Mark Scheme (Results)

October 2020

Pearson Edexcel GCE Further Mathematics
AdvancedSubsidiarey
in Further Core Pure Paper 8FM0_01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$\begin{vmatrix} k & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -k & -k \end{vmatrix} = k(k+k)-3(-3k+16)-1(-3k-16)$	M1	2.1
	Solves det = 0 $\Rightarrow 2k^2+12k-32=0$ or $k^2+6k-16=0$ To achieve k = 2 (k = – 8 must be rejected)	A1	1.1b
		(2)	
	Special case $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -2 & -2 \end{vmatrix} = 2(2+2)-3(-3\times2+16)-1(-3\times2-16)$ Shows det = 0, therefore when k = 2 there is no unique solution	M1 A0	2.1 1.1b
(b)	Eliminates z to achieve **two** equations in x and y e.g. $5x+2y=1$ $-10x-4y=-2$ $20x+8y=4$ Eliminates x to achieve **two** equations in y and z e.g. $11y-5z=13$ $22y-10z=26$ $-22y-10z=-26$ Eliminates y to achieve **two** equations in x and z e.g. $11x+2z=-3$ $22x+4z=-6$ $-44x-8z=12$	M1 A1	3.1a 1.1b
	Must give a **reason:** e.g. Two equations are a linear multiple of each other e.g. shows they are the same equation therefore the equations are **consistent**.	A1	2.4
	Alternative Eliminates two different variables to form two equations, should be one equation from two of the three sections in the main scheme. e.g $5x+2y=1$ and $11y-5z=13$ rearranges and substitutes in to one of the original equations in three variables. e.g. $2x+3\left(\frac{1-5x}{2}\right)-\left(\frac{-3-11x}{2}\right)=3$	M1	3.1a
	Correct equations e.g $5x+2y=1$ and $11y-5z=13$	A1	1.1b
	Shows that the equations are a solution e.g. 3 = 3 therefore **consistent**	A1	2.4
(c)	The three **planes** form a **sheaf.**	B1	2.2a
		(1)	
			(6 marks)

Notes:
(a) **M1:** Finds the determinant of the matrix corresponding to the system of equations. **A1:** Sets determinant = 0 and solves their 3TQ to achieve $k = 2$ ($k = -8$ must be rejected)
(a) Special case **M1A0:** Uses $k = 2$ and finds the determinant of the matrix corresponding to the system of equations Shows that determinant = 0 and concludes that when $k = 2$ there is no unique solution
(b) **M1:** A complete method eliminating one variable from the equations using two different pairs of equations. Condone if a different value of k is used **A1:** Achieves two equations in the same two variables **A1: Must give a reason,** shows that the equations are a linear multiple of each other therefore they are **consistent**.
(b) Alternative **M1:** A complete method eliminating one variable from the equations using two different pairs of equations. Substitutes these equations into one of the original equations in three variables. **A1:** Achieves two correct equations in two different variables **A1:** Shows that the equation works therefore they are **consistent**.
(c) **B1:** The three **planes** form a **sheaf.** They must have full marks in (b) to award this mark.

Question	Scheme	Marks	AOs
2(a)	$\lvert z_1\rvert=\sqrt{13}$ and $\arg z_1=\tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1=\sqrt{13}\left(\cos 0.9828+\mathrm{i}\sin 0.9828\right)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $\lvert z_1\rvert=\sqrt{13}$ and uses $\lvert z_1z_2\rvert=\lvert z_1\rvert\times\lvert z_2\rvert=39\sqrt{2}\Rightarrow\lvert z_2\rvert=3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)=\frac{\pi}{4}\Rightarrow\arg(z_2)=...$ $\arg(z_2)=\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4}-0.9828$ or $-0.1974...$	M1 A1	3.1a 1.1b
	$z_2=3\sqrt{26}\left(\cos('-0.1974...')+\mathrm{i}\sin('-0.1974...')\right)$ or $z_2=a+b\mathrm{i}\Rightarrow a^2+b^2=234$ and $\tan^{-1}(-0.1974)=\frac{b}{a}\Rightarrow\frac{b}{a}=-0.2$ $\Rightarrow a=...$ and $b=...$	ddM1	1.1b
	Deduces that $z_2=15-3\mathrm{i}$ only	A1	2.2a
	Alternative $z_1z_2=(a+b\mathrm{i})(2+3\mathrm{i})=(2a-3b)+(3a+2b)\mathrm{i}$		
	$(2a-3b)^2+(3a+2b)^2=\left(39\sqrt{2}\right)^2$ or 3042 $\Rightarrow a^2+b^2=234$ or $\lvert z_1z_2\rvert=\lvert z_1\rvert\times\lvert z_2\rvert=39\sqrt{2}\Rightarrow\lvert z_2\rvert=3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2+b^2=234$	M1 A1	3.1a 1.1b
	$\arg\left[(2a-3b)+(3a+2b)\mathrm{i}\right]=\frac{\pi}{4}\Rightarrow\tan^{-1}\left(\frac{3a+2b}{2a-3b}\right)=\frac{\pi}{4}\Rightarrow\frac{3a+2b}{2a-3b}=1$ $\Rightarrow a=-5b$	M1 A1	3.1a 1.1b
	Solves $a=-5b$ and $a^2+b^2=234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2=15-3\mathrm{i}$ only	A1	2.2a
		(6)	
			(8 marks)

Notes:
(a) **B1:** Correct exact value for $\lvert z_1\rvert=\sqrt{13}$ and $\arg z_1=\tan^{-1}\left(\frac{3}{2}\right)$. The value for $\arg z_1$ can be implied by sight of awrt 0.98 or awrt 56.3° **B1ft**: Follow through on $r=\lvert z_1\rvert$ and $\theta=\arg z_1$ and writes $z_1=r\left(\cos\theta+\mathrm{i}\sin\theta\right)$ where r is exact and θ is correct to 4 s.f. do not follow through on rounding errors.
(b) **M1:** A complete method to find the modulus of z_2 **A1:** $\lvert z_2\rvert=3\sqrt{26}$ **M1:** A complete method to find the argument of z_2 **A1:** $\arg\left(z_2\right)=\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4}-0.9828$ or $-0.1974...$ **ddM1:** Writes z_2 in the form $r\left(\cos\theta+\mathrm{i}\sin\theta\right)$, dependent on both previous M marks. Alternative forms two equations involving *a* and *b* using the modulus and argument of z_2 and solve to find values for *a* and *b* **A1**: Deduces that $z_2=15-3\mathrm{i}$ only
(b) Alternative: $z_1z_2=\left(a+b\mathrm{i}\right)\left(2+3\mathrm{i}\right)=\left(2a-3b\right)+\left(3a+2b\right)\mathrm{i}$ **M1:** A complete method to find an equation involving *a* and *b* using the modulus **A1:** Correct simplified equation $a^2+b^2=234$ o.e. **M1:** A complete method to find an equation involving *a* and *b* using the argument. Note $\tan^{-1}\left(\frac{2a-3b}{3a+2b}\right)=\frac{\pi}{4}$ this would score **M0 A0 ddM0 A0** **A1:** Correct simplified equation $a=-5b$ o.e. **ddM1:** Dependent on both the previous method marks. Solves their equations to find values for *a* and *b* **A1**: Deduces that $z_2=15-3\mathrm{i}$ only

Question	Scheme	Marks	AOs
3	$x^2 + y^2 = r^2$	B1	1.2
	$\{V\} = \pi\int_{-r}^{r} r^2 - x^2 \, dx$ or $\{V\} = 2\pi\int_{0}^{r} r^2 - x^2 \, dx$	B1	2.1
	Integrates to the form $\alpha x \pm \beta x^3$ $\left[\text{note: the correct integration gives } r^2 x - \frac{1}{3}x^3\right]$	M1	1.1b
	Substitutes limits of – *r* and *r* and subtracts the correct way round $\left(r^2(r) - \frac{1}{3}(r)^3\right) - \left(r^2(-r) - \frac{1}{3}(-r)^3\right)$ or Substitutes limits of 0 and *r* and subtracts the correct way round with twice the volume. Note the limit of 0 can be implied if gives and answer of 0 $\left(r^2(r) - \frac{1}{3}(r)^3\right) - (0)$	dM1	1.1b
	$V = \frac{4}{3}\pi r^3$ * cso	A1*	1.1b
		(5)	
			(5 marks)

Notes:

B1: Correct equation of the circle, may be implied by correct integral

B1: Correct expression for the volume, including limits, d*x* may be implied and if using limits *r* and 0 the 2 could appear later with reasoning

M1: Integrates to the form $\alpha x \pm \beta x^3$. Do not award if $r^2 \to \lambda r^3$

dM1: Dependent on previous method mark. Correct use of limits – *r* and *r* or limits of 0 and *r* with twice the volume.

A1*: $V = \frac{4}{3}\pi r^3$ * cso

Note: rotation about the *y*-axis all marks are available, however for the final accuracy mark must refer to symmetry

Question	Scheme	Marks	AOs
4(a)	Finds any two vectors $\pm\overrightarrow{LM}$, $\pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$ two out of three values correct is sufficient to imply the correct method	M1	3.3
	Applies the vector equation of the plane formula $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ Where **a** is any coordinate from L, M & N and vectors **b** and **c** come from an attempt at finding any two vectors that lie on the plane.	M1	1.1b
	A correct equation for the plane $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$ $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ **b** and **c** are any two vectors from $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$	A1	1.1b
		(3)	
(b)(i) **(ii)**	Applies 'their' $\mathbf{b}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **Alternative 1** Finds 'their **b**' – 'their **c**' or vice versa and applies the dot product with $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** one of their **b** or **c** **Alternative 2** Applies 'their' $\mathbf{b}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ and solves to find values of *x*, *y* and *z* **Alternative 3** Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$	M1	1.1b
	Show that both dot product(s) **= 0** therefore the lawn is **perpendicular** **Alternative 1** Shows results is **parallel** to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ therefore the lawn is **perpendicular** **Alternative 2** Achieves the value 2 and concludes as a **constant** therefore the lawn is **perpendicular**	A1	2.4
	Outside Specification for this paper – using the cross product Finds the cross product between 'their **b**' and 'their **c**' and either	M1	1.1b

	compares with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel or applies the dot product formula with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel		
	Concludes **parallel** therefore the lawn is **perpendicular**	A1	2.4
	Attempts $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ where $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-3\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ Allow $\mathbf{r}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ for this mark	M1	1.1b
	$x+2y-10z=2$ or $x+2y-10z-2=0$	A1	1.1b
		(4)	
(c)	Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ Two out three values correct is sufficient to imply the correct method	M1	3.3
	$\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ where $\mathbf{a}=\begin{pmatrix}-10\\8\\2\end{pmatrix}$ or $\begin{pmatrix}6\\4\\3\end{pmatrix}$ and $\mathbf{d}=\pm\begin{pmatrix}16\\-4\\1\end{pmatrix}$	A1	1.1b
		(2)	
(d)	For example: The lawn will not be flat The washing line will not be straight	B1	3.5b
		(1)	
(e)	Applies the distance formula $\dfrac{\lvert(2\times1)+5\times2+(2.75\times-10)-2\rvert}{\sqrt{1^2+2^2+(-10)^2}}$	M1	3.4
	= 1.71 m or 171 cm	A1	2.2b
		(2)	
(f)	Must have an answer to part (e). Compares their answer to part (e) with 1.5 m and makes an appropriate comment about the model that is consistent with their answer to part (e). If their answer to part (e) is close to 1.5 (e.g. 1.4 to 1.6) they must compare and conclude that the model therefore is good If their answer to part (e) is significantly different to 1.5 they must compare and conclude that the model therefore it is not a good model.	B1ft	3.5a

		(1)	

(13 marks)

Notes:

(a)

M1: Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ by subtracting relevant vectors. Two out three values correct is sufficient to imply the correct method

M1: Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ where **a** is any point on the plane and the vectors **b** and **c** are any two from their $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$

A1: Any correct equation for the plane. Must start with **r** =…

(b)(i)

M1: Applies the dot product between their vectors **b AND c** with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

(b)(i) Alternative 1

M1: Applies the dot product between their vector **b – c AND** one of their vectors **b** or **c** with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Shows both dot products = 0 and concludes that the lawn is **perpendicular** to the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

(b)(i) Alternative 2

M1: Applies the dot product between their vectors **b** and **c** $\begin{pmatrix}x\\y\\z\end{pmatrix}$ and attempts to find values of *x*, *y* and *z*

A1: Shows results is **parallel** to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ therefore the lawn is **perpendicular**

(b)(i) Alternative 3

M1: Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$

A1: Achieves the value 2 and concludes as a constant therefore the lawn is **perpendicular**

(b)(i) Outside Specification for this paper – using the cross product **M1:** Finds the cross product between 'their **b**' and 'their **c**' and shows parallel to $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$ **A1:** Concludes **parallel** therefore the **lawn** is **perpendicular**
(b)(ii) **M1:** Applies the formula $\mathbf{r.n} = \mathbf{a.n}$ where $\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$ and $\mathbf{a} = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$ **A1:** Correct Cartesian equation of the plane **Note**: If no method is shown then it must be correct to score **M1 A1**, if incorrect scores **M0 A0.** Look at part (i) to see if there is any method as long as it if used in (ii)
(c) **M1:** Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula. $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$. Two out three values correct is sufficient to imply the correct method **A1:** A correct equation including **r** =…
(d) **B1:** States an acceptable limitation of the model for the lawn or washing line
(e) **M1:** Applies the distance formula using the point (2, 5, 2.75) and the normal vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$ **A1:** 1.71 m or 171 cm
(f) **B1ft:** Compares their answer to part (e) with 1.5 and makes an assessment of the model with a reason with no contradictory statements.

Question	Scheme	Marks	AOs
5(a)	Volume = $r\times(r+1)\times(r+2)$	B1	1.1b
	A complete method for finding the total volume of *n* blocks and expressing it in sigma notation. This can be implied by later work. $\sum_{r=1}^{n}(r^3+3r^2+2r)$	M1	3.1b
	$V=\frac{1}{4}n^2(n+1)^2+3\times\frac{1}{6}n(n+1)(2n+1)+2\times\frac{n}{2}(n+1)$	M1	2.1
	$V=\frac{1}{4}n(n+1)[n(n+1)+2(2n+1)+4]$	dM1	1.1b
	$V=\frac{1}{4}n(n+1)[n^2+5n+6]$ $\Rightarrow V=\frac{1}{4}n(n+1)(n+2)(n+3)$ *	A1*	1.1b
		(5)	
(b)	Sets $\frac{1}{4}n(n+1)(n+2)(n+3)=n^4+6n^3-11710$ $\frac{1}{4}n^4+\frac{3}{2}n^3+\frac{11}{4}n^2+\frac{3}{2}n=n^4+6n^3-11710$ simplifies $(3n^4+18n^3-11n^2-6n-46840=0)$ and solves $n=\ \ldots$	M1	1.1b
	There are 10 blocks or *n* = 10	A1	3.2a
		(2)	
			(7 marks)

Notes:

(a)

B1: Correct volume of a block

M1: Expressing the total volume of all *n* blocks as a series in terms of r, r^2 and r^3

M1: Substitutes at least one of the standard formulae into their volume.

dM1: Attempts to factorise $\frac{1}{4}n(n+1)$ having used at least one standard formula correctly. Each term must contain a factor of $n(n+1)$

A1*: Obtains the printed result with no errors seen, no bracketing errors and following from $V=\frac{1}{4}n(n+1)[n^2+5n+6]$ o.e.

Note: Going from $\frac{1}{4}n(n^3+6n^2+11n+6)$ to $\frac{1}{4}n(n+1)(n+2)(n+3)$ with no reasoning shown scores **dM0 A0**

(b)

M1: Sets the printed answer $= n^4 + 6n^3 - 11710$, simplifies, collects terms and uses their calculator to solve a quartic equation to find a value for n.
A1: Selects n = 10 or states that there are **10 blocks** from a **correct equation**

Question	Scheme	Marks	AOs
6(i) (a)	Multiplies the matrix **A** by itself and sets equal to **I** to form one equation in *a* only and another equation involving both *a* and *b*. $\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 4+a(a-4)=1$ and either $2a+ab=0$ or $2(a-4)+b(a-4)=0$ or $a(a-4)+b^2=1$	M1	3.1a
	Solves a 3TQ involving only the constant *a*. This could come after a value of *b* is found and this value substituted into an equation involving both *a* and *b* $a^2-4a+3=0 \Rightarrow (a-3)(a-1)=0 \Rightarrow a=...$	dM1	1.1b
	$a=1, a=3$	A1	11b
	Substitutes a value for *a* into an equation involving both *a* and *b* and solves for *b*. e.g. $2(1)+(1)b \Rightarrow b=...$ $2(1-4)b+(1-4)=0 \Rightarrow b=...$ $(1)(1-4)+b^2=1 \Rightarrow b=...$ Alternatively uses $2a+ab=0$ $a(2+b)=0$ As $a \neq 0$ $2+b=0 \Rightarrow b=...$	dM1	1.1b
	$b=-2$	A1	1.1b
		(5)	
	Alternative (i) (a) Finds $\mathbf{A}^{-1}$ in terms of *a* and *b*, sets equal to **A** and attempts to find at least two different equations. Allow a single sign slip $\frac{1}{2b-a(a-4)}\begin{pmatrix} b & -a \\ -(a-4) & 2 \end{pmatrix}=\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$ One equation from $\frac{b}{2b-a(a-4)}=2$, $\frac{2}{2b-a(a-4)}=b$ One equation from $\frac{-a}{2b-a(a-4)}=a$, $\frac{-(a-4)}{2b-a(a-4)}=a-4$	M1	3.1a
	Uses their value of *b* and their value of the determinant to form and solve a 3TQ involving only the constant *a* $a^2-4a+3=0$ $\Rightarrow (a-3)(a-1)=0$ $\Rightarrow a=...$ | Eliminates *b* from their equations and solve a 3TQ involving only the constant *a* $a^2-4a+3=0$ $\Rightarrow (a-3)(a-1)=0$ $\Rightarrow a=...$	dM1	1.1b
	$a=1, a=3$	A1	1.1b

	$\frac{-a}{2b-a(a-4)}=a$ $\Rightarrow 2b-a(a-4)=-1 \Rightarrow \frac{b}{-1}=2$ Or $\frac{-(a-4)}{2b-a(a-4)}=a-4$ $\Rightarrow 2b-a(a-4)=-1$ $\Rightarrow \frac{2}{-1}=b$	Substitutes a value for *a* into an equation to find a value for *b*	dM1	1.1b
	$b=-2$		A1	1.1b
(b)	Uses their smallest value of **a** and their value for **b** to form two equations $\begin{pmatrix} 2 & 'a' \\ 'a-4' & 'b' \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x+ay=x$ and $(a-4)x+by=y$ $\begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2x+y=x$ and $-3x-2y=y$		M1	3.1a
	$2x+y=x \Rightarrow x+y=0$ o.e. and $-3x-2y=y \Rightarrow x+y=0$ o.e.		M1	1.1b
	$x+y=0$ o.e.		A1	2.1
			(3)	
(ii)(a)	Area of the triangle $T = 3$		B1	1.1b
	Complete method to find a value for *p*. Need to see an attempt at the determinant and setting equal to 15 divided by their area of *T*. The resulting 3TQ needs to be solved to find a value of *p*. Determinant $3p\times p-(-1)\times 2p=\frac{15}{\text{'their area'}} \Rightarrow p=...$		M1	3.1a
	$3p^2+2p-5(=0)$		A1	1.1b
	$p=1$ must reject $p=-\frac{5}{3}$		A1	1.1b
			(4)	
(b)	$\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$		B1 B1	1.1b 1.1b
			(2)	
(c)	(their matrix found in part (b))$\begin{pmatrix} 'p' & 2'p' \\ -1 & 3'p' \end{pmatrix}=\begin{pmatrix} ... & ... \\ ... & ... \end{pmatrix}$ $\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}=\begin{pmatrix} ... & ... \\ ... & ... \end{pmatrix}$		M1	1.1b

	$\begin{pmatrix} 3 & 6 \\ 2 & -6 \end{pmatrix}$	A1ft	1.1b
		(2)	
			(16 marks)

Notes:

(i)(a)

M1: Forming two equations, one involving a only and one involving a and b

dM1: Dependent on previous mark, solves a 3TQ involving a

A1: Correct values for a

dM1: Dependent on first method mark Substitutes one of their values of a into an equation involving a and b and solve to find a value for b. Alternatively factorises either $2a + ab = 0$ and uses $a \neq 0$ to find a value for b.

A1: Correct value for b

Alternative(i)(a)

M1: Finds $\mathbf{A}^{-1}$ and sets equal to **A** and forms two different equations

dM1: Dependent on previous mark. Eliminates b from their equations and solves a 3TQ involving only the constant a. Alternatively if the value of b is found first substitutes their value for b into their determinant = – 1 to form and solve a 3TQ for a

A1: Correct value for a

dM1: Dependent on first method mark. Substitutes a value for a into an equation to find a value for b. Alternatively uses one equation to find the determinant = – 1 and uses this to find a value of b.

A1: Correct values for b

(b)

M1: Extracts simultaneous equations using their matrix **A** with their smaller value of a.

M1: Gathers terms from their two equations.

A1: Achieves the correct equations and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same.

(ii)(a)

B1: Area of the triangle T = 3

M1: Full method. Finds the determinant, sets equal to 15/their area and solves the resulting 3TQ

A1: Correct quadratic

A1: $p = 1$ only

(b)

B1 One correct row or column

B1: All correct

(c)

M1: Multiplies the matrices **QP** in the correct order (if answer only then evidence can be taken from 3 correct or 3 correct ft elements)

A1ft: Correct matrix following through on their answer to part (b) and their value of p as long as it is a positive constant

Question	Scheme	Marks	AOs
7	$z_2 = 2-3\text{i}$	B1	1.1b
	$(z_3 =)\ p-3\text{i}$ and $(z_4 =)\ p+3\text{i}$ May be seen in an Argand diagram	M1	3.1a
	$(z_3 =)-4-3\text{i}$ and $(z_4 =)-4+3\text{i}$ May be seen in an Argand diagram, but the complex numbers used in their method takes precedence	A1	1.1b
	$\left(z^2-4z+13\right)\left(z^2+8z+25\right)$ or $\left(z-(2-3\text{i})\right)\left(z-(2+3\text{i})\right)\left(z-(-4-3\text{i})\right)\left(z-(-4+3\text{i})\right)$ or $a=-\left[(2-3\text{i})+(2+3\text{i})+(-4-3\text{i})+(-4+3\text{i})\right]$ and $b=(2-3\text{i})(2+3\text{i})+(2-3\text{i})(-4-3\text{i})+(2-3\text{i})(-4+3\text{i})$ $+(2+3\text{i})(-4-3\text{i})+(2+3\text{i})(-4+3\text{i})+(-4-3\text{i})(-4+3\text{i})$ and $c=-\left[(2-3\text{i})(2+3\text{i})(-4-3\text{i})+(2-3\text{i})(2+3\text{i})(-4+3\text{i}) +(2-3\text{i})(-4-3\text{i})(-4+3\text{i})+(2+3\text{i})(-4-3\text{i})(-4+3\text{i})\right]$ and $d=(2-3\text{i})(2+3\text{i})(-4-3\text{i})(-4+3\text{i})$ or Substitutes in one root from each conjugate pair and equates real and imaginary parts and solves simultaneously $(2\pm3\text{i})^4+a(2\pm3\text{i})^3+b(2\pm3\text{i})^2+c(2\pm3\text{i})+d=0$ $(-4\pm3\text{i})^4+a(-4\pm3\text{i})^3+b(-4\pm3\text{i})^2+c(-4\pm3\text{i})+d=0$	dM1	3.1a
	$a=4, b=6, c=4, d=325$ $\text{f}(z)=z^4+4z^3+6z^2+4z+325$	A1 A1	1.1b 1.1b
		(6)	
			(6 marks)

Notes:

B1: Seen $2-3\text{i}$

M1: Finds the third and fourth roots of the form $p\pm3\text{i}$. May be seen in an Argand diagram

A1: Third and fourth roots are $-4\pm3\text{i}$. May be seen in an Argand diagram

dM1: Uses an appropriate method to find $\text{f}(z)$. If using roots of a polynomial at least 3 coefficients must be attempted.

A1: At least two of *a*, *b*, *c*, *d* correct

A1: All *a*, *b*, *c* and *d* correct

Note: Using roots $2 \pm 3\text{i}$ and $-2 \pm 3\text{i}$ leads to $z^4 + 10z^2 + 169$ Maximum score **B1 M1 A0 M1 A0 A0**

Question	Scheme	Marks	AOs
8	**Way 1:** $\mathbf{f}(\boldsymbol{k}+1)-\mathbf{f}(\boldsymbol{k})$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ Shows the statement is true for $n=1$, allow 5(7)	B1	2.2a
	Assume true for $\boldsymbol{n}=\boldsymbol{k}$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)-\mathrm{f}(k)=2^{k+3}+3^{2k+3}-\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-2^{k+2}-3^{2k+1}$ $=2^{k+2}+8\times3^{2k+1}$ $=\mathrm{f}(k)+7\times3^{2k+1}$ or $8\mathrm{f}(k)-7\times2^{k+2}$	A1	1.1b
	$\mathrm{f}(k+1)=2\mathrm{f}(k)+7\times3^{2k+1}$ or $9\mathrm{f}(k)-7\times2^{k+2}$	A1	1.1b
	If **true for** $\boldsymbol{n}=\boldsymbol{k}$ then **true for** $\boldsymbol{n}=\boldsymbol{k}+1$ and as it is **true for** $\boldsymbol{n}=1$ the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
	Way 2: $\mathbf{f}(\boldsymbol{k}+1)$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)=2^{(k+1)+2}+3^{2(k+1)+1}$	M1	2.1
	$\mathrm{f}(k+1)=2^{k+3}+3^{2k+3}=2\times2^{k+2}+9\times3^{2k+1}$ $=2\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$ $=2\mathrm{f}(k)+7\times3^{2k+1}$or $9\mathrm{f}(k)-7\times2^{k+2}$	A1 A1	1.1b 1.1b
	If true for $\boldsymbol{n}=\boldsymbol{k}$ then **true for** $\boldsymbol{n}=\boldsymbol{k}+1$ and as it is **true for** $\boldsymbol{n}=1$ the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
	Way 3: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\mathrm{f}(k+1)-m\mathrm{f}(k)=2^{k+3}+3^{2k+3}-m\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-m\times2^{k+2}-m\times3^{2k+1}$ $=(2-m)2^{k+2}+9\times3^{2k+1}-m\times3^{2k+1}$ $=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$	A1	1.1b
	$\mathrm{f}(k+1)=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}+m\mathrm{f}(k)$	A1	1.1b

	If true for $\boldsymbol{n=k}$ then **true for** $\boldsymbol{n=k+1}$ and as it is **true for** $\boldsymbol{n=1}$ the statement is **true for all** (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
			(6 marks)

Notes:

Way 1: $\mathbf{f}(\boldsymbol{k}+1)-\mathbf{f}(\boldsymbol{k})$

B1: Shows that f(1) = 35 and concludes or shows divisible by 7. This may be seen in the final statement.

M1: Makes a statement that assumes the result is true for some value of n

M1: Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$

A1: Achieves a correct expression for $\mathrm{f}(k+1)-\mathrm{f}(k)$ in terms of $\mathrm{f}(k)$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks, look out for scoring B1 in this statement. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

Way 2: $\mathbf{f}(\boldsymbol{k}+1)$

B1: Shows that f(1) = 35 and concludes divisible by 7

M1: Makes a statement that assumes the result is true for some value of n

M1: Attempts $\mathrm{f}(k+1)$

A1: Correctly obtains either $2\mathrm{f}(k)$ **or** $7\times3^{2k+1}$ or either $9\mathrm{f}(k)$ or $-7\times2^{k+2}$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

Way 3: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$

B1: Shows that f(1) = 35 and concludes divisible by 7

M1: Makes a statement that assumes the result is true for some value of n

M1: Attempts $\mathrm{f}(k+1)-m\mathrm{f}(k)$

A1: Achieves a correct expression for $\mathrm{f}(k+1)-m\mathrm{f}(k)$ in terms of $\mathrm{f}(k)$

A1: Reaches a correct expression for $\mathrm{f}(k+1)$ in terms of $\mathrm{f}(k)$

A1: Correct conclusion. This mark is dependent on all previous marks. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution.

Question	Scheme	Marks	AOs
9(a)	$\alpha\beta\gamma = -\frac{1}{3}$ and $\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{4}{3}$	B1	3.1a
	$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{-4/3}{-1/3}$	M1	1.1b
	$= 4$	A1	1.1b
		(3)	
(b)	$\left\{\alpha + \beta + \gamma = -\frac{1}{3}\right\}$ New product $= \frac{1}{\alpha} \times \frac{1}{\beta} \times \frac{1}{\gamma} = \frac{1}{\alpha\beta\gamma} = \frac{1}{-1/3} = ...(-3)$ New pair sum $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{-1/3}{-1/3} = ...(1)$	M1	3.1a
	$x^3 - (\text{part (a)})x^2 + (\text{new pair sum})x - (\text{new product})(= 0)$	M1	1.1b
	$x^3 - 4x^2 + x + 3 = 0$	A1	1.1b
		(3)	
	Alternative e.g. $z = \frac{1}{x} \Rightarrow \frac{3}{x^3} + \frac{1}{x^2} - \frac{4}{x} + 1 = 0$	M1	3.1a
	$x^3 - 4x^2 + x + 3 = 0$	M1 A1	1.1b 1.1b
		(3)	
			(6 marks)

Notes:

(a)

B1: Correct values for the product and pair sum of the roots

M1: A complete method to find the sum of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. Must substitute in their values of the product and pair sum

A1: correct value 4

Note: If candidate does not divide by 3 so that $\alpha\beta\gamma = -1$ and $\alpha\beta + \alpha\gamma + \beta\gamma = -4$ the maximum they can score is B0 M1 A0

(b)

M1: A correct method to find the value of the new pair sum and the value of the new product

M1: Applies $x^3 - (\text{part (a)})x^2 + (\text{their new pair sum})x - (\text{their new product})(= 0)$

A1: Fully correct equation, in any variable, including = 0

(b) Alternative

M1: Realises the connection between the roots and substitutes into the cubic equation

M1: Manipulates their equation into the form $x^3 + ax^2 + bx + c = 0$

A1: Fully correct equation in any variable, including = 0

Question	Scheme	Marks	AOs
10	$(x-3)^2+(y-5)^2=(2r)^2$ and $y=-x+2$	B1	1.1b
	$(x-3)^2+(-x+2-5)^2=(2r)^2$ or $(-y+2-3)^2+(y-5)^2=(2r)^2$	M1	3.1a
	$2x^2+18-4r^2=0$ or $2y^2-8y+26-4r^2=0$	A1	1.1b
	$b^2-4ac>0 \Rightarrow 0^2-4(2)(18-4r^2)>0 \Rightarrow r>...$ or $x^2=9-2r^2 \Rightarrow 9-2r^2>0 \Rightarrow r>...$ or $b^2-4ac>0 \Rightarrow (-8)^2-4(2)(26-4r^2)>0 \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
	Alternative Using a circle with centre (3, 5) and radius 2r and $y=-x+2$	B1	1.1b
	$y-5=1(x-3) \Rightarrow y=x+2$ $x+2=-x+2 \Rightarrow x=...$	M1	3.1a
	$(0,2)$	A1	1.1b
	$2r>\sqrt{(3-0)^2+(5-2)^2} \Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2 \Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
		(7)	
			(7 marks)

Notes:

B1: Correct equations for each loci of points
M1: A complete method to find a 3TQ involving one variable using equations of the form $(x\pm3)^2+(y\pm5)^2=(2r)^2$ or $2r^2$ or r^2 and $y=\pm x\pm2$
A1: Correct quadratic equation

dM1: Dependent on previous method mark. A complete method uses $b^2 - 4ac > 0$ or rearranges to find $x^2 = \mathrm{f}(r)$ and uses $\mathrm{f}(r) > 0$ to the minimum value of *r*. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ condone $(r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality
Alternative **B1:** Using a circle with centre (3, 5) and radius 2*r* and $y = -x + 2$ **M1:** A complete method to find the point of intersection of the line $y = \pm x \pm 2$ and circle where the line is a tangent to the circle. **A1:** Correct point of intersection **dM1:** Finds the distance between the point of intersection and the centre and uses this to find the minimum value of *r*. Condone radius of r. **M1:** Realises there will be an upper limit for *r* and uses Pythagoras theorem $(2r)^2 = (y \text{ coord of centre})^2 + (x \text{ coord of centre} - 2)^2$ **A1:** One correct limit, either $\frac{3\sqrt{2}}{2} < r$ or $r < \frac{\sqrt{26}}{2}$ o.e. **A1:** Fully correct inequality

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