Rewarding Learning

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General Certificate of Education

2015

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

AMM31

[AMM31]

FRIDAY 12 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Six uniform rods, each of the same material, are rigidly joined to form an L shape as shown in **Fig. 1** below.

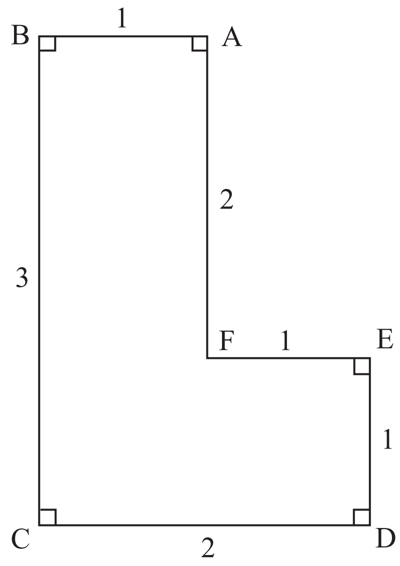

Fig. 1

AB and DC are perpendicular to BC.
ED is perpendicular to CD.
AF is perpendicular to FE.
The lengths of the rods are:
AB = 1 m, BC = 3 m, CD = 2 m, DE = 1 m, EF = 1 m and FA = 2 m.

(i) Find the distances of the centre of mass of this L shape from BC and CD. [7]

This L shape is freely suspended from B.

(ii) Find the angle AB makes with the vertical. [3]

2 Take **i** to be a unit vector in the direction East and **j** to be a unit vector in the direction North.
A ship A is moving with constant velocity $(6\mathbf{i} + 2\mathbf{j})\,\text{km h}^{-1}$
A second ship B is moving with constant velocity $(-2\mathbf{i} + 10\mathbf{j})\,\text{km h}^{-1}$

(i) Find the velocity of A relative to B. [2]

At noon B is 10 km from A on a bearing of 150°

(ii) Find the shortest distance between the two ships. [4]

(iii) Find the time at which A is due east of B. [5]

3 **Fig. 2** below shows a light elastic string attached to a fixed point A, hanging vertically in equilibrium supporting a particle of mass 3 kg. The string has modulus of elasticity 980 N and natural length 2 m. The extension of the string is *e* metres.

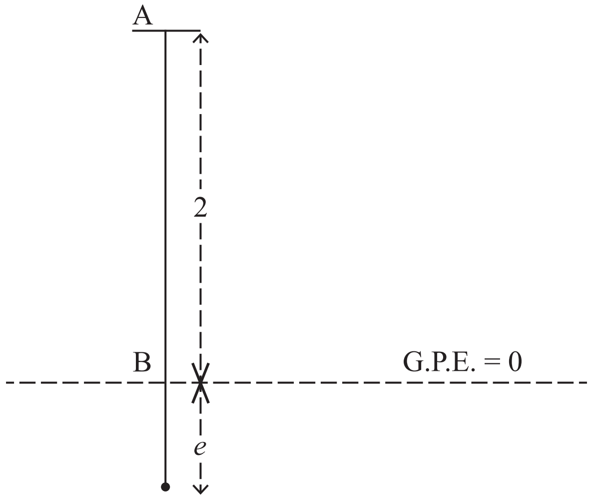

Fig. 2

(i) Find *e*. [3]

The particle is pulled vertically downwards until the total extension is 0.15 m. It is then released from rest. Take the zero level for gravitational potential energy to be at B, 2 m vertically below A.

(ii) Find the speed of the particle at B. [5]

The string is replaced by a light spring of the same natural length and modulus of elasticity as the string.
The particle, attached to the spring, is again pulled vertically downwards until the extension is 0.15 m. It is then released from rest.

(iii) Find the height the particle rises above B before coming to rest. [5]

4 A boat moored in a harbour moves vertically up and down. The boat moves 1 metre from its highest point to its lowest point in 6 seconds.
Model the boat as a particle and its vertical motion as simple harmonic.

(i) Find the amplitude and period of the motion. [2]

Take O to be the equilibrium position of the motion.

(ii) Show that the speed of the boat when it is displaced 0.4 m from O is approximately 0.157 m s^{-1} [4]

A and B are points 0.4 m above O and 0.2 m below O respectively.

(iii) Find the least time taken for the boat to move from A to B. [5]

5 **Fig. 3** below shows two small rings A and B, each of weight W, threaded on a fixed rough horizontal wire.
The rings are connected by a light inextensible string of length $2l$.
A small block of weight $3W$ is attached to the midpoint, C, of the string.
The block and the rings are in equilibrium with angle ACB = 2θ.

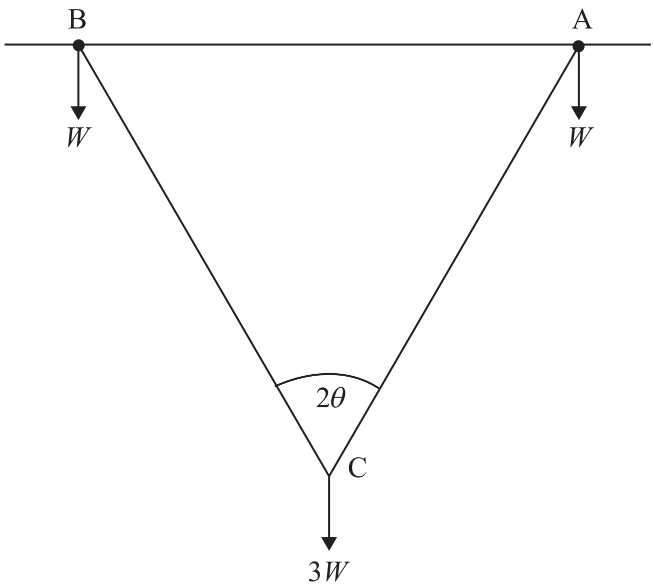

Fig. 3

(i) Draw a diagram which shows all the external forces acting on the ring A and the block at C. [2]

(ii) Show that the tension in the string AC is $\dfrac{3W}{2\cos\theta}$ [2]

(iii) Express the normal reaction of the wire on A in terms of W. [2]

(iv) Express the frictional force at A in terms of θ and W. [2]

The coefficient of friction between each ring and the wire is $\dfrac{1}{4}$

(v) Show that

$$AB \leqslant \frac{10l}{13}$$ [5]

6 **(a)** Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ newtons move a particle from A to B where

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix}$$

AB is measured in metres.
The forces are:

$$\mathbf{F}_1 = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad \mathbf{F}_2 = \begin{pmatrix} -3 \\ 8 \\ 10 \end{pmatrix} \quad \mathbf{F}_3 = \begin{pmatrix} 3\alpha + 5 \\ \alpha - 3 \\ 5 - 7\alpha \end{pmatrix}$$

(i) Find the value of α for which the resultant force is parallel to $\overrightarrow{AB}$. [4]

(ii) Using this value of α, find the work done by the resultant force in moving the particle from A to B. [2]

(b) A particle P is moving to the right of a fixed point O along a horizontal line through O.
The distance of P from O is x metres.
A variable force F newtons acts on P where

$$F = \frac{2}{x^2} - \frac{8}{x^3}$$

The work done by F in moving P from $x = 1$ to $x = a$, where $a > 1$, is $-\frac{35}{16}$ J.

(i) Find the values of a. [6]

The mass of P is $\frac{1}{8}$ kg.
The speed of P at $x = 1$ is $8\,\text{m}\,\text{s}^{-1}$

(ii) Show that the minimum speed of P is $\sqrt{28}\,\text{m}\,\text{s}^{-1}$ [5]

THIS IS THE END OF THE QUESTION PAPER
