Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D1 answer book provided – **there may be more space than you need**.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – **use this as a guide as to how much time to spend on each question**.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
Write your answers in the D1 answer book for this paper.

1.

(a) Define the term ‘bipartite graph’.

Figure 1 shows the possible allocations of five people, Larry (L), Monisha (M), Nina (N), Phil (P) and Theo (T), to five activities, A, B, C, D and E.

Figure 2 shows an initial matching.

(b) Starting from this initial matching, use the maximum matching algorithm to find a complete matching. You should list the alternating path you use and state your complete matching.

(Total 5 marks)
2. Draw the activity network described in the precedence table below, using activity on arc and exactly three dummies.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Immediately preceding activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
</tr>
<tr>
<td>B</td>
<td>–</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
</tr>
<tr>
<td>E</td>
<td>B</td>
</tr>
<tr>
<td>F</td>
<td>B</td>
</tr>
<tr>
<td>G</td>
<td>A, E, F</td>
</tr>
<tr>
<td>H</td>
<td>F</td>
</tr>
<tr>
<td>I</td>
<td>C</td>
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<tr>
<td>J</td>
<td>D, G</td>
</tr>
<tr>
<td>K</td>
<td>D, G</td>
</tr>
</tbody>
</table>

(Total 5 marks)
3. 59 45 18 55 47 11 63 17 15 42

(a) The list of numbers above is to be sorted into descending order. Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

The numbers in the list represent the lengths, in cm, of some pieces of copper wire. The copper wire is sold in one metre lengths.

(b) Use the first-fit decreasing bin packing algorithm to determine how these pieces could be cut from one metre lengths. (You should ignore wastage due to cutting.)

(3)

(c) Determine whether your solution to (b) is optimal. Give a reason for your answer.

(2)

(Total 9 marks)
4. Figure 3 represents a network of tram tracks. The number on each edge represents the length, in miles, of the corresponding track. One day, Sarah wishes to travel from A to F. She wishes to minimise the distance she travels.

(a) Use Dijkstra’s algorithm to find the shortest path from A to F. State your path and its length. (6)

On another day, Sarah wishes to travel from A to F via J.

(b) Find a route of minimal length that goes from A to F via J and state its length. (2)

(c) Use Prim’s algorithm, starting at G, to find the minimum spanning tree for the network. You must clearly state the order in which you select the edges of your tree. (3)

(d) State the length, in miles, of the minimum spanning tree. (1)

(Total 12 marks)
5.

An algorithm is described by the flow chart shown in Figure 4.

Given that \( x = 27 \) and \( y = 5 \),

(a) complete the table in the answer book to show the results obtained at each step when the algorithm is applied. Give the final output.

(b) (i) State, giving a reason, which number should be input as \( x \).

(ii) State the output.

(Total 7 marks)
6.

Figure 5 models a network of corridors in an office complex that need to be inspected by a security guard. The number on each arc is the length, in metres, of the corresponding section of corridor.

Each corridor must be traversed at least once and the length of the inspection route must be minimised. The guard must start and finish at vertex A.

(a) Use the route inspection algorithm to find the length of the shortest inspection route. State the arcs that should be repeated. You should make your method and working clear.

(5)

It is now possible for the guard to start at one vertex and finish at a different vertex. An inspection route that traverses each corridor at least once is still required.

(b) Explain why the inspection route should start at a vertex with odd degree.

(2)

The guard decides to start the inspection route at F and the length of the inspection route must still be minimised.

(c) Determine where the guard should finish. You must give reasons for your answer.

(2)

(d) State a possible route and its length.

(2)

(Total 11 marks)
The network in Figure 6 shows the activities that need to be undertaken by a company to complete a project. Each activity is represented by an arc and the duration, in days, is shown in brackets. Each activity requires exactly one worker. The early event times and late event times are shown at each vertex.

Given that the total float on activity D is 1 day,

(a) find the values of $w$, $x$, $y$ and $z$. 

(b) On Diagram 1 in the answer book, draw a cascade (Gantt) chart for the project.

(c) Use your cascade chart to determine a lower bound for the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to times and activities.

It is decided that the company may use up to 36 days to complete the project.

(d) On Diagram 2 in the answer book, construct a scheduling diagram to show how the project can be completed within 36 days using as few workers as possible.

(Total 12 marks)
8. Charlie needs to buy storage containers.

There are two different types of storage container available, standard and deluxe.

Standard containers cost £20 and deluxe containers cost £65. Let \( x \) be the number of standard containers and \( y \) be the number of deluxe containers.

The maximum budget available is £520

(a) Write down an inequality, in terms of \( x \) and \( y \), to model this constraint.

Three further constraints are:

\[
\begin{align*}
    x & \geq 2 \\
    -x + 24y & \geq 24 \\
    7x + 8y & \leq 112
\end{align*}
\]

(b) Add lines and shading to Diagram 1 in the answer book to represent all four constraints. Hence determine the feasible region and label it \( R \).

The capacity of a deluxe container is 50% greater than the capacity of a standard container. Charlie wishes to maximise the total capacity.

(c) State an objective function, in terms of \( x \) and \( y \).

(d) Use the objective line method to find the optimal vertex, \( V \), of the feasible region. You must make your objective line clear and label the optimal vertex \( V \).

(e) Calculate the exact coordinates of vertex \( V \).

(f) Determine the number of each type of container that Charlie should buy. You must make your method clear and calculate the cost of purchasing the storage containers.

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END
1. 

Figure 1

Figure 2

(Total 5 marks)
2. 

(Total 5 marks)
Question 3 continued

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(Total 9 marks)
4. 

**Key:**

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Order of labelling</th>
<th>Final value</th>
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**Working values**

Shortest path: .................................................................

Length of shortest path: .............................................
Question 4 continued

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(Total 12 marks)
5. (a) You may not need to use all the rows in this table. It may not be necessary to complete all boxes in each row.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$t$</th>
<th>Is $x$ odd?</th>
<th>Is $x = 0$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>5</td>
<td>0</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>
Question 5 continued

Final output

(b)

(Total 7 marks)
6.

Figure 5

[The total weight of the network is 384]
Question 6 continued

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(Total 11 marks)
7. (a) ___________________________________________________________________
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(b) Diagram 1

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32
Question 7 continued

(c) 
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(d) 

Diagram 2

(Total 12 marks)
Question 8 continued

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(Total 14 marks)