Rewarding Learning

ADVANCED
General Certificate of Education
2016

Mathematics

AMM31

Assessment Unit M3
assessing
Module M3: Mechanics 3

[AMM31]
TUESDAY 21 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows two light elastic strings AP and PB used to keep a particle of weight 68 N in equilibrium at P.
The ends A and B are attached to points 1.7 m apart on a fixed horizontal beam.
AP is stretched to 0.8 m and BP is stretched to 1.5 m.

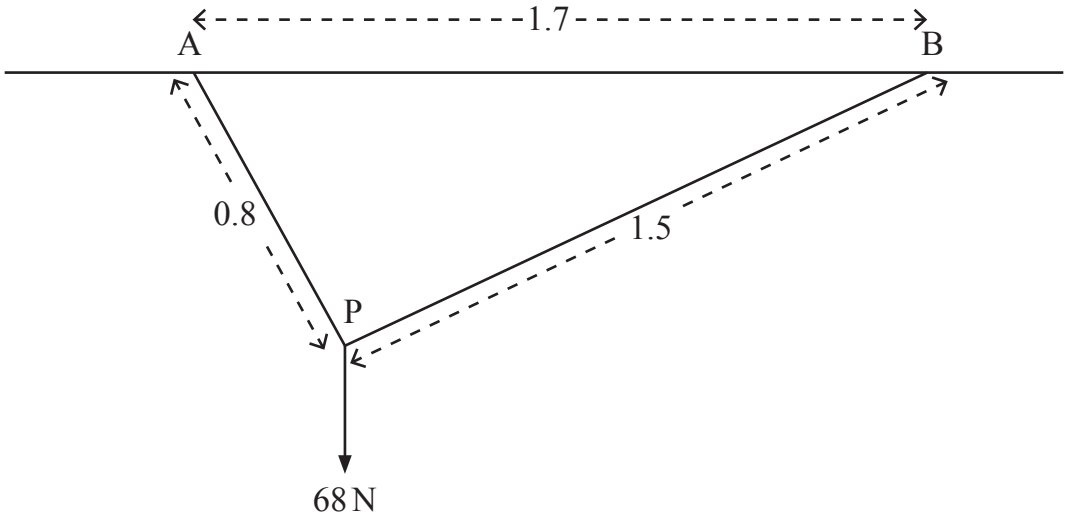

Fig. 1

(i) By resolving in the direction of AP, or otherwise, show that the tension in AP is 60 N. [4]

The extension of AP is 0.2 m.

(ii) Find the modulus of elasticity of AP. [2]

The stored elastic energy in BP is 8 J.

(iii) Find the extension in BP. [6]

2 A particle P is moving along the line whose vector equation is

$$\mathbf{r} = \lambda \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

under the action of two constant forces, $\mathbf{F_1}$ and $\mathbf{F_2}$ newtons where

$$\mathbf{F_1} = \begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix} \text{ and } \mathbf{F_2} = \begin{pmatrix} 2k - 4 \\ 4 - k \\ k - 9 \end{pmatrix}$$

A and B are two points on the line where λ takes the values 1 and –2 respectively.
The distance AB is measured in metres.

(i) Show that the work done by $\mathbf{F_1}$ as P is moved from A to B is 54 J. [5]

The mass of P is 4 kg.
P is moving at $1\,\text{m s}^{-1}$ at A. P is moving at $8\,\text{m s}^{-1}$ at B.

(ii) Use the Work–Energy Principle to find k. [5]

3 The equation of motion of a particle moving in a straight line with S.H.M. of amplitude a is given by

$$\ddot{x} = -\omega^2 x$$

where x is the displacement from the centre of oscillation.

A particle P moving with S.H.M., of period 4π seconds, has velocity $1.4\,\text{m s}^{-1}$ when x is 4.5 m.

(i) Find ω and a. [4]

(ii) Find the maximum speed of P during the motion. [1]

(iii) Find the maximum magnitude of the acceleration of P during the motion. [1]

(iv) Find the total time, in one complete oscillation, for which the speed of P is less than or equal to $1.4\,\text{m s}^{-1}$ [5]

4 **Fig. 2** below shows a design for a uniform metal logo ABCD in the shape of a trapezium with a rectangle PQRS removed.

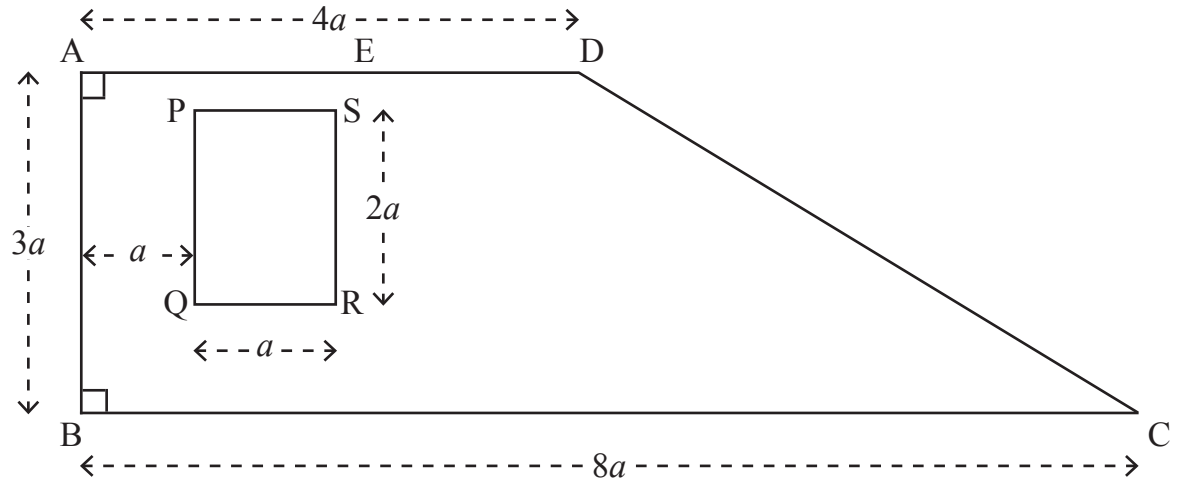

Fig. 2

$A\hat{B}C = D\hat{A}B = 90°$
$AB = 3a$ cm, $AD = 4a$ cm and $BC = 8a$ cm.
PQ is parallel to AB.
The distance between the parallel sides PQ and AB is a cm.
$PQ = 2a$ cm and $QR = a$ cm.
Model the logo as a lamina.
The centre of mass of the logo is at G.

(i) Show that the distance of G from AB is $\frac{53a}{16}$ cm. [6]

E is the point on AD such that $AE = 2.5a$ cm.
The mass of the removed metal rectangle PQRS is m kg.

A small metal stud of mass M kg is attached at A.

(ii) When the logo is freely suspended from E, AD is horizontal.
Find M in terms of m. [4]

(iii) If, instead, the logo is freely suspended from A, find the minimum force in terms of m and g required to keep AD horizontal. [3]

5 O is a fixed point on a straight horizontal line. A particle P of mass 6 kg is a distance x metres from O, where $x \geqslant 0$
A variable force F newtons acts on P in the direction OP. F is given by

$$F = 10 + 0.5e^{\frac{x}{2}} - 2x$$

(i) Find the acceleration of P at O. [2]

(ii) Show that the work done by F for $0 \leqslant x \leqslant 6$ is approximately 43.1 J [5]

The speed of P at O is $5\,\text{m}\,\text{s}^{-1}$

(iii) Find the speed of P when $x = 6$ [2]

(iv) Find x when F is a minimum. [6]

6 A ship B is 32 km away from a ship A on a bearing of 240° as shown in **Fig. 3** below.

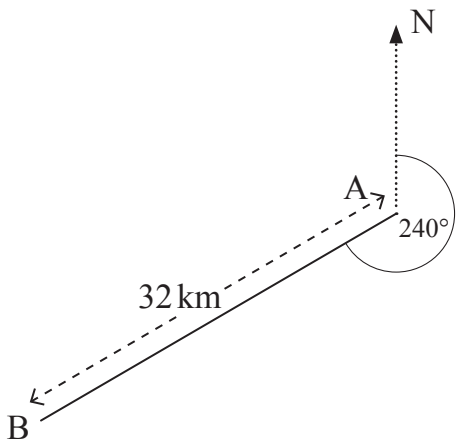

Fig. 3

Ship B is sailing due east at $16\,\text{km}\,\text{h}^{-1}$
A sets out to intercept B.
A travels at $v\,\text{km}\,\text{h}^{-1}$ on a bearing of $\theta°$ as shown in **Fig. 4** below.

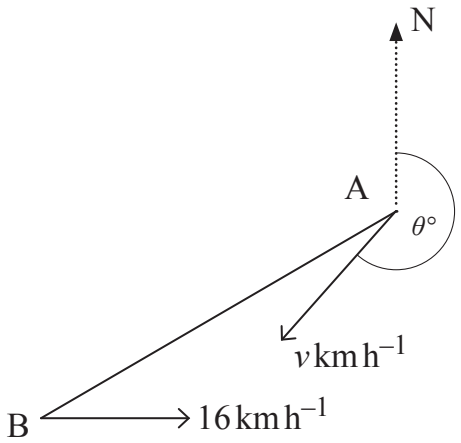

Fig. 4

(i) When $v = 24$, show that $\theta°$ is approximately 221° [5]

(ii) Find the least value of v for which A can intercept B. [1]

(iii) When $v = 12$, show, by drawing a velocity diagram, that A has a choice of two courses to intercept B.
Hence, find the greater time taken to intercept. [8]

THIS IS THE END OF THE QUESTION PAPER