Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
1. A new diet has been designed. Its designers claim that following the diet for a month will result in a mean weight loss of more than 2 kg. In a trial, a random sample of 10 people followed the new diet for a month. Their weights, in kg, before starting the diet and their weights after following the diet for a month were recorded. The results are given in the table below.

<table>
<thead>
<tr>
<th>Person</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight before diet (kg)</td>
<td>96</td>
<td>110</td>
<td>116</td>
<td>98</td>
<td>121</td>
<td>91</td>
<td>98</td>
<td>106</td>
<td>110</td>
<td>116</td>
</tr>
<tr>
<td>Weight after diet (kg)</td>
<td>91</td>
<td>101</td>
<td>111</td>
<td>96</td>
<td>121</td>
<td>91</td>
<td>90</td>
<td>101</td>
<td>104</td>
<td>110</td>
</tr>
</tbody>
</table>

(a) Using a suitable \( t \)-test, at the 5% level of significance, state whether or not the trial supports the designers’ claim. State your hypotheses and show your working clearly.

(b) State an assumption necessary for the test in part (a).
Question 1 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
2. The weights of piglets at birth, $M$ kg, are normally distributed $N(\mu, \sigma^2)$

A random sample of 9 piglets is taken and their weights at birth, $m$ kg, are recorded. The results are summarised as

$$\sum m = 11.6 \quad \sum m^2 = 15.2$$

Stating your hypotheses clearly, test at the 5% level of significance

(a) whether or not the mean weight of piglets at birth is greater than 1.2 kg,  

(b) whether or not the standard deviation of the weights of piglets at birth is different from 0.3 kg.

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________

___________________________________________________________________________
Question 2 continued
Question 2 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 13 marks)
3. A jar contains a large number of sweets which have either soft centres or hard centres. The jar is thought to contain equal proportions of sweets with soft centres and sweets with hard centres. A random sample of 20 sweets is taken from the jar and the number of sweets with hard centres is recorded.

(a) Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that there are equal proportions of sweets with soft centres and sweets with hard centres in the jar.

(b) Calculate the probability of a Type I error for this test.

Given that there are 3 times as many sweets with soft centres as there are sweets with hard centres,

(c) calculate the probability of a Type II error for this test.
Question 3 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 6 marks)
4. A manufacturer produces boxes of screws containing short screws and long screws. The manufacturer claims that the probability, $p$, of a randomly selected screw being long, is 0.5.

A shopkeeper does not believe the manufacturer’s claim. He designs two tests, $A$ and $B$, to test the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$.

In test $A$, a random sample of 10 screws is taken from a box of screws and $H_0$ is rejected if there are fewer than 3 long screws.

In test $B$, a random sample of 5 screws is taken from a box of screws and $H_0$ is rejected if there are no long screws, otherwise a second random sample of 5 screws is taken from a box of screws. If there are no long screws in this second sample $H_0$ is rejected, otherwise it is accepted.

(a) Find the size of test $A$. (1)

(b) Find the size of test $B$. (3)

(c) Find an expression for the power function of test $B$ in terms of $p$. (2)

Some values, to 2 decimal places, of the power function for test $A$ and the power function for test $B$ are given in the table below.

<table>
<thead>
<tr>
<th>$p$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power test $A$</td>
<td>0.93</td>
<td>$r$</td>
<td>0.38</td>
<td>0.17</td>
</tr>
<tr>
<td>Power test $B$</td>
<td>0.83</td>
<td>0.55</td>
<td>0.31</td>
<td>0.15</td>
</tr>
</tbody>
</table>

(d) Find the value of $r$. (1)

The shopkeeper believes that the value of $p$ is less than 0.4.

(e) Suggest which of the tests the shopkeeper should use. Give a reason for your answer. (2)
Question 4 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

Leave blank
Question 4 continued
5. Fire brigades in cities $X$ and $Y$ are in similar locations. The response times, in minutes, during a particular month, for randomly selected calls are summarised in the table below.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Sample mean</th>
<th>Standard deviation $s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>9</td>
<td>14.8</td>
</tr>
<tr>
<td>$Y$</td>
<td>6</td>
<td>7.2</td>
</tr>
</tbody>
</table>

You may assume that the response times are from independent normal distributions.

Stating your hypotheses and showing your working clearly

(a) test, at the 10% level of significance, whether or not the variances of the populations from which the response times are drawn are the same, (5)

(b) test, at the 5% level of significance, whether or not the mean response time for the fire brigade in city $X$ is more than 5 minutes longer than the mean response time for the fire brigade in city $Y$. (8)

(c) Explain why your result in part (a) enables you to carry out the test in part (b). (1)
Question 5 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

Leave blank
Question 5 continued
6. A random sample of size \( n \) is taken from the random variable \( X \), which has a continuous uniform distribution over the interval \([0, a], a > 0\)

The sample mean is denoted by \( \bar{X} \)

(a) Show that \( Y = 2 \bar{X} \) is an unbiased estimator of \( a \)

(b) Find \( E(M) \)

(c) Show that \( \text{Var}(M) = \frac{na^2}{(n + 2)(n + 1)^2} \)

The estimator \( S \) is defined by \( S = \frac{n + 1}{n} M \)

Given that \( n > 1 \)

(d) state which of \( Y \) or \( S \) is the better estimator for \( a \). Give a reason for your answer.
Question 6 continued
7. The times taken to travel to school by sixth form students are normally distributed. A head teacher records the times taken to travel to school, in minutes, of a random sample of 10 sixth form students from her school.

Based on this sample, the 95% confidence interval for the mean time taken to travel to school for sixth form students from her school is

\[ [28.5, 48.7] \]

Calculate a 99% confidence interval for the variance of the time taken to travel to school for sixth form students from her school.

(9)
Question 7 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

Leave blank
Question 7 continued

___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END