Rewarding Learning

ADVANCED
General Certificate of Education
2017

Mathematics

AMM21

Assessment Unit M2
assessing
Module M2: Mechanics 2

[AMM21]
WEDNESDAY 14 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 At time t seconds, where $t \geqslant 0$, a particle P is moving in a horizontal plane with acceleration $\mathbf{a}$ m s^{-2}, where

$$\mathbf{a} = (3t^2 + 4t)\mathbf{i} + (6t - 5)\mathbf{j}$$

When $t = 3$, the velocity of P is $(48\mathbf{i} + 10\mathbf{j})$ m s^{-1}

(i) Find the velocity of P at time t seconds. [5]

(ii) Find the speed of P when it is moving parallel to the vector $\mathbf{i}$ [4]

2 A block of mass 10 kg is pulled along a rough, straight, horizontal road by a constant horizontal force of magnitude 70 N.
The block moves in a straight line passing through two points X and Y on the road, where XY = 50 m.
The work done against friction in moving the block from X to Y is 2800 J.
Model the block as a particle.

(i) State why you can ignore the effect of air resistance in this model. [1]

(ii) Show that the coefficient of friction between the block and the road is $\frac{4}{7}$ [4]

The velocity of the block at Y is six times its velocity at X.

(iii) Using the work–energy principle, find the velocity of the block at X. [4]

3 A car of mass 1300 kg accelerates along a straight, horizontal road.
The power produced by the car's engine is 32.5 kW.
Model the car as a particle and the resistance to its motion as negligible.

(i) Show that the acceleration of the car is $\frac{25}{v}\,\text{m s}^{-2}$, where $v\,\text{m s}^{-1}$ is the speed of the car at time t seconds. [4]

(ii) Find the distance travelled by the car as it accelerates from $1\,\text{m s}^{-1}$ to $5\,\text{m s}^{-1}$ [6]

4 Ben and Rachel are walking through a field.
At time $t = 0$ seconds, Ben is at a fixed point O in the field and Rachel is at the point with position vector $(-3\mathbf{i} + 6\mathbf{j})$ m relative to O.
Ben is moving with a constant speed of $1.25\,\text{m s}^{-1}$ towards the point with position vector $(6\mathbf{i} + 8\mathbf{j})$ m relative to O.

(i) Find, in vector form, Ben's velocity. [4]

Rachel is moving with a constant speed of $\sqrt{10}\,\text{m s}^{-1}$ towards the point with position vector $(15\mathbf{i})$ m relative to O.

(ii) Show that Rachel's velocity is $(3\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ [4]

At $t = 2$ seconds, Rachel is at the point P.

(iii) Show that Ben will also pass through the point P. [4]

5 A particle P is projected from a point A on horizontal ground with speed $u\,\text{m s}^{-1}$ at an angle of elevation α and moves freely under gravity.
P hits the ground at the point B.

(i) Show that $\text{AB} = \frac{u^2}{g}\sin 2\alpha$ [6]

An archer fires an arrow with an initial speed of $45\,\text{m s}^{-1}$ to hit a target which is level with the point of projection.
The arrow travels a horizontal distance of 80 m to hit the target.
You may assume that air resistance is negligible.

(ii) Find the two possible angles of projection. [4]

(iii) Find the shortest possible time of flight of this arrow. [3]

(iv) State one modelling assumption you have made in answering this question. [1]

6 **Fig. 1** below shows a light inelastic string OP of length 0.5 m.
The end O of the string is fixed to a point 0.3 m vertically above the point C which lies on a smooth horizontal surface. A particle of mass 5 kg is fastened to the end P.
The particle rotates on the surface in horizontal circles about the point C.

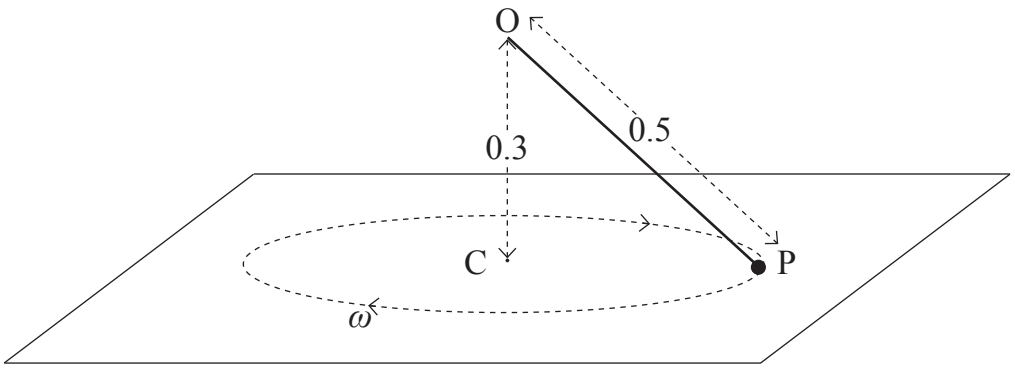

Fig. 1

The particle moves with angular speed ω rad s^{-1}

(i) Draw a diagram showing the external forces acting on the particle. [2]

(ii) Find, in terms of ω, an expression for the reaction between the particle and the surface. [7]

(iii) Find the maximum value of ω at which the particle is on the point of leaving the surface. [2]

7 **In this question take $g = 10\,m\,s^{-2}$**

Fig. 2 below shows a smooth bead, B, of mass 0.05 kg threaded on to a smooth fixed vertical rod.
The bead is attached to one end of a light inextensible string of length 2 m.
The string passes over a smooth fixed peg P. A particle C, of mass of 0.1 kg, is attached to the other end of the string.
The horizontal distance between P and the rod is 0.4 m.
Initially, B is held at the same horizontal level as P.

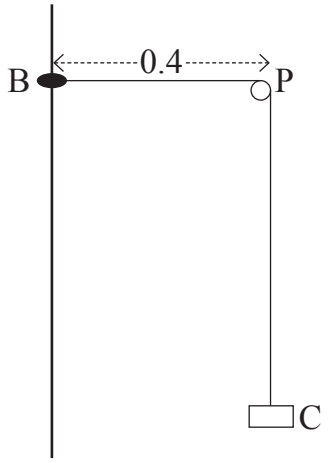

Fig. 2

Take the gravitational potential energy to be zero at the horizontal through P.

(i) Find the initial gravitational potential energy of the system. [2]

B is then released from rest.
Fig. 3 below shows the system when B has fallen a distance of x metres from its original position.

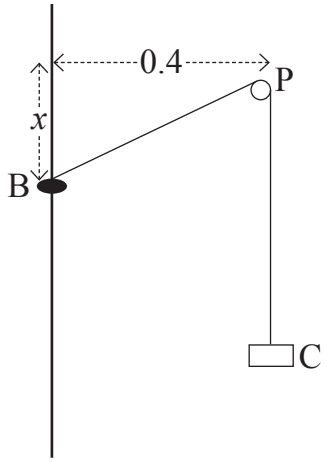

Fig. 3

(ii) Show that when B has fallen a distance of x metres, the distance PC is now

$$2 - \sqrt{x^2 + 0.16}$$ [1]

(iii) Find the gravitational potential energy of the system when B has fallen a distance of x metres. [4]

(iv) By using the principle of conservation of mechanical energy, find the value of x at which B momentarily comes to rest. [3]

THIS IS THE END OF THE QUESTION PAPER