant rate of  $b$  litres per minute. You may assume that the chemical mixes instantly and uniformly with the water. After  $t$  minutes, the mixture in the vessel contains  $x$  litres of the chemical.
- (a) (i) Show that the proportion of chemical present in the vessel after  $t$  minutes is  $\frac{x}{1 + (a-b)t}$ . [2]
- (ii) Hence show that  $\frac{dx}{dt} + \frac{bx}{1 + (a-b)t} = a$ . [2]
- (b) First, consider the case where  $b = a$ .
- (i) Solve the differential equation to find  $x$  in terms of  $a$  and  $t$ . [4]
- (ii) Given that after 1 minute the vessel contains equal amounts of water and chemical, find the rate of inflow of chemical. [2]
- (c) Now consider the case where  $b = 2a$ .
- (i) Explain why the differential equation in part (a)(ii) is now invalid for  $t \geq \frac{1}{a}$ . [1]
- (ii) Find the maximum amount of chemical in the vessel. [9]

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