Rewarding Learning

ADVANCED
General Certificate of Education
2017

Centre Number

Candidate Number

Mathematics

Assessment Unit C4
assessing
Module C4:
Core Mathematics 4

AMC41

[AMC41]
WEDNESDAY 7 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eight** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an H.B. pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

10346

1 **(i)** Write

$$12\cos\theta + 5\sin\theta$$

in the form $R\cos(\theta - \alpha)$, where R is a positive integer and $0 < \alpha < \frac{\pi}{2}$ [3]

The depth d (m) of water in a reservoir at time t (hours) can be modelled by the equation

$$d = 12\cos t + 5\sin t + 20$$

(ii) Find the minimum depth of water in the reservoir. [2]

2 The functions f and g are defined as:

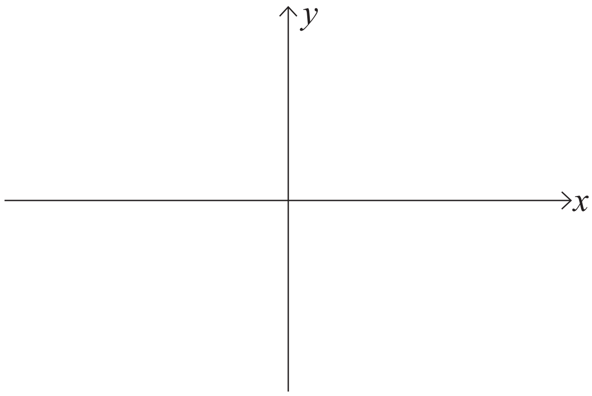
$$f(x) = 2x^2 - 4 \qquad x > 0$$

$$g(x) = \sec x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(i) Find the inverse function $f^{-1}(x)$ and state its domain. [4]

10346

(ii) On the axes below sketch the graph of $y = \mathrm{g}(x)$. [2]



(iii) State the range of the function $\mathrm{g}(x)$. [1]

(iv) Find the composite function $\mathrm{fg}(x)$. [2]

3 The points A, B and C have position vectors

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \qquad \overrightarrow{OC} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix}$$

(i) Find $\overrightarrow{BA}$ [2]

10346

20AMC4106

(ii) Find the angle ABC. [5]

4 The value V of a car decreases at a rate proportional to V.

(i) Model this by a differential equation. [2]

After 2 years a car, with an initial value of £25 000, has a value of £15 000

(ii) Find the value of the car after 5 years. [9]

5 **Fig. 1** below shows a sketch of part of the curve $y = x^2 \ln 3x$

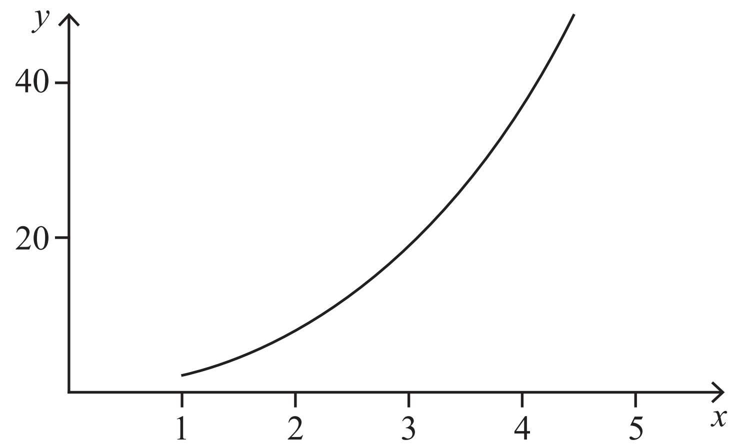


Fig. 1

A sail for a boat can be modelled by the area between the curve $y = x^2 \ln 3x$, the x-axis and the lines $x = 1$ and $x = 4$

Find the area of the sail. [8]

6 A curve is defined by the parametric equations

$$x = 2 + 3\sin\theta \qquad \text{and} \qquad y = \sin\theta - \cos\theta$$

(i) Find $\dfrac{dy}{dx}$ in terms of θ. [4]

(ii) Find $\dfrac{d^2y}{dx^2}$ in terms of θ. [4]

(iii) Hence find and classify the stationary point (x, y) in the range $0 < \theta < \pi$ [4]

10346

[Turn over

7 **(a)** Solve

$$\sin\left(x - \frac{\pi}{3}\right) = \sqrt{3}\sin x$$

for $-\pi \leqslant x \leqslant \pi$ [5]

10346

(b) Prove the identity

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} \equiv \sec 2\theta + \tan 2\theta$$

[6]

8 Use the substitution $u = \sin x$ to find

$$\int \frac{3\cos x \sin^2 x}{4 - \sin^2 x}\, dx$$

[12]

THIS IS THE END OF THE QUESTION PAPER

DO NOT WRITE ON THIS PAGE

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
Total Marks	

Examiner Number