Rewarding Learning

ADVANCED

General Certificate of Education

2018

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

TUESDAY 19 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find, in radians, the general solution of the equation

$$\sin 2\theta = 1 + \cos 2\theta$$ [6]

2 **(i)** Show that for $r \geqslant 1$

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)}$$ [1]

(ii) Hence or otherwise show that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$ [3]

(iii) Using **(ii)** evaluate

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$$ [1]

3 Using partial fractions, show that

$$\int_0^1 \frac{x+3}{(x+1)(x^2+4x+5)}\,dx = \frac{1}{2}\ln 2$$ [10]

4 **(i)** Given that

$$f(x) = e^{-mx} - (1+2x)^{-n}$$

find the Maclaurin expansion for $f(x)$ up to and including the term in x^2 [6]

(ii) Given that the first non-zero term in this expansion is $-4x^2$, find the values of m and n. [3]

5 The equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at any point ($a \cos \theta$, $b \sin \theta$) on the ellipse is given by

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

(i) Tangents are drawn to this ellipse at the points ($a \cos \alpha$, $b \sin \alpha$) and ($a \cos \beta$, $b \sin \beta$). The tangents are perpendicular.

Show that

$$\tan \alpha \tan \beta = \frac{-b^2}{a^2}$$ [3]

(ii) A line joins P ($a \cos \theta$, $b \sin \theta$) to the fixed point S (a, 0) on this ellipse. As θ varies, find the cartesian equation of the locus of the midpoint of the chord PS. [6]

6 Using the principle of mathematical induction, prove that for $n \geqslant 0$

$$5^n + 11^{n+1} \text{ is divisible by } 6$$ [6]

7 **(i)** Given that

$$(\cos\theta + \mathrm{i}\sin\theta)^n \equiv \cos n\theta + \mathrm{i}\sin n\theta$$

when n is a positive integer, deduce that the statement is also true when n is a negative integer. [4]

(ii) Using **(i)**, show that if $Z = \cos\theta + \mathrm{i}\sin\theta$, then

$$Z^n + Z^{-n} = 2\cos n\theta \qquad [2]$$

(iii) By considering $(Z + Z^{-1})^4$, show that

$$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3) \qquad [3]$$

(iv) Hence or otherwise find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos^4\theta \, \mathrm{d}\theta \qquad [5]$$

8 **(i)** Find the general solution of the differential equation

$$\lambda\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + (\lambda^2 + 1)\frac{\mathrm{d}y}{\mathrm{d}x} + \lambda y = 0$$

(a) when $\lambda \neq 0$ and $\lambda \neq 1$ [4]

(b) when $\lambda = 1$ [2]

(ii) Hence or otherwise determine the solution of

$$2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 2x^2 - 1$$

where $y = 9$ when $x = 0$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$ [10]

THIS IS THE END OF THE QUESTION PAPER