



Rewarding Learning

ADVANCED

General Certificate of Education

2016

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3



AMC31

[AMC31]

FRIDAY 20 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Solve the equation

$$2 \sec^2 \theta = 3 + \tan \theta$$

for $0^\circ \leq \theta \leq 180^\circ$

[6]

2 (i) Use the binomial theorem to find, in ascending powers of x , the expansion of

$$(1 + 2x)^{-\frac{1}{2}}$$

as far as the term in x^3

[4]

(ii) State the range of values of x for which the expansion is valid.

[1]

3 (a) On separate diagrams sketch the graphs of

(i) $y = |x + 2|$

[2]

(ii) $y = e^x + 2$

[2]

(iii) $y = |\ln x|$

[2]

labelling any relevant points.

(b) Write

$$\frac{x^3 + 1}{x^3 - x^2}$$

in partial fractions.

[10]

4 Fig. 1 below shows the logo for an opticians.

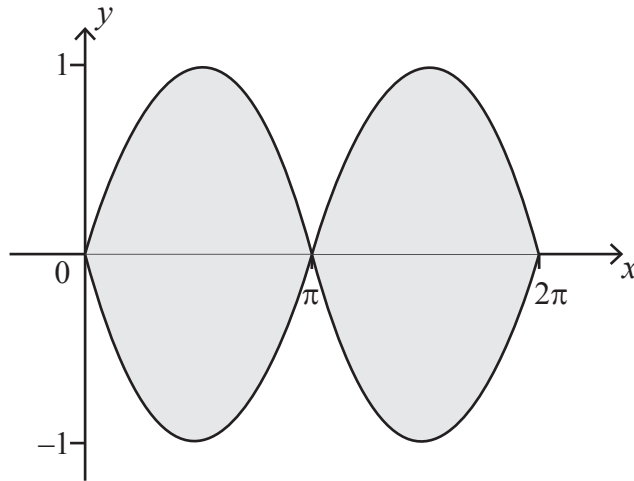


Fig. 1

The area of the logo can be modelled as the area bounded by the curves $y = \sin x$ and $y = -\sin x$ between $x = 0$ and $x = 2\pi$.

Find this area.

[5]

5 (i) Find the gradient of the curve

$$y = x^2 + x \ln x \quad x > 0$$

at any point (x, y) .

[4]

(ii) Show that a stationary point of the curve occurs between $x = 0.2$ and $x = 0.3$

[4]

(iii) By taking $x = 0.2$ as a first approximation to the x coordinate of this stationary point and applying the Newton-Raphson method once, find a better approximation to this coordinate.

[4]

6 A sample of radium loses mass at a rate of 4% per century.

Find, in years, the half-life of radium, i.e. the time taken for its mass to be halved.

[6]

7 (i) Show that

$$(x^2 - x + 2)$$

is positive for all values of x .

[4]

(ii) Given that

$$y = (x^2 + 2)e^{-2x}$$

show that $\frac{dy}{dx} < 0$ for all values of x .

[7]

8 (a) Differentiate with respect to x

(i) $4 \sin x - \ln(1 - x^2)$

[3]

(ii) $\frac{\operatorname{cosec}^2 x}{\tan 3x}$

[6]

(b) Show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{3 - 3 \sin^2 2x} dx = \frac{-4\sqrt{3}}{3}$$

[5]

* Please see note below regarding question 8 part (b).

This question is designed to assess the application of the techniques of integration to a definite integral involving trigonometric functions. In this case, the integral does not have a finite value due to the presence of a singularity in the integrand within the specified limits.

THIS IS THE END OF THE QUESTION PAPER
