



Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. [1]

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

- (b) Show that the lines intersect, stating the point of intersection. [5]

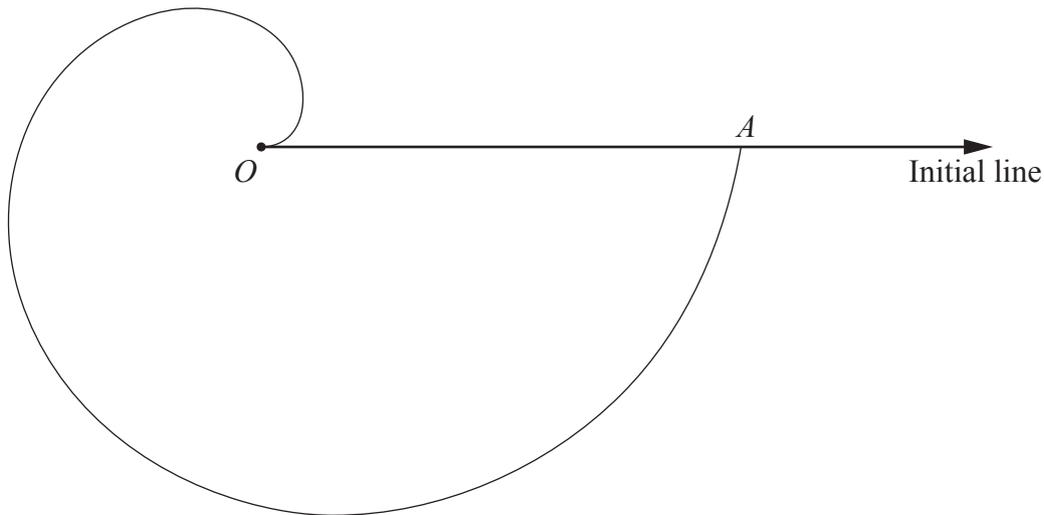
- 2 Two polar curves, C_1 and C_2 , are defined by $C_1: r = 2\theta$ and $C_2: r = \theta + 1$ where $0 \leq \theta \leq 2\pi$.

C_1 intersects the initial line at two points, the pole and the point A .

- (a) Write down the polar coordinates of A . [2]

- (b) Determine the polar coordinates of the point of intersection of C_1 and C_2 . [2]

The diagram below shows a sketch of C_1 .



- (c) On the copy of this sketch in the Printed Answer Booklet, sketch C_2 . [1]

- 3 In this question you must show detailed reasoning.

The roots of the equation $4x^3 + 6x^2 - 3x + 9 = 0$ are α , β and γ .

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. [6]

4 In this question you must show detailed reasoning.

Determine the smallest value of n for which $\frac{1^2 + 2^2 + \dots + n^2}{1 + 2 + \dots + n} > 341$. [4]

5 (a) By using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$. [2]

(b) Hence find an expression for $\cosh 2x$ in terms of $\cosh x$. [1]

(c) Determine the solutions of the equation $5\cosh 2x = 16\cosh x + 21$, giving your answers in exact logarithmic form. [4]

- 6 A particle, P , positioned at the origin, O , is projected with a certain velocity along the x -axis. P is then acted on by a single force which varies in such a way that P moves backwards and forwards along the x -axis.

When the time after projection is t seconds, the displacement of P from the origin is x m and its velocity is v ms⁻¹.

The motion of P is modelled using the differential equation $\ddot{x} + \omega^2 x = 0$, where ω rad s⁻¹ is a positive constant.

- (a) Write down the general solution of this differential equation. [1]

D is the point where $x = d$ for some positive constant, d . When P reaches D it comes to instantaneous rest.

- (b) Using the answer to part (a), determine expressions, in terms of ω , d and t only, for the following quantities

- x
- v

[3]

- (c) Hence show that, according to the model, $v^2 = \omega^2(d^2 - x^2)$. [1]

The quantity z is defined by $z = \frac{1}{v}$.

- (d) Using part (c), determine an expression for z_m , the mean value of z **with respect to the displacement**, as P moves directly from O to D . [2]

One measure of the validity of the model is consideration of the value of z_m . If z_m exceeds 8 then the model is considered to be valid.

The value of d is measured as 0.25 to 2 significant figures. The value of ω is measured as 0.75 ± 0.02 .

- (e) Determine what can be inferred about the validity of the model from the given information. [1]
- (f) Find, according to the model, the least possible value of the velocity with which P was initially projected. Give your answer to 2 significant figures. [2]

7 You are given that a is a parameter which can take only real values.

The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ -3 & 10-4a & 9 \\ 7 & 4 & 4 \end{pmatrix}$.

(a) Find an expression for the determinant of \mathbf{A} in terms of a . [2]

You are given the following system of equations in x , y and z .

$$\begin{array}{rcl} 2x + & 4y - 6z = & 6 \\ -3x + (10-4a)y + 9z = & & -9 \\ 7x + & 4y + 4z = & 11 \end{array}$$

The system can be written in the form $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 11 \end{pmatrix}$.

(b) (i) In the case where \mathbf{A} is **not** singular, solve the given system of equations by using \mathbf{A}^{-1} . [5]

(ii) In the case where \mathbf{A} is singular describe the configuration of the planes whose equations are the three equations of the system. [3]

The transformation represented by \mathbf{A} is denoted by T .

A 3-D object of volume $|5a - 20|$ is transformed by T to a 3-D image.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. [1]

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. [2]

8 In this question you must show detailed reasoning.

It is given that $\sum_{r=k}^{98} \frac{5r+2}{r(r+1)(r+2)} = \frac{20539}{34650}$ for some k .

Determine the value of k . [7]

9 In this question you must show detailed reasoning.

(a) Show that $\operatorname{Re}(e^{4i\theta}(e^{i\theta} + e^{-i\theta})^4) = a \cos 4\theta \cos^4 \theta$, where a is an integer to be determined. [3]

(b) Hence show that $\cos \frac{1}{12}\pi = \frac{1}{2} \sqrt[4]{b+c\sqrt{3}}$, where b and c are integers to be determined. [6]

10 The coordinates of the points A and B are $(3, -2, -1)$ and $(13, 10, 9)$ respectively.

- The plane Π_A contains A and the plane Π_B contains B .
- The planes Π_A and Π_B are parallel.
- The x and y components of any normal to plane Π_A are equal.
- The shortest distance between Π_A and Π_B is 2.

There are **two** possible solution planes for Π_A which satisfy the above conditions.

Determine the acute angle between these two possible solution planes. [8]

END OF QUESTION PAPER

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