



Oxford Cambridge and RSA

GCE

Further Mathematics A

Y544/01: Discrete Mathematics

A Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

1. Annotations and abbreviations

| Annotation in RM assessor | Meaning |
|------------------------------------|---|
| ✓ and ✕ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| BP | Blank Page |
| Seen | |
| Highlighting | |
| | |
| Other abbreviations in mark scheme | Meaning |
| dep* | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This question included the instruction: In this question you must show detailed reasoning. |

2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
- Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

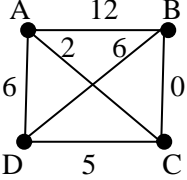
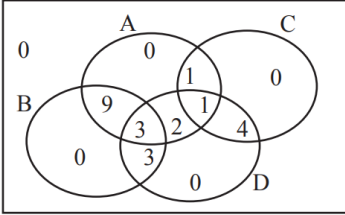
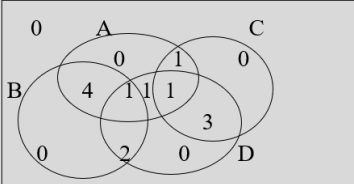
g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

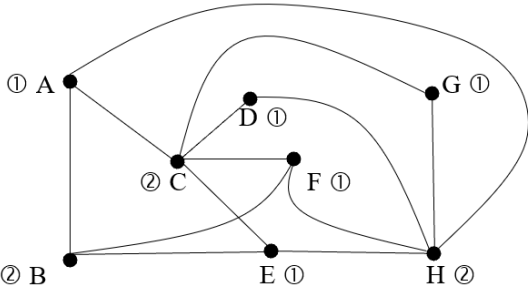
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

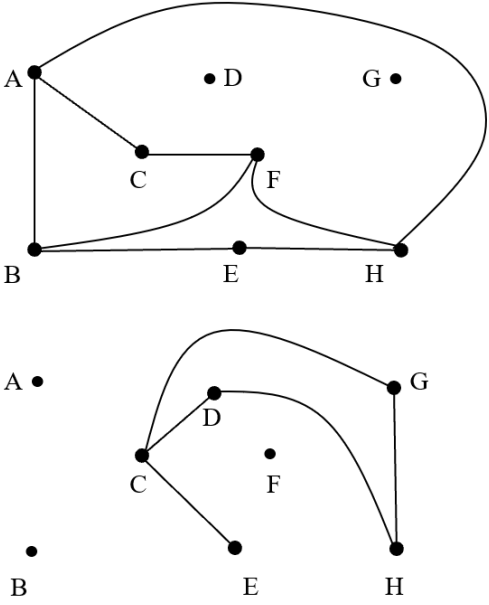
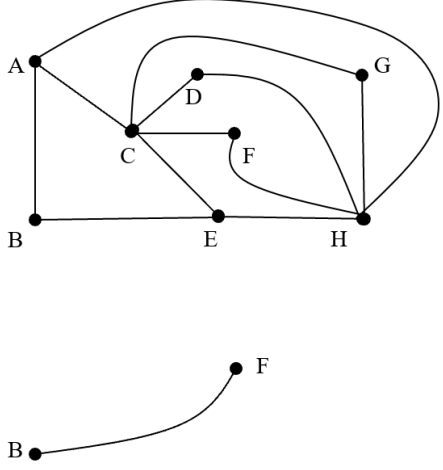
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | Answer | Marks | AO | Guidance |
|--------------|---|---|----------------------------|---|
| <p>1 (a)</p> |  | <p>M1 M1 A1</p> <p>[3]</p> | <p>3.3 3.4 1.1</p> | <p>Graph K_4 with vertices labelled A, B, C, D (in any order)</p> <p>K_4 with any correct non-zero arc weight</p> <p>Fully correct graph with all six arc weights correct</p> <p>For reference</p>  |
| <p>1 (b)</p> | <p>e.g. no parties attended by only one these people</p> | <p>B1</p> <p>[1]</p> | <p>3.5b</p> | <p>An appropriate reference to the number of parties attended by 0, 1, 3 or 4 of A, B, C, D OR the total number of parties</p> |
| <p>1 (c)</p> | <p>$(8 + 7 + 5 + 8) - (5 + 2 + 3 + 3 + 4) + (1 + 1)$ $= 28 - 17 + 2$ $= 13$</p> <p>Alternative method 1</p> <p>$23 = (16+15+6+13+n(E)) - (12+2+6+6+5+8+7+5+8) + (3+1+5+2+3+3+4) - (1+1)$ $23 = (50+n(E)) - 59 + 21 - 2$ $n(E) = 13$</p> <p>Alternative method 2</p>  | <p>M1 A1</p> <p>M1 A1</p> <p>SC B1</p> <p>[2]</p> | <p>1.1 1.1</p> | <p>Evidence of using inclusion-exclusion with E</p> <p>May also show 0's</p> <p>From correct working</p> <p>Using inclusion-exclusion for all 5 sets</p> <p>13 from a Venn diagram showing how many attended with E, but without evidence of using inclusion-exclusion</p> <p>Or $4 + 1 + 1 + 1 + 1 + 2 + 3 = 13$</p> <p>13 from valid working</p> |

| Question | | Answer | Marks | AO | Guidance | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|------------------------|---|---------------------------|-----------------------|--|----------|------------------------|--------------------|---|---|---|---|---|---|---|---|---|---|------|---|---|---|---|---|------|---|---|------|---|---|------|---|
| 2 | (a) | | M1 A1 A1 [3] | 3.1a 1.1 1.1 | Durations and working for parts (b) and (c) may be seen Single start with activities A, B and single end with activities G, H (and no others at either end) Precedences for activities C, D, E, F all correct All dummies directed and correct with at most 1 extra | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | (b) | <p>Minimum project completion time = 14 minutes</p> | M1 A1 B1 [3] | 3.4 3.4 1.1 | Forward pass for their network, provided it includes at least one burst and at least one merge (other than at start and finish) All forward pass labels correct for a correct network For reference: <table border="1"> <thead> <tr> <th>Activity</th> <th>Immediate predecessors</th> <th>Duration (minutes)</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>-</td> <td>4</td> </tr> <tr> <td>B</td> <td>-</td> <td>1</td> </tr> <tr> <td>C</td> <td>A</td> <td>2</td> </tr> <tr> <td>D</td> <td>A, B</td> <td>5</td> </tr> <tr> <td>E</td> <td>D</td> <td>1</td> </tr> <tr> <td>F</td> <td>B, C</td> <td>2</td> </tr> <tr> <td>G</td> <td>D, F</td> <td>5</td> </tr> <tr> <td>H</td> <td>E, F</td> <td>4</td> </tr> </tbody> </table> 14 cao | Activity | Immediate predecessors | Duration (minutes) | A | - | 4 | B | - | 1 | C | A | 2 | D | A, B | 5 | E | D | 1 | F | B, C | 2 | G | D, F | 5 | H | E, F | 4 |
| Activity | Immediate predecessors | Duration (minutes) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A | - | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B | - | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| C | A | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| D | A, B | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| E | D | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| F | B, C | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| G | D, F | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| H | E, F | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | (c) | See answer to part (b) C = 1, F = 1 B = 3 | M1 A1 A1 [3] | 3.4 3.4 1.1 | Their backward pass used to find any one non-zero float Both cao cao If all three are correct but also any other activities are listed with non-zero floats give M1 A1 A0 | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question | | Answer | Marks | AO | Guidance |
|----------|-----|--|---|-------------------------------|---|
| 3 | (a) | 35 | B1 [1] | 1.1 | 7C_4 or $\frac{7 \times 6 \times 5 \times 4}{4!}$ o.e. |
| 3 | (b) | 3 men, 1 woman = ${}^5C_3 \times {}^2C_1 = 10 \times 2 = 20$ 4 men = ${}^5C_4 = 5$ $20 + 5 = 25$ | M1 A1 | 2.1 1.1 | Attempting either case (some appropriate working seen) 25 |
| | | Alternative method 2 men, 2 women = ${}^5C_2 \times {}^2C_2 = 10 \times 1 = 10$ $35 - 10 = 25$ | M1 A1 FT | | Or equivalent 25 or (their 35) – 10 |
| | | | [2] | | |
| 3 | (c) | 3 4 4 with 6, 7, 8 or 9 3 and one of the 4's with 6, 7 4 ways for 3, 4, 4, with 6, 7, 8 or 9 2 ways for 3 one of the 4's with 6, 7 Total = 6 | M1 A1 [2] | 3.1b 2.1 | Identifying at least 4 valid cases {3, 4, 4', X} with X = 6, 7, 8 or 9 {3, 4, 6, 7}, {3, 4', 6, 7} 6 from valid working |
| 3 | (d) | Each team can be arranged in $4! = 24$ ways $6 \times 24 = 144$ | B1 FT [1] | 1.1 | 144 or (their 6) $\times 24$ |
| 3 | (e) | 3 4 4 with 6, 7, 8 or 9 = $2 \times 4 = 8$ 3 4 6 7 with either 4 = 2 Total = 10 | M1 A1 FT [2] | 2.2a 1.1 | Recognising that the two 4's are distinguishable (some appropriate working seen, e.g. allow $2 \times$ (their 6) or implied from answer 10 if (c) was correct) 10 or follow through their (incomplete) list of valid cases with no extras (= $2 \times$ (their 4) + (their 2)) |

| Question | | Answer | Marks | AO | Guidance |
|----------|-----|---|--|--|---|
| 4 | (a) | e.g. A B F C D H E | B1 [1] | 1.2 | A valid path through 7 of the vertices (no repeats) |
| 4 | (b) | e.g. A B E H G C A | B1 [1] | 1.2 | A valid cycle through 6 of the vertices (no repeats, closed) |
| 4 | (c) | e.g. D and G have degree sum $2 + 2 = 4$ which is less than 8 | M1 A1 [2] | 2.1 2.4 | Identifying any pair of <u>non-adjacent</u> vertices, not including C or H Demonstrating that sum of vertex degrees is less than 8 |
| 4 | (d) | e.g. D \Rightarrow CDH (or reversed) G \Rightarrow CGH (or reversed) F \Rightarrow at least one of FC or FH (or reversed) so at least one of C and H must be repeated | M1 A1 [2] | 2.1 2.1 | Or equivalent valid explanation of why there cannot be a cycle through all the vertices <u>without repeating a vertex</u> Complete valid explanation |
| 4 | (e) |  <p>Output: bipartite</p> | M1 A1 [2] | 1.1 1.1 | A, D, E, F, G = ⊙ and B, C, H = ⊙ Leading to conclusion ‘bipartite’ |
| 4 | (f) | Graph contains $K_{3,3}$ as a subgraph {A, E, F}, {B, C, H} Hence graph is non-planar | M1 B1 A1 [3] | 2.2a 2.2a 1.1 | $K_{3,3}$ only (not ‘graph contains $K_{3,3}$ or K_5 ’) Identifying the sets that form $K_{3,3}$ (without D and G) Non-planar (or not planar or equivalent) with an attempt at identifying the sets |

| 4 | (g) | Question | Answer | Marks | AO | Guidance | | | | | | | | | | | | | | | | | |
|--------|-----|---|--|-------------------------------------|---|----------|---|---|---|---|---|---|---|---|--------|---|---|---|---|---|---|---|---|
| | | <p>Non-planar so thickness is at least 2</p> <p>Can be drawn using two planes</p> <p>e.g.</p>  <p>Drawn on 2 planes so thickness is at most 2 Hence thickness = 2</p> | <p>B1</p> <p>B1</p> <p>[2]</p> | <p>2.1</p> <p>2.1</p> | <p>Deducing that thickness $\neq 1$ or thickness > 1</p> <p>Representing the graph using exactly two planes Using a diagram or described in words (e.g. one graph with all arcs except CE and a second with just CE)</p> <p>e.g.</p>  <p>Vertices used must be labelled or description must make it obvious which vertices are connected in each subgraph. All 13 arcs included once only, and no extras</p> <p>Conclusion may be implied</p> <p>For reference:</p> <table border="1" data-bbox="1234 1246 2040 1315"> <thead> <tr> <th>vertex</th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> </tr> </thead> <tbody> <tr> <td>degree</td> <td>3</td> <td>3</td> <td>5</td> <td>2</td> <td>3</td> <td>3</td> <td>2</td> <td>5</td> </tr> </tbody> </table> | vertex | A | B | C | D | E | F | G | H | degree | 3 | 3 | 5 | 2 | 3 | 3 | 2 | 5 |
| vertex | A | B | C | D | E | F | G | H | | | | | | | | | | | | | | | |
| degree | 3 | 3 | 5 | 2 | 3 | 3 | 2 | 5 | | | | | | | | | | | | | | | |

| Question | | | Answer | Marks | AO | Guidance | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|---------|----------|---|-------------------------------------|----------------------------|--|--|--|---------|--|--|---|---|---|--|----------|---|--------|--------|--------|---|---|--------|--------|--------|---|---|--------|--------|--------|---|--|---------|---|---|---|--|-------------------------------|------------------------|---|
| 5 | (a) | | <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td colspan="3" style="text-align: center;">Player 2</td> <td style="text-align: right;">row min</td> </tr> <tr> <td></td> <td></td> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 2px;">X</td> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 2px;">Y</td> <td style="border-bottom: 1px solid black; padding: 2px;">Z</td> <td></td> </tr> <tr> <td rowspan="3" style="vertical-align: middle;">Player 1</td> <td style="border-right: 1px solid black; padding: 2px;">A</td> <td style="padding: 2px;">(6, 0)</td> <td style="padding: 2px;">(1, 7)</td> <td style="padding: 2px;">(5, 6)</td> <td style="text-align: right; padding: 2px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">B</td> <td style="padding: 2px;">(9, 4)</td> <td style="padding: 2px;">(2, 6)</td> <td style="padding: 2px;">(8, 1)</td> <td style="text-align: right; padding: 2px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">C</td> <td style="padding: 2px;">(6, 8)</td> <td style="padding: 2px;">(1, 3)</td> <td style="padding: 2px;">(7, 2)</td> <td style="text-align: right; padding: 2px;">1</td> </tr> <tr> <td></td> <td style="text-align: center;">col min</td> <td style="text-align: center; padding: 2px;">0</td> <td style="text-align: center; padding: 2px;">3</td> <td style="text-align: center; padding: 2px;">1</td> <td></td> </tr> </table> <p>Play-safe for player 1 (rows) is B Play-safe for player 2 (cols) is Y</p> | | | Player 2 | | | row min | | | X | Y | Z | | Player 1 | A | (6, 0) | (1, 7) | (5, 6) | 1 | B | (9, 4) | (2, 6) | (8, 1) | 2 | C | (6, 8) | (1, 3) | (7, 2) | 1 | | col min | 0 | 3 | 1 | | M1 A1 A1 [3] | 1.1 1.1 | Calculating row minima for player 1 or column minima for player 2 B from sight of 1, 2, 1 as row minima Y from sight of 0, 3, 1 as col minima SC B1 Both play-safes correct but no (or insufficient) working |
| | | Player 2 | | | row min | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | X | Y | Z | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Player 1 | A | (6, 0) | (1, 7) | (5, 6) | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | B | (9, 4) | (2, 6) | (8, 1) | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | C | (6, 8) | (1, 3) | (7, 2) | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | col min | 0 | 3 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | (b) | | Column Z is (strictly) dominated by column Y $7 > 6, 6 > 1$ and $3 > 2$ | B1 B1 [2] | 1.1 2.2a | Identifying Y (only) (as better / dominating) [NOT X and Y] Three appropriate comparisons, or equivalent in words, e.g. Y gives player 2 more points than Z, for <u>each</u> of player 1's choices | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | (c) | | <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td colspan="3" style="text-align: center;">Player 2</td> <td></td> </tr> <tr> <td></td> <td></td> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 2px;">X</td> <td style="border-right: 1px solid black; border-bottom: 1px solid black; padding: 2px;">Y</td> <td style="border-bottom: 1px solid black; padding: 2px;">Z</td> <td></td> </tr> <tr> <td rowspan="3" style="vertical-align: middle;">Player 1</td> <td style="border-right: 1px solid black; padding: 2px;">A</td> <td style="padding: 2px;">(6, 0)</td> <td style="padding: 2px;">(1, 7)</td> <td style="padding: 2px;">(5, 6)</td> <td style="text-align: right; padding: 2px;">Y</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">B</td> <td style="padding: 2px;">(9, 4)</td> <td style="padding: 2px;">(2, 6)</td> <td style="padding: 2px;">(8, 1)</td> <td style="text-align: right; padding: 2px;">Y</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">C</td> <td style="padding: 2px;">(6, 8)</td> <td style="padding: 2px;">(1, 3)</td> <td style="padding: 2px;">(7, 2)</td> <td style="text-align: right; padding: 2px;">X</td> </tr> <tr> <td></td> <td></td> <td style="text-align: center; padding: 2px;">B</td> <td style="text-align: center; padding: 2px;">B</td> <td style="text-align: center; padding: 2px;">B</td> <td></td> </tr> </table> <p>Nash equilibrium at (B, Y)</p> | | | Player 2 | | | | | | X | Y | Z | | Player 1 | A | (6, 0) | (1, 7) | (5, 6) | Y | B | (9, 4) | (2, 6) | (8, 1) | Y | C | (6, 8) | (1, 3) | (7, 2) | X | | | B | B | B | | B1 B1 [2] | 1.1 1.1 | Identifying at least 4 of (A, Y), (B, Y), (C, X), (B, X), (B, Y), (B, Z) or at least 4 of B B B and Y Y X (with X for row C) May be seen in table May be convincingly argued in words (B, Y) |
| | | Player 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | X | Y | Z | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Player 1 | A | (6, 0) | (1, 7) | (5, 6) | Y | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | B | (9, 4) | (2, 6) | (8, 1) | Y | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | C | (6, 8) | (1, 3) | (7, 2) | X | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | B | B | B | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | (d) | (i) | $6(p) + 1(1 - p) = 0(p) + 7(1 - p)$ $\Rightarrow 1 + 5p = 7 - 7p$ $p = 0.5$ | M1 * M1 dep* A1 [3] | 3.1a 1.1 1.1 | Finding expressions for the expected number of points won by <u>each</u> player using <u>row A</u> (not two or more rows for one player) Equate these expressions or sketch graph o.e. and solve for p (or implied from correct expressions seen and $p = 0.5$) 0.5, cao from valid working | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | (d) | (ii) | Player 1 gets $9(0.5) + 2(0.5) = 5.5$ Player 2 gets $4(0.5) + 6(0.5) = 5.0$ Hence player 1 | M1 A1 [2] | 1.1 2.2a | Calculating $9p + 2(1 - p)$ or $4p + 6(1 - p)$ for their p (seen) SC B1 only for both $2 + 7p$ and $6 - 2p$ o.e. without numerical p '1' from valid correct working <u>seen</u> using $p = 0.5$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question | | | Answer | Marks | AO | Guidance | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------------|-------------|---|---|----------------|-------------|---|----------|----------|----------|----------|-----|---|----|---------------|----------------|---------------|---|---|----|---|---|---------------|----------------|---------------|---|---|----|---|---|----------------|----------------|----------------|---|---|----|---|----|---------------|---------------|---------------|---|---|----|-------------|------------|---|
| 6 | (a) | (i) | <table border="1"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>z</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>3</td> <td>1</td> <td>-4</td> <td>1</td> <td>0</td> <td>0</td> <td>24</td> </tr> <tr> <td>0</td> <td>5</td> <td>0</td> <td>-3</td> <td>0</td> <td>1</td> <td>0</td> <td>60</td> </tr> <tr> <td>0</td> <td>-1</td> <td>2</td> <td>3</td> <td>0</td> <td>0</td> <td>1</td> <td>12</td> </tr> </tbody> </table> | <i>P</i> | <i>x</i> | <i>y</i> | <i>z</i> | <i>s</i> | <i>t</i> | <i>u</i> | RHS | 1 | -2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | -4 | 1 | 0 | 0 | 24 | 0 | 5 | 0 | -3 | 0 | 1 | 0 | 60 | 0 | -1 | 2 | 3 | 0 | 0 | 1 | 12 | B1 | 1.1 | Objective row correct |
| | | | <i>P</i> | <i>x</i> | <i>y</i> | <i>z</i> | <i>s</i> | <i>t</i> | <i>u</i> | RHS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 1 | -2 | 1 | 0 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | 0 | 3 | 1 | -4 | 1 | 0 | 0 | 24 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 5 | 0 | -3 | 0 | 1 | 0 | 60 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | -1 | 2 | 3 | 0 | 0 | 1 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B1 | 1.1 | Three constraint rows correct | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [2] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | (a) | (ii) | Pivot on the 3 in row 2 of the <i>x</i> column <table border="1"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>z</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th>RHS</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>$\frac{5}{3}$</td> <td>$-\frac{8}{3}$</td> <td>$\frac{2}{3}$</td> <td>0</td> <td>0</td> <td>16</td> </tr> <tr> <td>0</td> <td>1</td> <td>$\frac{1}{3}$</td> <td>$-\frac{4}{3}$</td> <td>$\frac{1}{3}$</td> <td>0</td> <td>0</td> <td>8</td> </tr> <tr> <td>0</td> <td>0</td> <td>$-\frac{5}{3}$</td> <td>$\frac{11}{3}$</td> <td>$-\frac{5}{3}$</td> <td>1</td> <td>0</td> <td>20</td> </tr> <tr> <td>0</td> <td>0</td> <td>$\frac{7}{3}$</td> <td>$\frac{5}{3}$</td> <td>$\frac{1}{3}$</td> <td>0</td> <td>1</td> <td>20</td> </tr> </tbody> </table> | <i>P</i> | <i>x</i> | <i>y</i> | <i>z</i> | <i>s</i> | <i>t</i> | <i>u</i> | RHS | 1 | 0 | $\frac{5}{3}$ | $-\frac{8}{3}$ | $\frac{2}{3}$ | 0 | 0 | 16 | 0 | 1 | $\frac{1}{3}$ | $-\frac{4}{3}$ | $\frac{1}{3}$ | 0 | 0 | 8 | 0 | 0 | $-\frac{5}{3}$ | $\frac{11}{3}$ | $-\frac{5}{3}$ | 1 | 0 | 20 | 0 | 0 | $\frac{7}{3}$ | $\frac{5}{3}$ | $\frac{1}{3}$ | 0 | 1 | 20 | B1FT | 1.1 | Positive pivot element for their initial tableau May be seen in answer to (a)(i) |
| <i>P</i> | <i>x</i> | <i>y</i> | <i>z</i> | <i>s</i> | <i>t</i> | <i>u</i> | RHS | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | $\frac{5}{3}$ | $-\frac{8}{3}$ | $\frac{2}{3}$ | 0 | 0 | 16 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 1 | $\frac{1}{3}$ | $-\frac{4}{3}$ | $\frac{1}{3}$ | 0 | 0 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | $-\frac{5}{3}$ | $\frac{11}{3}$ | $-\frac{5}{3}$ | 1 | 0 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | $\frac{7}{3}$ | $\frac{5}{3}$ | $\frac{1}{3}$ | 0 | 1 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M1FT | 1.1 | Pivot row correct (for their positive pivot value) and four different basis columns including <i>P</i> and pivot col | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A1 | 1.1 | cao | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [3] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | (b) | (i) | <i>P, x, z, u</i> | B1 | 2.5 | Allow <i>P</i> missing | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [1] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | (b) | (ii) | $x = 15\frac{3}{11}, y = 0, z = 5\frac{5}{11}$ | B1 | 1.1 | cao | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [1] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | (b) | (iii) | After two iterations $P = 30\frac{6}{11}$ (<i>z</i> value is) negative in the objective row so solution is not yet optimal, hence $P_{\max} \geq 30\frac{6}{11}$ | M1 | 1.1 | $30\frac{6}{11}$ or $\frac{336}{11}$ or 30.5 (3 s.f.) or better | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A1 | 2.4 | 'Negative in top row' so 'not optimal' or 'at least $30\frac{6}{11}$ ' or 'greater than $30\frac{6}{11}$ ' (since no 0's in RHS) o.e. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [2] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | (c) | | $3x + y - 4(9 - x - y) \leq 24$ $\Rightarrow 7x + 5y \leq 60$ $5x - 3(9 - x - y) \leq 60$ $\Rightarrow 8x + 3y \leq 87$ $-x + 2y + 3(9 - x - y) \leq 12$ $\Rightarrow 4x + y \geq 15$ $x \geq 0, y \geq 0$ and $9 - x - y \geq 0 \Rightarrow x + y \leq 9$ | M1 | 3.1a | Substitute $z = 9 - x - y$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A1 | 1.1 | Any of the first three constraints correct (in form $ax + by \leq$ or $\geq c$), allow negative values of <i>c</i> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| A1 | 1.1 | All three correct (in form $ax + by \leq$ or $\geq c$), allow $c < 0$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| B1 | 3.1a | Dealing with non-negativity for <i>z</i> (may imply $x \geq 0, y \geq 0$) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| [4] | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question | | Answer | Marks | AO | Guidance | | | | | | | | | | | | |
|----------|-----|--|--|-------------------------------|---|---|---|---|----|----|-----|-----|-----|-----|--|---|--|
| 6 | (d) | The constraint $4x + y \geq 15$ (o.e.) is not of the form $ax + by \leq c$ with $c \geq 0$ | M1 FT A1 [2] | 3.2b 2.4 | Identifying that this constraint is the problem (FT their constraints provided one is like this) Explaining that this is not of the required form ($\dots \leq$ non-negative) | | | | | | | | | | | | |
| 7 | (a) | | B1 [1] | 3.3 | Correct network (arcs and weights) | | | | | | | | | | | | |
| 7 | (b) | Correct use of boxes at vertices <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>B</td> <td>C</td> <td>D</td> <td>E</td> <td>F</td> <td>G</td> </tr> <tr> <td>64</td> <td>60</td> <td>111</td> <td>143</td> <td>118</td> <td>193</td> </tr> </table> | B | C | D | E | F | G | 64 | 60 | 111 | 143 | 118 | 193 | M1* M1dep* A1 [3] | 3.1a 1.1 1.1 | Evidence of using Dijkstra's algorithm (order of labelling and permanent labels at all 7 vertices) Updating temporary label at E correctly All correct (if not listed in table check on diagram) |
| B | C | D | E | F | G | | | | | | | | | | | | |
| 64 | 60 | 111 | 143 | 118 | 193 | | | | | | | | | | | | |
| 7 | (c) | First pass: $(0) + 2$ Second pass: $2 + 1$ Third pass: $1 + (0)$ $2 + (2 + 1) + 1$ or $2 + 2 + 1 + 1$ $= 2(2 + 1) = 6$ AG | M1 A1 [2] | 2.1 1.1 | Correct comps for any (identified) pass correct or a type (column) but NOT $3 + 2 + 1$ Verifying the given value 6 (from correct comps) But not using the given result in stem to part (d) [i.e. NOT $(4 - 1)(4 - 2) = 3 \times 2 = 6$] | | | | | | | | | | | | |

| Question | Answer | Marks | AO | Guidance | | |
|----------|--------|--|--|-------------------------|-----------------------------------|--|
| 7 | (d) | $0.03 \times \frac{69 \times 68}{6 \times 5}$ | M1 | 2.2a | Or implied from answer | |
| | | = 4.7 seconds | A1 | 1.1 | 4.7 or better (4.69, 4.692) | |
| | | Alternative answer $10^2 \times 0.03$ = 3 | M1 A0 | | Working must be seen, not implied | |
| | | | [2] | | | |
| 7 | (e) | (i) | MST with D deleted = 58 + 60 + 64 + 75 + 82 = 339 339 + 32 + 66 = 437 Lower bound = 437 metres | M1 | 3.4 | Attempting MST with D deleted, soi from 339 or 437 |
| | | | | A1 [2] | 3.4 | 437 |
| 7 | (e) | (ii) | D – E – G – F – C – A – B – D = 32 + 82 + 75 + 58 + 60 + 64 + 72 Upper bound = 443 metres | B1 [1] | 3.4 | D – E – G – F – C – A – B – D and 443 |
| 7 | (e) | (iii) | $437 \leq \text{length} \leq 443$ To make the lower bound into a tour we need to use BD = 72 instead of CD = 66 Hence 443 (metres) | B1 [1] | 3.1b | 443 with some valid reasoning about lower bound not being a tour |

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