Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2017

Mathematics

AMF11

Assessment Unit F1
assessing
Module FP1: Further Pure Mathematics 1

[AMF11]
TUESDAY 13 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 S is the set of non-zero numbers of the form $p + q\sqrt{3}$, where p, q are rational.

(i) Prove that S is closed under multiplication. [3]

Assume that the identity, under multiplication, of S is 1

(ii) Find the inverse of $p + q\sqrt{3}$ under multiplication, giving your answer in the form $a + b\sqrt{3}$
You may assume that $p^2 \neq 3q^2$ [3]

2 The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & -2 \\ 0 & -3 & 0 \\ 2 & 0 & -2 \end{pmatrix}$$

(i) Show that the only real eigenvalue of **M** is –3 [6]

(ii) For the eigenvalue –3, find a corresponding unit eigenvector. [4]

3 The circles C_1 and C_2 are given by the following equations

$$\begin{aligned} C_1 \quad & x^2 + y^2 + 2x - 8y + 4 = 0 \\ C_2 \quad & x^2 + y^2 - 10x - 26y + 142 = 0 \end{aligned}$$

(i) Show that these circles touch externally. [7]

The line $y = 3x + k$ is a tangent to the circle C_1

(ii) Find the exact values of k. [6]

4 **(a)** Describe fully the transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ [3]

(b) **(i)** Find the image of the circle

$$x^2 + y^2 = 9$$

under the transformation represented by the matrix $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ [8]

(ii) Find the area enclosed by the image curve. [3]

5 Let $\mathbf{N} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & 2 \\ -1 & p & 1 \end{pmatrix}$

(i) Find the rational value of p for which this matrix does not have an inverse. [4]

(ii) If $p = 3$, find the inverse of $\mathbf{N}$ [7]

(iii) Hence solve the following system of equations

$$\begin{aligned} -y + 2z &= -5 \\ x + y + 2z &= 1 \\ -x + 3y + z &= 19 \end{aligned}$$ [4]

6 **(a)** The complex number z is such that $|z| = 8$, $\arg z = \frac{\pi}{6}$

Express z in the form $a + b\mathrm{i}$, where a and b are real numbers. [4]

(b) **(i)** Sketch on an Argand diagram the locus of those points u which satisfy

$$|u - (7 + 2\mathrm{i})| = \sqrt{20}$$ [3]

(ii) On the same diagram sketch the locus of those points v which satisfy

$$\arg\{v - (1 + 2\mathrm{i})\} = \frac{\pi}{4}$$ [3]

(iii) Find the points of intersection of these loci. [7]

THIS IS THE END OF THE QUESTION PAPER