GCE

Further Mathematics B (MEI)

Y421/01: Mechanics major

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Question			Answer	Marks	AOs	Guidance	
1			$J = 0.25(4.2-(-5))$	M1	3.3	Use of Impulse = change in momentum	
			$J = 0.02F$	M1	3.3	Use of Impulse = Ft	
			$F = \frac{2.3}{0.02} = 115$ (N)	A1	1.1	cao	
				[3]			
2			$10m\bar{x} = 1(3m) + 2(5m) + 5(2m)$	M1	1.1	Use of $\bar{x}\sum m_i = \sum x_i m_i$	
			$\bar{x} = 2.3$	A1	1.1	cao	
			$10m\bar{y} = 2(3m) + (-2)(5m) + 3(2m)$	M1	1.1	Use of $\bar{y}\sum m_i = \sum y_i m_i$	
			$\bar{y} = 0.2$	A1	1.1	cao	
				[4]			
3	(a)		$T = 4g$	B1	1.1	Resolve vertically (possibly implied by subsequent working)	
			$\frac{\lambda(0.02)}{0.3} = 4g$	M1	3.3	Use of Hooke's law with their $4g$	
			$\lambda = 588$(N)	A1	1.1	cao oe e.g. $60g$	
				[3]			
3	(b)		e.g. spring stretched beyond its elastic limit e.g. Hooke's law no longer applies	B1	2.2b	oe (any correct equivalent statement for why the extension of the spring may not be 0.1 m)	
				[1]			

Question			Answer	Marks	AOs	Guidance	
4			**DR**				
			$A=\int_0^1\left(4-x^2\right)-3\sqrt{x}\,\mathrm{d}x=\left[4x-\frac{1}{3}x^3-2x^{\frac{3}{2}}\right]_0^1$	M1*	2.1	Correct integral expression for the area and attempt to integrate (at least two terms correct)	Ignore limits for first two M marks
			$A=4-\frac{1}{3}-2=\frac{5}{3}$	A1	1.1		**SC M1 A0** if correct integral and value seen but with no intermediate working
			$A\bar{x}=\int_0^1 4x-x^3-3x^{\frac{3}{2}}\,\mathrm{d}x=\left[2x^2-\frac{1}{4}x^4-\frac{6}{5}x^{\frac{5}{2}}\right]_0^1$	M1*	1.1	Correct integral expression for $A\bar{x}$ and attempt to integrate (at least two terms correct)	
			$A\bar{x}=2-\frac{1}{4}-\frac{6}{5}=\frac{11}{20}$	A1	1.1		**SC M1 A0** if correct integral and value seen but with no intermediate working
			$\bar{x}=\frac{A\bar{x}}{A}=\frac{11/20}{5/3}$	M1dep*	1.1	Correct use of $\bar{x}=\frac{A\bar{x}}{A}$	Dependent on both previous M marks
			$=\frac{33}{100}$	A1	2.2a	oe	This mark can be awarded even if the two previous **A** marks were not awarded
				[6]			

Question			Answer	Marks	AOs	Guidance	
5			Let w_A and w_B be the horizontal components of the velocity of A and B after collision				
			$w_B = 2.5$	B1	1.2		
				M1	3.3	Use of conservation of linear momentum (parallel to the line of centres) – correct number of terms	
			$2(6) + 4(0) = 2w_A + 4(2.5)$	A1	1.1	Allow with w_B instead of 2.5	For reference: $w_A = 1$
				M1	3.3	Use of Newton's experimental law (parallel to the line of centres) – correct number of terms	
			$w_A - 2.5 = -e(6 - 0)$	A1	1.1	Use of NEL must be consistent with CLM – allow with w_B instead of 2.5 and possibly their w_A	
			$e = 0.25$	A1 [6]	1.1		

Question			Answer	Marks	AOs	Guidance	
6	(a)		$[F] = \mathrm{MLT}^{-2}$	B1 [1]	1.2		
6	(b)		$[G] = \mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$	B1 [1]		May use $F = \frac{Gm_1m_2}{d^2}$ to obtain the dimensions of G	
6	(c)		$G = \left(6.67 \times 10^{-11}\right) \times 0.454 \times \frac{1}{(0.305)^3}$ $G = 1.07 \times 10^{-9}$ ($\mathrm{lb}^{-1}\ \mathrm{ft}^3\ \mathrm{s}^{-2}$)	M1 A1 [2]	3.1a 1.1	**SC B1** for $G = \left(6.67 \times 10^{-11}\right) \times \frac{1}{0.454} \times (0.305)^3$ $= 4.17 \times 10^{-12}$ awrt 1.07×10^{-9}	
6	(d)		$\left[\frac{kGM}{r}\right] = \frac{\left(\mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}\right)\mathrm{M}}{\mathrm{L}}$ $\left[\sqrt{\frac{kGM}{r}}\right] = \mathrm{LT}^{-1}$ $[v] = \mathrm{LT}^{-1}$ so the formula is dimensionally consistent	M1 A1 A1 [3]	2.1 1.1 2.2a	Attempt to calculate the dimension of either $\frac{kGM}{r}$ or its square root with $[k] = 1$ and two other terms correct Or $\left[\frac{kGM}{r}\right] = \mathrm{L}^2\mathrm{T}^{-2}$ Or allow showing consistency for $v^2 = \frac{kGM}{r}$	

Question			Answer	Marks	AOs	Guidance	
6	(e)		$11186=\sqrt{\dfrac{k\left(6.67\times10^{-11}\right)\left(5.97\times10^{24}\right)}{6\,371000}}$	M1	3.4		
			$k\approx 2$	A1	1.1		$k=2.0019677\ldots$
			$v=\sqrt{\dfrac{2\left(6.67\times10^{-11}\right)\left(6.39\times10^{23}\right)}{3\,389\,500}}$	M1	1.1		
			$v=5015\ (\text{m s}^{-1})$	A1	2.2a	Allow to 3 sf or better (allow 5015 to 5017 inclusive)	If using $k=2.0019677\ldots$ expect to see 5017.346122…
				[4]			
7	(a)		Driving force of engine is $\dfrac{kmg}{v}$	B1	1.1		
			$\dfrac{kmg}{v}-mg=mv\dfrac{dv}{dx}$	M1	3.3	Use of N2L, correct number of terms, allow D (oe) for $\dfrac{kmg}{v}$ and a (oe) for the acceleration	
			$kg-gv=v^2\dfrac{dv}{dx}\Rightarrow v^2\dfrac{dv}{dx}=(k-v)g$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[3]			

Question			Answer	Marks	AOs	Guidance	
7	(b)		$gx = k^2 \ln\left(\frac{k}{k-v}\right) - kv - \frac{1}{2}v^2$				
			$x = 0, v = 0 \Rightarrow g(0) = k^2 \ln\left(\frac{k}{k-0}\right) - k(0) - \frac{1}{2}(0)^2$ so initial conditions are consistent with given equation	B1	1.1		
			$g\frac{dx}{dv} = k^2\left[\frac{1}{\left(\frac{k}{k-v}\right)} k(k-v)^{-2}\right] - k - v$	M1*	2.1	Attempt to differentiate using chain rule	Or equivalent (e.g. solving using separation of variables)
				A1	1.1	cao oe e.g. $g = k^2\left(\frac{k-v}{k}\right)\left(\frac{-k\left(-\frac{dv}{dx}\right)}{(k-v)^2}\right) - k\frac{dv}{dx} - v\frac{dv}{dx}$	
			$g\frac{dx}{dv} = \frac{-kv + v^2 - k^2 + kv + k^2}{(k-v)}$	M1dep*	1.1	Correct method to obtain an expression for $\frac{dx}{dv}$ as a single fraction or as a single fraction with $\frac{dv}{dx}$ e.g. $g = \left(\frac{k^2 - k^2 + kv - kv + v^2}{k-v}\right)\frac{dv}{dx}$	
			$v^2 = g(k-v)\frac{dx}{dv} \Rightarrow v^2\frac{dv}{dx} = (k-v)g$	A1	2.2a	**AG** – sufficient working required as answer given	
				[5]			

Question			Answer	Marks	AOs	Guidance	
7	(c)		Work done by engine is $kmgt$	B1	1.1		
			$kgmt = \frac{1}{2}mV^2 + mgx$	M1*	3.3	Use work-energy principle – correct number of terms	
			$kgt = \frac{1}{2}V^2 + k^2\ln\left(\frac{k}{k-V}\right) - kV - \frac{1}{2}V^2$	M1dep*	3.4	Use given result from **(b)** in work-energy equation to eliminate x	
			$kgt = k^2\ln\left(\frac{k}{k-V}\right) - kV \Rightarrow t = \frac{k}{g}\ln\left(\frac{k}{k-V}\right) - \frac{V}{g}$	A1	2.2a	**AG** – sufficient working required as answer given	
				[4]		**SC** if correctly found by solving $\frac{kmg}{v} - mg = m\frac{dv}{dt}$ this can score 3/4 max.	
8	(a)			B1	1.2	All remaining forces adding on correctly (with arrows to indicate directions) to the figure in the Printed Answer Booklet	
				[1]			
8	(b)			M1*	3.3	Resolve horizontally and vertically (correct number of terms in both equations)	
			$F_D + R_C = W$ $R_D = F_C$	A1	1.1	Where R_C is the normal contact force at C, etc.	
			$F_D = \frac{1}{3}R_D$ and $F_C = \frac{1}{3}R_C$	B1	3.4	Correct use of $F = \mu R$ at C and D	
			$\frac{1}{3}F_C + R_C = W \Rightarrow \frac{1}{9}R_C + R_C = W$	M1dep*	3.4	Combine results to get an equation in R_C only	
			$R_C = \frac{9}{10}W$	A1	1.1		
				[5]			

Question			Answer	Marks	AOs	Guidance	
8	(c)			M1*	3.1b	Taking moments about D (or any other equivalent point) – correct number of terms	
			$(r+h\sin\theta)W+(r+2h\cos\theta)F_C=(r+2h\sin\theta)R_C$	A1	1.1	oe	
			$(r+h\sin\theta)W+(r+2h\cos\theta)\left(\frac{3}{10}W\right)=(r+2h\sin\theta)\left(\frac{9}{10}W\right)$	M1dep*	3.4	Substitute expressions for F_C and R_C	
			$r=h(2\sin\theta-1.5\cos\theta)$	A1	1.1		
			$2h\sin\theta-1.5h\cos\theta>0$	M1	2.3	Setting their expression for $r>0$	
			$4\sin\theta-3\cos\theta>0\Rightarrow\tan\theta>\frac{3}{4}$	A1	2.2a	**AG**	
				[6]			

Question			Answer	Marks	AOs	Guidance	
9	(a)		$\ddot{x} = -g\sin\alpha,\ \ \ddot{y} = -g\cos\alpha$	B1	2.1		
				M1*	3.4	Attempt to integrate (twice) and use of initial conditions	
			$\dot{x} = 5\cos\theta - gt\sin\alpha,\ \ \dot{y} = 5\sin\theta - gt\cos\alpha$	A1	1.1		
			$x = 5t\cos\theta - 0.5gt^2\sin\alpha$ $y = 5t\sin\theta - 0.5gt^2\cos\alpha$	A1	1.1	Or **M1** for use of $s = ut + \frac{1}{2}at^2$ parallel to line of greatest slope and then **A1** for correct expression for x	Similarly **M1 A1** for correct expression for y (following SUVAT perpendicular to slope)
			$y = 0 \Rightarrow t = \ldots$	M1dep*	3.3	Sets $y = 0$ and solve for t	
			$t = \dfrac{10\sin\theta}{g\cos\alpha}$	A1	1.1		
			$x = 5\left(\dfrac{10\sin\theta}{g\cos\alpha}\right)\cos\theta - 0.5g\left(\dfrac{10\sin\theta}{g\cos\alpha}\right)^2\sin\alpha$	M1	3.4	Substitute expression for t into equation for x	Dependent on both previous **M** marks
			$x = \dfrac{50\sin\theta}{g\cos^2\alpha}(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ $\Rightarrow \text{OR} = \dfrac{50\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}$	A1	2.2a	**AG**	
				[8]			

Question		Answer	Marks	AOs	Guidance	
9	(b)	$\sin\theta\cos(\theta+\alpha)=\frac{1}{2}(\sin(2\theta+\alpha)-\sin\alpha)$	M1	1.1	Use of given identity to re-write numerator from **(a)** as a difference of two sines	
		$OR=\frac{25}{g\cos^2\alpha}(\sin(2\theta+\alpha)-\sin\alpha)$	A1	1.1		
		$R_{max}=\frac{25}{g(1-\sin^2\alpha)}(1-\sin\alpha)$	A1	3.1a	Use of correct trig. identity and setting $\sin(2\theta+\alpha)$ equal to 1 – oe e.g. $R_{max}=\frac{25}{g(1+\sin\alpha)}$	R_{max} occurs when $\sin(2\theta+\alpha)=1$
			[3]			
9	(c)	$\frac{25}{g(1+\sin\alpha)}=1.8$ or $\frac{25(1-\sin\alpha)}{g(1-\sin^2\alpha)}=1.8$	M1*	3.4	Setting their expression equal to 1.8	Expression must only contain $\sin\alpha$ terms
		$\frac{25}{g(1+\sin\alpha)}=1.8\Rightarrow\sin\alpha=...$	M1dep*	1.1	Attempting to solve for $\sin\alpha$ or α - for reference $\sin\alpha=\frac{184}{441}$ or $\alpha=24.660053...$ (or 0.430399… in radians)	If solving a 3TQ in sine then must solve using a correct method
		$\theta=45-0.5\alpha$	M1	3.1a	Follow through their α	
		$\theta=32.7$	A1	1.1		32.6699733… or 0.5701986… (in radians)
			[4]			

Question			Answer	Marks	AOs	Guidance	
10	(a)		[At B,] KE = $\frac{1}{2}mu^2$, PE = 0	B1	1.1		Note that the reference level for zero GPE might be taken at C
			[At θ,] KE = $\frac{1}{2}mv^2$, PE = $mga\left(1-\cos\theta\right)$	B1	1.1		
				M1*	3.3	Use of conservation of energy – correct number of terms	
			$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga\left(1-\cos\theta\right)$	A1	1.1	cao	
			$R - mg\cos\theta = \frac{mv^2}{a}$	M1*	3.3	N2L radially with correct number of terms and weight resolved	
			$R - mg\cos\theta = \frac{m}{a}\left(u^2 - 2ga\left(1-\cos\theta\right)\right)$	M1dep*	3.4	Substitute an expression for v^2	
			$R = m\left(3g\cos\theta - 2g + \frac{u^2}{a}\right)$	A1	1.1		
				[7]			

Question			Answer	Marks	AOs	Guidance	
10	(b)		Before collision at C, $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mga$	M1	3.4	Substituting $\theta = \frac{\pi}{2}$ into their conservation of energy equation from **(a)**	
			After collision at C, speed of P is $e\sqrt{u^2 - 2ga}$	A1	1.1		
			$\frac{1}{2}mv_B^{\ 2} = mga + \frac{1}{2}m\left(e\sqrt{u^2 - 2ga}\right)^2$	M1	3.1b	Conservation of energy to find an expression for the speed of P at B	Where v_B is the speed of P at B
			$v_B^{\ 2} = 2ga + e^2\left(u^2 - 2ga\right)$				
			$\frac{1}{2}mv_B^{\ 2} - \frac{1}{2}mv_A^{\ 2} = Fb$	M1	3.1b	Work-energy principle for motion between B and A	
				M1	2.5	Set $v_A \geq 0$ and substitute for v_B^2	
			$m\left(2ga + e^2\left(u^2 - 2ga\right)\right) - 2bF \geq 0$	A1	2.2a	k need not be stated explicitly	
			$Fb \leq mga + \frac{1}{2}me^2u^2 - me^2ga$				
			$\Rightarrow Fb \leq \frac{1}{2}m\left[e^2u^2 + 2\left(1 - e^2\right)ga\right]$, so $k = 2$	[6]			
11	(a)			M1*	3.3	Conservation of linear momentum with correct number of terms	Where v_A is the speed of A after 1st impact and similarly for v_B
			$4V = 4v_A + 3v_B$	A1	1.1	cao	
				M1*	3.3	Newton's experimental law with correct number of terms	
			$v_A - v_B = -eV$	A1	1.1	Must be consistent with CLM	
				M1dep*	1.1	Solve the simultaneous equations to find both speeds	
			$v_A = \frac{V(4-3e)}{7}$ and $v_B = \frac{4V(1+e)}{7}$	A1	1.1		
				[6]			

Question			Answer	Marks	AOs	Guidance	
11	(b)		Let θ be the angle subtended by A in time t For A, $t = \dfrac{r\theta}{\dfrac{V(4-3e)}{7}}$	M1	3.1b	Use of $s = ut$ with their v_A and $s = r\theta$	Where r is the radius of the circular groove
			For B, $t = \dfrac{2\pi r + r\theta}{\dfrac{4V(1+e)}{7}}$	M1	1.1	Use of $s = ut$ with their v_B and $s = 2\pi r + r\theta$	
			$\dfrac{2\pi + \theta}{4V(1+e)} = \dfrac{\theta}{V(4-3e)}$	M1	3.4	Equate expressions for t to form an equation in terms of θ, V and e	
			$\theta = \dfrac{2\pi(4-3e)}{7e}$	A1	2.2a	AG	
				[4]			
			Alternative method **ALT:** $v_B - v_A = \dfrac{4V(1+e)}{7} - \dfrac{V(4-3e)}{7} = eV$	M1*		Difference in speeds calculated	
			Time for B to catch up to A is $\dfrac{2\pi r}{eV}$	M1dep*		Using their eV	Where r is the radius of the circular groove
			$d_A = \dfrac{2\pi r}{eV}\left(\dfrac{V(4-3e)}{7}\right) = \dfrac{2\pi r}{7e}(4-3e)$	M1		Where d_A is the distance travelled by A	
			$\theta = \dfrac{2\pi r(4-3e)}{7er} = \dfrac{2\pi(4-3e)}{7e}$	A1		AG	

Question			Answer	Marks	AOs	Guidance	
11	(c) (i)		$3w_B + 4w_A = \frac{12}{7}V(1+e) + \frac{4}{7}V(4-3e)$	M1*	3.3	CLM correct number of terms using their expressions from **(a)**	Where w_A is the speed of A after the second collision
			$w_B - w_A = -e\left(\frac{4}{7}V(1+e) - \frac{1}{7}V(4-3e)\right)$	M1*	3.3	NEL correct number of terms	
			$3w_B + 4w_A = 4V$ and $w_B - w_A = -e^2V$	A1	1.1	oe	
				M1dep*	1.1	Solve simultaneously for w_B	
			$w_B = \frac{4}{7}V(1-e^2)$	A1	1.1	cao	For reference: $w_A = \frac{1}{7}V(4+3e^2)$
				[5]			
11	(c) (ii)		If the collision is perfectly elastic ($e = 1$) B is brought to rest by the second collision and A is moving with speed V (which is the situation before the first collision)	B1	3.5a	oe correct statement	
				[1]			
12	(a)		$PE = -mg(l+e)$ (while P is at rest)	B1	1.1	Where e is the extension in the string	Taking the horizontal through O as the reference level for zero GPE
			$EPE = \frac{12mge^2}{2l}$	B1	1.1		
			$\frac{6mge^2}{l} - mg(l+e) = 0$	M1*	3.3	Conservation of energy with correct number of terms	
			$6e^2 - el - l^2 = 0$ $(3e+l)(2e-l) = 0$	M1dep*	1.1a	Solving three-term quadratic in e	
			$e = \frac{l}{2} \Rightarrow$ length of string is $\frac{1}{2}l + l = \frac{3}{2}l$	A1	2.2a	**AG**	
				[5]			

Question		Answer	Marks	AOs	Guidance	
12	(b)	$mg - T = m\ddot{x}$	M1	3.3	N2L vertically with correct number of terms	
		$mg - \frac{12mgx}{l} = m\ddot{x}$	M1	3.4	Use of Hooke's law and substitute for T in N2L	
		$\ddot{x} + \frac{12g}{l}x = g$ so $\ddot{x} + \omega^2 x = g$ where $\omega^2 = \frac{12g}{l}$	A1	2.2a	**AG**	
			[3]			
12	(c)	$x = y + \frac{g}{\omega^2} \Rightarrow \ddot{y} + \omega^2 y = 0$	M1	1.1	Use given substitution to form differential equation in y	
		$y = A\cos\omega t + B\sin\omega t$	A1ft	1.2	Correctly solves their differential equation in y	
		$x = A\cos\omega t + B\sin\omega t + \frac{g}{\omega^2}$	A1	1.1	oe e.g. $x = A\cos\omega t + B\sin\omega t + \frac{l}{12}$	
		$t = 0, x = 0 \Rightarrow A = -\frac{g}{\omega^2}$	M1	3.4	Use correct initial conditions in their expression for x	
		$\frac{1}{2}mv_P^2 = mgl$	M1*	3.1b	Use conservation of energy to find speed v_P of P at time $t = 0$	
		$v_P = \sqrt{2gl}$	A1	1.1		
		$t = 0, \dot{x} = \sqrt{2gl} \Rightarrow B = \frac{\sqrt{2gl}}{\omega}$	M1dep*	3.4	Use initial speed in an expression for $\dot{x}$	
		$x = -\frac{g}{\omega^2}\cos\omega t + \frac{\sqrt{2gl}}{\omega}\sin\omega t + \frac{g}{\omega^2}$	A1	1.1	oe e.g. $x = \frac{l}{12}\left(1 - \cos\omega t + \sqrt{24}\sin\omega t\right)$	
		$\frac{l}{12}\left(1 - \cos\omega t + \sqrt{24}\sin\omega t\right) = 0$	M1	3.1b	Sets $x = 0$ and replaces $\omega^2 = \frac{12g}{l}$	Dependent on all previous **M** marks
		$\cos\omega t - \sqrt{24}\sin\omega t = 1$ so $k = 24$	A1	2.2a	k need not be stated explicitly	
			[10]			

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